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Monetary Policy REVIEW

Optimal Level of Inflation Target, ZLB, and Equilibrium Real Interest Rate

Working paper

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Abstract

In this paper, I explore the optimal inflation target level in the New Keynesian DSGE-model with imperfect price indexation for non-zero trend inflation and a zero lower bound on interest rates. In addition, I study the impact of the real interest rate on the choice of the optimal inflation target and discuss the costs of adopting a new target level. As a criterion for determining the optimal target level, I use a structural, consumer utility-based loss function. My model is calibrated for the Russian economy but may also be relevant for other resource-rich emerging market countries. I have found out that the optimal inflation target level in this setting of the problem is below the current target of the Bank of Russia of 4%, and this conclusion is robust to the model parameters. In addition, I have ascertained a stable negative relationship between the real interest rate and optimal inflation rate.

Keywords: monetary policy, inflation targeting, interest rate zero lower bound, ZLB, equilibrium real interest rate, optimal inflation target level, DSGE, structural models.

JEL-classification: E12, E31, E52, E58, C68.

CONTENTS

1. Introduction

Inflation targeting has proven itself over the past decades, prompting central banks in both advanced economies and emerging market countries to adopt to this regime. There is an extensive body of research describing inflation targeting in comparison to other monetary policy regimes. Meanwhile, the main issue of inflation targeting, the inflation target choice, has not been studied thoroughly enough.

On the one hand, if the target is chosen too high, the economy bears the cost of high inflation. But setting a target too low can also be problematic. A lot of inflation-targeting central banks have recently faced the issue of Zero Lower Bound (ZLB). The ZLB issue means a situation where, in response to a shock(s), the central bank needs to lower its key rate, but the optimal rate is in the negative area and cannot actually be set. In this situation, the central bank temporarily loses the opportunity to stabilise the economy with the help of its main tool. This issue was initially faced by advanced economies, but in the context of recent significant and prolonged shocks leading to a decline in consumption and production, this topic is becoming relevant for emerging market countries as well. Although the Russian economy has not experienced the issue so far, the possibility of hitting the ZLB should be taken into account when choosing a long-term inflation target.

Thus, the choice of inflation target is a key matter of inflation targeting and is made taking into account the trade-off between the risk of facing the ZLB issue (and losses from the inability to stabilise the economy) and the costs of economic agents from high inflation.

In the course of the research, I was looking for answers to several questions for the Russian economy. What is the probability of facing the ZLB issue depending on inflation targeting? What is the relationship between the probability of being at the ZLB and the probability of being in the negative area of interest rates[1]? What is the optimal level of inflation target in terms of consumer welfare? How does such optimal level depend on the real neutral interest rate? What is the loss of output when moving to a new target level?

I answer the questions posed on the basis of the DSGE model. My DSGE model is New Keynesian in nature and, compared to standard models of this type (Smets and Wouters, 2003, 2007, Christiano et al., 2005), includes several features that are important for understanding the functioning of the ZLB mechanism, choosing the optimal inflation target, and the relevance of the results to the Russian economy.

First, as shown in the Bank of Russia Analytical Note (2017), most firms in Russia prefer to change prices not all the time, but once in a certain period. In addition to inflation, firms are guided by production costs, the structure of contracts and other factors. Thus, part of the prices in the economy does not change or changes partially for some time. In modeling terms, this means that there is price rigidity and imperfect indexation. The imperfect indexation of producer prices causes distortions in relative prices, thus creating costs from high inflation.

Second, the zero lower bound on interest rates is a natural constraint on inflation targeting. Although emerging market countries have not often encountered this issue, for example, the experience of Chile in 2008-2010 demonstrates the issue in practice (Céspedes et al., 2014).

Together, these two mechanisms create a trade-off between the losses from high inflation (by targeting too high) and the losses from ZLB (by choosing too low inflation target, which increases the probability of ZLB).

Accounting for the peculiarities of the Russian economy is ensured by including the oil sector and calibrating the parameters.

The academic literature describing the issue of choosing the optimal level of inflation target amid a zero lower bound using the general equilibrium approach is very limited. For the US economy, such DSGE

[1] Here and below, when I talk about the probability of being at the ZLB, I mean the probability calculated from the model that includes the ZLB cap, and when I talk about the probability of being in the negative area of interest rates, I mean the probability from the model that does not include such a cap.

models are built, for example, by Andrade et al. (2019) and Coibion et al. (2012). However, these are models of a closed economy. Thus, the key difference between my paper and the existing ones is that it considers the relationship of the domestic economy with the external sector when choosing an inflation target.

A number of papers explore the probabilities of being at the ZLB for the US economy, such as Chung et al. (2012). Kiley and Roberts (2017) and Bernanke et al. (2019) show that choosing a higher inflation target reduces the probability of being close to the ZLB. To my knowledge, there are no studies examining the choice of the optimal inflation target given the ZLB for the Russian economy or other emerging market countries. For the Russian economy, the issue of a zero lower bound is described by Andreev and Polbin (2021), in which ZLB probabilities are calculated. The authors find that the probability of ZLB at the optimal level is in the range from 6.0% to 20.1%. As an optimal level criterion, a semi-structural rule is used, which assumes the minimisation of inflation, key rate, and output dispersions. For the current target of 4%, the authors get a ZLB probability of 0.3%, which is lower than what I have in my calculations. This is probably due to the fact that the authors include only two shocks in their work, while my model includes 14 shocks.

In my research, I find a negative relationship between the chosen inflation target and the probability of being at the ZLB. So, with a target inflation of 4%, the ZLB probability is about 1%, and with a target of 0.5%, about 17%. At the same time, if there is no ZLB in the model, this probability will slightly decrease for each similar target level. This is due to the fact that in a situation where the central bank cannot lower the rate below zero, it needs more time to stabilise the economy than in a situation where there is no such restriction.

To talk about optimal level, we first need to define what is meant by this term. The literature on the optimal inflation target often assumes that the optimal level will be the one that minimises the squared inflation and output deviations from their natural levels à la Woodford (2003), meaning that a problem of the form is solved: $\psi^{PI} * \hat{\pi}^2{}_t + \psi^Y \hat{y}^2{}_t \rightarrow min$. Whereas in non-structural and semi-structural models, the parameters ψ^{PI} and ψ^Y are usually calibrated or estimated, that is, in general, the loss function doesn't have micro-foundations. For structural models of a closed economy with full price indexation[2], a function of this kind can be derived from the utility function of consumers. In such case, structural coefficients will already be obtained before the inflation gap and the output gap. When imperfect indexing is added to the model, the loss function will take on a more complex form (see, for example, Andrade et al.; 2019, Coibion et al., 2012). When the model is extended to an open economy, this relationship will become even more complex, primarily because the assumption that consumption equals output is no longer satisfied. The derivation of the loss function for such a formulation of the model is given in this paper.

Based on the constructed base model and the loss function above, I find that the optimal inflation target for the Russian economy is 1.1%. This target level corresponds to an 11% probability of being at the ZLB.

Another important matter in choosing the optimal inflation target level is understanding the value of the real neutral interest rate. Given that the nominal interest rate is the sum of the real rate and inflation, the higher the real rate, the lower target can be chosen, ceteris paribus, without changing the probability of being at the ZLB. Meanwhile, the choice of the calibrated level of the real neutral interest rate for the model is not obvious, since this value is unobservable.

According to existing works, the real interest rate for the Russian economy is likely to be in the range of 1% to 3%. For example, Kreptsev et al. (2016) – 1%–3.2%, IMF (2019) – 1%–3%, Isakov and Latypov (2019) – 1.5%–2.5%. The Monetary Policy Report of the Bank of Russia (October 2022) suggests a range of 1% to 2% for the long-term real neutral interest rate. While the Monetary Policy Report of the Bank of Russia (May 2022) has noted that the Central Bank of the Russian Federation expects a real rate increase in the near future due to uncertainty in the economy. I calibrate the equilibrium rate based on fundamental factors and set it at 1.78% per annum.

To my knowledge, Andrade et al. (2019) is the only paper to explicitly examine the relationship between the real rate and optimal target level. The authors show that a decrease in the real rate by 1pp should be

[2] Imperfect indexation means that some firms cannot fully adjust prices for inflation of the previous period or for the equilibrium inflation rate.

offset by an increase in the inflation target by approximately the same amount. Coibion et al. (2012) focuses on the choice of optimal inflation given ZLB but assume that the real interest rate is constant.

In my paper, I present the model-calculated optimal inflation target for different levels of the real rate. I have concluded that, first, as the theory predicts, a higher real rate corresponds to a lower optimal inflation target, and the decreased real rate by 1pp requires an increased inflation target by about 0.5pp; second, for each target level, the lower the probability of ZLB is, the higher the real interest rate.

When adopting a new inflation target, the strategy of such adoption, its duration and the amount of output losses are also important to focus on. There are a number of studies showing that the adopted lower inflation is accompanied by an output fall, such as Ball (1994b), Cecchetti and Rich (2001), Gordon and King (1982).

An indicator commonly used in the literature to measure the negative impact on output from disinflation is the sacrifice ratio (SR) (such as Ascari and Ropele, 2012). This ratio is the cumulative percentage loss of the output gap (the difference between the current value of output and its trend) divided by the difference between the old and new negative inflation targets. Thus, the coefficient shows the loss of output relative to the size of the target change. This indicator depends on the number of periods it takes for the economy to move to a new equilibrium, the magnitude of the change in target, as well as the strength of the transmission mechanism. According to empirical studies (such as Gordon and King, 1982; Cecchetti and Rich, 2001; Durand et al., 2008), this coefficient ranges from 0.5 to 3.

Ascari and Ropele (2012) is focused on the output effect of a permanent decline in inflation based on a medium-scale New Keynesian model such as Smets and Wouters (2003, 2007), Christiano et al. (2005). The authors obtain a coefficient value from 0.95 to 1.13, depending on the current and new target (three options for reducing the inflation target are being studied, from 4% to 2%, from 6% to 2%, from 8% to 2%), and the central bank policy rigidity parameter (1.5 and 3).

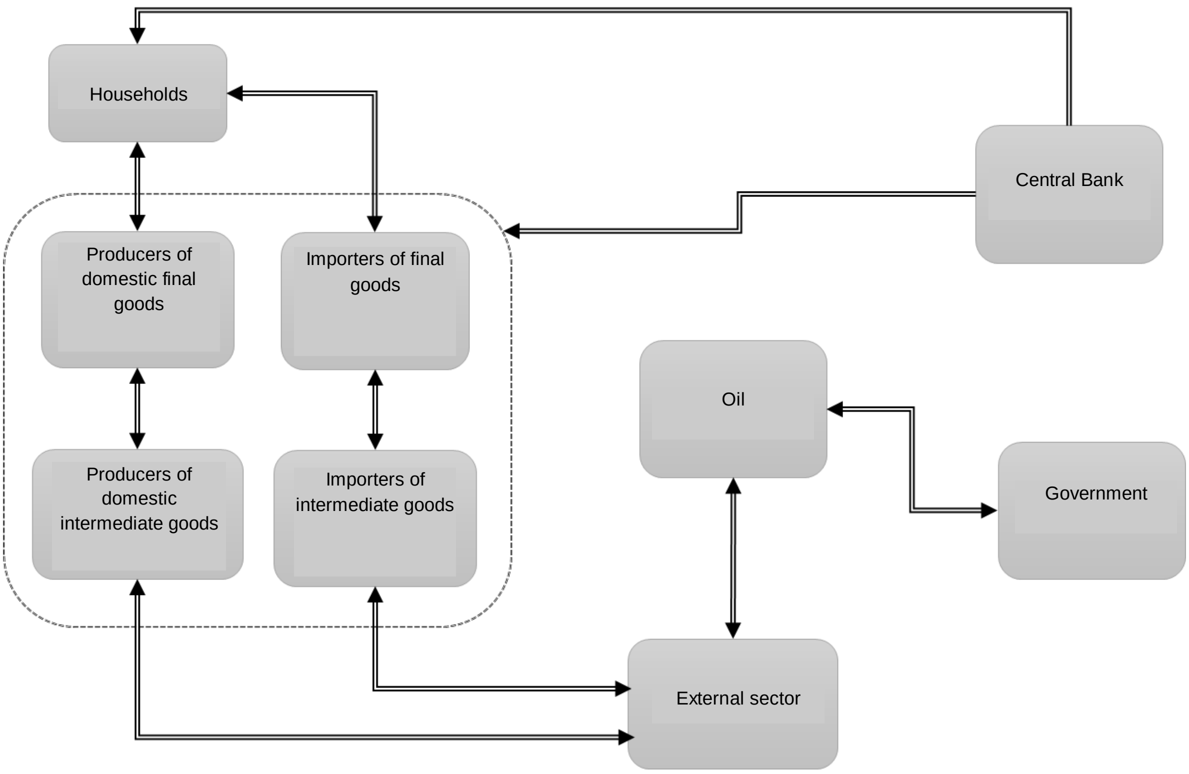
In my work, I find that the SR ratio is 1.03. Thus, for Russia this coefficient is closer to the lower bound of the range discussed in Ascari and Ropele (2012) [0.5; 3], which contains this coefficient for other works. It can be explained by the Russian economy structure. The range is based on calculations for European countries and the US. The fact that I'm getting a fairly low ratio suggests that lowering the inflation target comes at a lower cost than the average for other countries.

The rest of the paper is structured as follows. The second part describes the model. The third discusses the calibration of the model, its properties, and the fit of the model to the data. The fourth part of the paper describes the theoretical basis for choosing the optimal inflation target and related estimates. The fifth part gives conclusions.

2. MODEL

My DSGE model is similar to Smets and Wouters (2003, 2007). I also draw on the Medina and Soto (2007) model, which takes into account the characteristics of a resource-based economy. My model is a medium scale DSGE- model for a small open economy, calibrated for the Russian economy. In addition to the sectors of households, businesses and the state that are standard for DSGE models, this model includes the sector of natural resources (oil). To simplify the structure of the model and facilitate the interpretation of the results, capital was excluded from the model. This model is New Keynesian in nature and includes nominal rigidities. In addition, an important feature of my model is the imperfect indexation of prices, which ensures that there is a distortion in relative prices (that is, the costs of high inflation), and a zero lower bound on the interest rate. Figure 2.1 shows the model structure.

Chart 2.1. Model chart



2.1. HOUSEHOLDS

The consumer sector is modeled as a continuum of households $h \in [0;1]$. Households buy consumer goods, providing labour to firms. Each household offers a certain type of labour service to the producers of intermediate products. Households live indefinitely and maximise their utility function of the form:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(e^{\zeta^c_{t+s}} \log(C_{t+s} - \eta C_{t+s-1}) - \frac{1}{1+\sigma^L} \int_0^1 L_{t+s}(h)^{1+\sigma^L} dh \right),$$

where utility depends positively on consumption C_t and negatively on the number of hours worked $L_t(h)$. The parameter η characterises consumption habits. The parameter σ^L is the inverse Frisch elasticity of labour supply. ζ^c_{t+s} is a preference shock, which is a first-order autoregressive process:

$$\zeta^C_t = \rho_C \zeta^C_{t-1} + \varsigma^C_t,$$

where $\varsigma^C_t \sim i.i.d$ is innovation with mean zero.

Households consume a composite good in which the share ν_C is domestic goods and $(1-\nu_C)$ is foreign goods:

$$C_t = \left(\nu_C^{\frac{1}{\eta^C}} * (C_t^H)^{\frac{\eta^C-1}{\eta^C}} + (1-\nu_C)^{\frac{1}{\eta^C}} * (C_t^F)^{\frac{\eta^C-1}{\eta^C}} \right)^{\frac{\eta^C}{\eta^C-1}},$$

where C_t^H is consumption of domestic goods, C_t^F is consumption of foreign goods, η^C is elasticity between domestic and foreign goods.

I assume that there are two types of households, non-Ricardian (constituting a proportion λ of all households) and Ricardian (constituting a proportion $(1-\lambda)$), of those differing in access to financial assets.

Ricardian households can buy one-period bonds B_t with payments in the next period in rubles at the rate i_t and in foreign currency B_t^F at the rate of i_t^{rrF}. In addition, they receive dividends D_t, paid by monopoly firms. Thus, they maximise their utility subject to the following budget constraint:

$$P_t C_t + \frac{1}{i_t} B_t + \frac{1}{i_t^{rrF}} \mathcal{E}_t B_t^F \le \int_0^1 W_t(h) L_t(h) dh + B_{t-1} + \mathcal{E}_t B_{t-1}^F + D_t,$$

where P_t is the price level in the economy, $W_t(h)$ is the nominal wage of a type *h* household.

The bond rate in foreign currency i_t^{rrF} is risky and depends on the risk-free rate i_t^F and the risk premium θ:

$$i_t^{rrF} = i_t^F * \theta_t,$$

where θ_t is the risk premium, defined as:

$$\theta = \left(\frac{B_t^F}{P_t^Y Y_t}\right)^{\rho^{AY}} * \left(\frac{P_t^{Oil}}{P_t^Y}\right)^{\rho^{Oil}},$$

where ρ^{AY} is the elasticity of the risk premium with respect to the net position in foreign assets to output, ρ^{Oil} is the elasticity of the risk premium with respect to the oil price.

Thus, as a result of solving the optimisation problem, the following relations are obtained:

$$\frac{e^{\xi_{c,t}}}{C_t - \eta C_{t-1}} - \beta\eta \frac{e^{\xi_{c,t+1}}}{C_{t+1} - \eta C_t} = \Lambda_t$$ ← equation for the Lagrange multiplier

$$\Lambda_t = \beta \frac{\Lambda_{t+1}}{\Pi_{t+1}} i_t$$ ← Euler equation

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$

$$i_t = i_t^F \theta_t \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$$ ← uncovered interest rate parity (UIP)

Non-Ricardian households spend all their labour income on consumption. In addition, they receive oil revenues (paid by the state as transfers), which are also spent on consumption:

$$P_t C_t \le \int_0^1 W_t(h) L_t(h) dh + \chi * P_t^{OilRef} O_t,$$

where χ is the state share in oil revenues, P_t^{OilRef} is the base price of oil, and O_t is the physical volume of oil.

2.2. Producers of final domestic and imported goods

Final goods are produced under conditions of perfect competition from intermediate goods. The production function is the Dixit–Stiglitz function:

$$Y_t^i = \left(\int_0^1 Y_t^i(f)^{\frac{\epsilon_i-1}{\epsilon_i}} df\right)^{\frac{\epsilon_i}{\epsilon_i-1}}, \qquad (2.2.1)$$

where Y_t^i is the output of producers of final goods, $Y_t^i(f)$ is the output f-ro of a producer of intermediate goods, ϵ_i is the elasticity of substitutes between two intermediate goods and $i \in \{HD, F, HF\}$, HD are domestic goods sold domestically, $F$is foreign goods sold domestically, HF is domestic goods sold abroad.

The prices of individual firms $P_t^i(f)$ are aggregated into a general price index using the Dixit-Stiglitz function:

$$P_t^i = \left(\int_0^1 P_t^i(f)^{\epsilon_i-1} df\right)^{\frac{1}{\epsilon_i-1}}. \qquad (2.2.2)$$

A representative firm maximises profit of the form:

$$P_t^i Y_t^i - \int_0^1 P_t^i(f) Y_t^i(f) = P_t^i \left(\int_0^1 Y_t^i(f)^{\frac{\epsilon_i-1}{\epsilon_i}} df\right)^{\frac{\epsilon_i}{\epsilon_i-1}} - \int_0^1 P_t^i(f) Y_t^i(f),$$

where P_t^i is the price of the final product, $P_t^i(f)$ is the price of the intermediate product of the fth producer.

From the condition of equality of profit to zero, we obtain the demand for intermediate products:

$$Y_t^i(f) = \left(\frac{P_t^i}{P_t}\right)^{-\epsilon_i} Y_t^i. \qquad (2.2.3)$$

2.3. PRODUCERS OF INTERMEDIATE DOMESTIC GOODS

Intermediate goods are produced by firms under monopolistic competition in accordance with the production function:

$$Y_t^{HD}(f) = Z_t L_t(f),$$

where Z_t is the stochastic performance trend and

$$Z_t = Z_{t-1} e^{\zeta_t^a}.$$

Some of the goods are sold as raw materials to producers of final domestic products, and some are sold as non-primary exports.

Intermediate goods are produced with nominal à la Calvo price rigidities. This means that firms are ϕ^i probability to face an inability to optimise prices, $i \in \{HD, HF\}$, HD are domestic goods sold domestically, HF is domestic goods sold abroad.

If the firm cannot optimise its price in period *t*, then it sets it according to the following rule:

$$P_t^i(f) = \left(\Pi_{t-1}^i\right)^{\gamma_i} P_{t-1}^i(f),$$

$$where\ i \in \{HD, HF\}, \Pi_t^i \equiv \frac{P_t^i}{P_{t-1}^i},$$

$\Pi^i - steady\ state\ value\ of\ inflation, \gamma_i - degree\ of\ price\ indexation\ parameter\ and\ \ 0 \leq \gamma_i < 1.$

Essential in the context of choosing the optimal inflation target is the imperfect indexation in the model, that is, the fact that the coefficient γ_i is calibrated strictly less than one. Imperfect indexation means that some firms cannot fully adjust prices from the previous period to past or equilibrium inflation. The imperfect indexing mechanism allows modeling the relationship between price dispersion and trend inflation and results in high inflation costs.

If the firm can revise its price for domestic goods sold domestically in period *t*, then it chooses it based on the profit maximisation condition:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi^{HD}) \Lambda_{t+s} \left(\frac{V_{t,t+s}^{HD} P_t^{HD*}(f)}{P_{t+s}} Y_{t,t+s}^{HD} - \frac{W_{t+s}}{P_{t+s}} \frac{Y_{t,t+s}^{HD}}{Z_{t,t+s}} \right),$$

where Λ_t – is the marginal utility of consumers and $Y_{t,t+s}^i(f)$ is the demand for the products of a monopolist who fixed the price in period *t*, in period *t* + *s*, which has the form:

$$Y_{t,t+s}^i(f) = \left(\frac{V_{t,t+s}^i P_t^{i*}}{P_{t+s}} \right)^{-\epsilon_i} Y_{t+s}^i,$$

where V_t^i is the cumulative effect of price indexation on inflation in previous periods:

$$V_{t,t+s}^i = \prod_{j=t}^{t+s-1} (\Pi_j)^{\gamma_i}.$$

Λ_t in the firms problem is due to the fact that monopolistic firms are owned by consumers, and consumers receive the profit they earn as dividends.

The first order condition for this problem is[3]:

$$\sum_{s=0}^{\infty} (\beta \phi^{HD}) \Lambda_{t+s} \left(\frac{\left(V_{t,T}^{HD} P_t^{HD*}(f)\right)^{1-\epsilon^{HD}}}{P_{t+s}} \left(\frac{1}{P_{t+s}^{HD}} \right)^{-\epsilon^{HD}} Y_{t+s}^{HD} - \frac{\epsilon^{HD}}{\epsilon^{HD}-1} \frac{W_{t+s}}{P_{t+s}} \left(\frac{V_{t,t+s}^{HD} P_t^{HD*}(f)}{P_{t+s}^{HD}} \right)^{-\epsilon^{HD}} \frac{Y_{t+s}^{HD}}{Z_{t+s}} \right) = 0.$$

Similarly, the price is chosen for domestic goods sold abroad:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi^{HF}) \Lambda_{t+s} \left(\frac{V_{t,t+s}^{HF} P_t^{HF*}(f)}{P_{t+s}} Y_{t+s}^{HF} - \frac{1}{\mathcal{E}_{t+s}} \frac{W_{t+s}}{P_{t+s}} \frac{Y_{t+s}^{HF}}{Z_{t+s}} \right).$$

The first order condition for this problem is:

$$\sum_{s=0}^{\infty} (\beta \phi^{HF}) \Lambda_{t+s} \left(\frac{\left(V_{t,T}^{HF} P_t^{HF*}(f)\right)^{1-\epsilon^{HF}}}{P_{t+s}} \left(\frac{1}{P_{t+s}^{HF}} \right)^{-\epsilon^{HF}} Y_{t+s}^{HF} - \frac{\epsilon^{HF}}{\epsilon^{HF}-1} \frac{1}{\mathcal{E}_{t+s}} \frac{W_{t+s}}{P_{t+s}} \left(\frac{V_{t,t+s}^{HF} P_t^{HF*}(f)}{P_{t+s}} \right)^{-\epsilon^{HF}} \frac{Y_{t+s}^{HF}}{Z_{t+s}} \right) = 0.$$

2.4. PRODUCERS OF INTERMEDIATE IMPORTED GOODS

Intermediate goods are produced by firms under monopolistic competition from foreign goods.

Just like domestic goods, foreign goods are produced with nominal à la Calvo price rigidities. This means that firms are ϕ^F probability to be unable to change prices.

[3] A complete derivation of the model equations is contained in Appendix I.

If the firm can revise its price for domestic goods sold domestically in period *t*, then it chooses it based on the profit maximisation condition:

$$\mathbb{E}\sum_{s=0}^{\infty}(\beta\phi^F)\,\Lambda_{t+s}\left(\frac{V_{t,t+s}^F P_t^{F*}}{P_{t+s}}Y_{t,t+s}^F-\mathcal{E}_{t+s}\frac{P_t^{For}}{P_{t+s}}\frac{Y_{t,t+s}^F}{Z_{t,t+s}}\right),$$

where P_t^{For} is the price of intermediate goods abroad, P_t^{F*} is the effective price of importers.

If the firm cannot optimise its price in period *t*, then it sets it according to the following rule:

$$P_t^F(f)=\ (\Pi_{t-1}^F)^{\gamma_F}P_{t-1}^F(f),$$

$$\Pi_t^F\equiv\frac{P_t^F}{P_{t-1}^F}\ ,\Pi^F-\text{равновесное значение},0\leq\ \gamma_F<1.$$

As in the case of producers of domestic goods, price indexation for importers is imperfect.

The first order condition for this problem is:

$$\sum_{s=0}^{\infty}(\beta\phi^F)\,\Lambda_{t+s}\left(\frac{\left(V_{t,T}^F P_t^{F*}(f)\right)^{1-\epsilon^F}}{P_{t+s}}\left(\frac{1}{P_{t+s}^F}\right)^{-\epsilon^F}Y_{t+s}^F-\frac{\epsilon^F}{\epsilon^F-1}\mathcal{E}_{t+s}\frac{P_t^{For}}{P_{t+s}}\left(\frac{V_{t,t+s}^F P_t^{F*}(f)}{P_{t+s}}\right)^{-\epsilon^F}\frac{Y_{t+s}^F}{Z_{t+s}}\right)=0.$$

2.5. LABOUR SUPPLY

Each household h offers its own specific type of work $N_t(h)$. The labour of individual households is aggregated into the total labour supply N_t using the CES function:

$$N_t=\left(\int_0^1 N_t(h)^{\frac{\epsilon_L-1}{\epsilon_L}}dh\right)^{\epsilon_L/(\epsilon_L-1)}, \tag{2.5.1}$$

where ϵ_L is the elasticity between the types of labour of different firms.

$$N_t=\int_0^1 L_t(f)df, \tag{2.5.2}$$

where $L_t(f)$ is the firm's demand for labour f, L_t is the total demand for labour in the economy, aggregated by firms, $N_t(h)$ is the household's labour supply, and h, N_t are the labour supply aggregated across all households.

$$W_t=\left(\int_0^1 W_t(h)^{1-\epsilon_L}dh\right)^{1/(1-\epsilon_L)}, \tag{2.5.3}$$

where W_t is the nominal total wage, $W_t(h)$ is the wage paid to a household of type h.

I assume the presence of nominal à la Calvo rigidity in wages, that is, with the ϕ^L probability that households cannot optimise wages. In this case, the wage is set according to the following indexation rule:

$$W_t(h)=\ (\Pi_{t-1})^{\gamma_L}W_{t-1}(h),$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, Π is the equilibrium value of inflation, and the γ_L degree of indexation parameter lies in the range $0 \leq \gamma_L < 1$, that is, the indexation is imperfect.

In the case when households can choose a wage, it is found from the solution of the following optimisation problem:

$$\mathbb{E}\sum_{s=0}^{\infty}(\beta\phi^L)\,\Lambda_{t+s}\left(\frac{V_{t,t+s}^L W_t^*}{W_{t+s}}L_{t,t+s}-\frac{1}{v}L_{t,t+s}^{1+v}\right),$$

where the demand function for labour in a period $t+s$ for a household that changed the price in a period t, is:

$$N_{t,t+s}=\left(\frac{V_{t,t+s}^L W_t^*}{W_{t+s}}\right)^{-\epsilon_L} N_{t+s},$$

where is the cumulative effect on wages from indexation:

$$V_{t,t+s}^L=\prod_{j=t}^{t+s-1}(\Pi_j)^{\gamma_L}.$$

2.6. Oil sector

It is assumed that the oil firm produces a homogeneous commodity and all of it is exported. Oil production O_t depends on production in the previous period and foreign demand Y_t^F:

$$O_t = (O_{t-1})^{\rho^O} * (Y_t^F)^{\alpha^O},$$

where α^O is the elasticity of oil production to foreign demand.

The price of oil is determined exogenously:

$$\widehat{prO}_t = \rho^{pro}\widehat{prO}_{t-1} + \varsigma_t^{pro}.$$

The state receives a share χ of the sale of oil and pays it to households as transfers.

2.7. Base oil price

The fiscal rule is a mechanism for reducing the volatility of a country's income. This policy tool is often used in countries where natural resources make up a significant part of their exports. The meaning of this mechanism is to establish a long-term (base) price for the exported resource. If the actual price is higher than the base price, then the excess is transferred to a special fund for storage. If the actual price is lower than the base price, then the state budget uses the fund to finance the missing part of the planned expenditures. Thus, more stable government spending is balanced against more volatile government commodity revenues.

In Russia, the fiscal rule was originally introduced in 2004 and has since been revised several times, such as in 2008 when the Stabilisation Fund of the Russian Federation was split into the Reserve Fund and the National Wealth Fund, or in 2008 when the rule was changed in connection with the global financial crisis. In addition, the rule was suspended in order to be able to respond more flexibly to the situation in 2015, 2020 and 2022. At the time of this writing, a law has been passed to change the fiscal rule from 2023. The new fiscal rule proposes to understand 8 trillion rubles as basic oil and gas revenues, and income above this value is considered super income.

Considering that at the time of writing the new fiscal rule has not yet entered into force, it is difficult to assess its possible impact on the economy and the optimal level of the key rate as the main issue of my research. However, in order to illustrate that the fiscal rule has an impact on the choice of the optimal target level (by offsetting some of the shocks that affect the economy), I consider two versions of the model, without a fiscal rule and with a fiscal rule. The mechanism associated with the cut-off price, which was used in Russia from 2017 to 2022, is considered as a fiscal rule.

For simplicity, the model does not explicitly describe the reserve fund. It is assumed that each period the government saves/borrows the difference between the base and actual oil revenues in foreign currency $\chi * \left(P_t^{Oil} - P_t^{OilRef}\right)O_t$. In this formulation of the problem, it is implicitly assumed that the fund is inexhaustible. Basic oil revenues $\chi * P_t^{OilRef} O_t$ are paid to non-Ricardian households as dividends.

2.8. External sector

The external sector is modeled exogenously with respect to the domestic economy. The foreign interest rate $\hat{\imath}_t^F$ and foreign inflation $\hat{\pi}_t^F$ are respectively equal to:

$$\hat{\imath}_t^F = \rho^{iF}\hat{\imath}_{t-1}^F + \varepsilon_t^{iF} \sim AR(1),$$

$$\hat{\pi}_t^F = \rho^{piF}\hat{\pi}_{t-1}^F + \varepsilon_t^{piF} \sim AR(1).$$

2.9. Monetary policy and the zero lower bound of interest rates

The central bank policy rule is:

$$i_t^{ZLB} - \bar{\imath}_t = \psi_R(i_{t-1}^{ZLB} - \bar{\imath}_{t-1}) + (1-\psi_R)\psi_{PI} * \hat{\pi}_{t+1} + \zeta_t^{MP},$$

$$i_t^{ZLB} = \begin{cases} i_t^{ZLB}, if\ i_t^{ZLB} \geq 0 \\ 0, otherwise \end{cases},$$

where $i_t^{ZLB} \equiv \log(i_t^{ZLB})$ and $\bar{\imath}_t$ is the interest rate trend, ζ_t^{MP} is the monetary policy shock, ψ_R is the monetary policy response smoothing factor, ψ_{PI} is the coefficient of monetary policy response to inflation deviation from the target level.

2.10. Markets clearing

Equilibrium in the economy is determined by the following relations.

Demand for the goods of producers of intermediate products is equal to the total supply of these goods:

$$Y_t^H(f) = \left(\frac{P_t^H(f)}{P_t^H}\right)^{-\epsilon_H} Y_t^H + \left(\frac{P_t^{HF}(f)}{P_t^{HF}}\right)^{-\epsilon_{HF}} Y_t^{HF}.$$

The demand for domestic goods is equal to their supply:

$$Y_t^H = C_t^H + \left(1 - \frac{O}{X}\right) Y_t^{HF}.$$

Output represents household consumption expenditure C_t, oil and non-oil exports X_t, imports M_t:

$$Y_t = C_t + X_t - M_t.$$

The balance of payments is as follows:

$$\frac{\mathcal{E}_t B_t^F}{(1+i_t^F)} = \mathcal{E}_{t-1} B_{t-1}^F - (1-\chi) * P_t^{Oil} O_t - \chi * \left(P_t^{Oil} - P_t^{OilRef}\right) O_t + P_t^X X_t - P_t^M M_t,$$

where P_t^{OilRef} is the base oil price.

In the model with the fiscal rule, the impact on the exchange rate occurs through the balance of payments. In the model with the rule $P_t^{Oil} \neq P_t^{OilRef}$, therefore, part of the influence of the oil price on the exchange rate is leveled. In the model without the rule $P_t^{Oil} = P_t^{OilRef}$, therefore, the additional element associated with the base price of oil in the balance of payments is set to zero, and this effect does not occur.

3. Calibration

In this section, I discuss the parameter calibration used in this paper. To calibrate the parameters, I use Russian and foreign empirical works, as well as direct statistical data for Russia. The decision was made not to evaluate the model using Bayesian estimation for several reasons. First, the ability to adequately estimate micro-parameters, such as degrees of price rigidity or degree of price indexation, for individual types of firms using aggregated data series is questionable. Second, in many papers using Bayesian parameter estimation, the choice of priors and standard deviations is not described in detail, which, in fact, makes the distinction between estimation and calibration blurry. In this regard, I prefer to use calibration, while checking the robustness of the estimates in the corresponding section.

3.1. Basic calibration

Basic parameter calibration is given in Table 3.1.

The GDP growth rate is assumed to be 1.5% per year (i.e. $1{,}015^{0.25}$ $per\ quarter,\ \overline{g_y} = 1{,}015^{0.25}$) The mid-term consensus for Russia's potential GDP growth is estimated at 1.5–2%. For example, IMF (2021) - 1.6%, World Bank (2021) - 1.8%.

The value of the discount factor β is set at 0.999, which corresponds to a real interest rate of 1.78% per annum. As discussed above, according to empirical studies, the real rate for Russia lies in the range of 1 to 3%.

I calibrate the elasticity of substitution between domestic and imported goods at 0.9. This value is close to one, since the share of imports in GDP for Russia is stable and has not changed much over the past two decades and is about 20%.

I calibrate the share of imports in consumption at 0.43. I define this indicator as the ratio of the shares of imports to GDP and consumption to GDP, focusing on Rosstat data on the components of GDP from 2014 to 2020.

The degree of utility loss to the consumer from an additional unit of labour (the inverse of Frisch elasticity) ν indicates how much additional compensation workers need in order for them to be willing to provide an additional small unit of labour, and the higher this figure, the more compensation is required. I set this ratio at 1.04.

Estimates of elasticity of substitution between intermediate substitutes, as discussed in Leif et al. (2005), for the US, euro area and UK range from 3 to 11. The higher this value, the closer the market is to perfect competition. I set 6 for domestic goods sold domestically θ^{HD},, 6 for domestic goods sold abroad θ^{HF}, and 6 for foreign goods sold domestically θ^{F},, i.e. corresponding to a moderate level of competition, with a markup of 20%.

Calvo coefficients φ^{HD} are calibrated at the level of 0.4, φ^{HF} – 0.6, φ^{F} – 0.4. To my knowledge, there are no works for the Russian economy that explicitly estimate these coefficients on microeconomic data. I calibrate the Calvo coefficient for goods sold domestically to be lower than for industries where goods are sold abroad. This is due to the fact that I assume that exporters' prices are more rigid due to the influence of external factors. Thus, producers of goods sold domestically change prices on average every 2 months, while exporters change prices on average once every 4.5 months.

As for degree of indexation, γ^{i}, $where\ i \in \{HD, F, HF, L\}$ as discussed above, this parameter should be less than one to model price dispersion between firms. I calibrate this parameter at 0.4, which provides a sufficient level of rigidity in the economy, but still allows prices to be adjusted for previous inflation.

I calibrate the coefficient of monetary policy response to inflation deviation ψ^{PI} at 2.5. The monetary policy smoothing coefficient ψ^{R} is set at 0.75.

Table 3.1. Calibration of model parameters

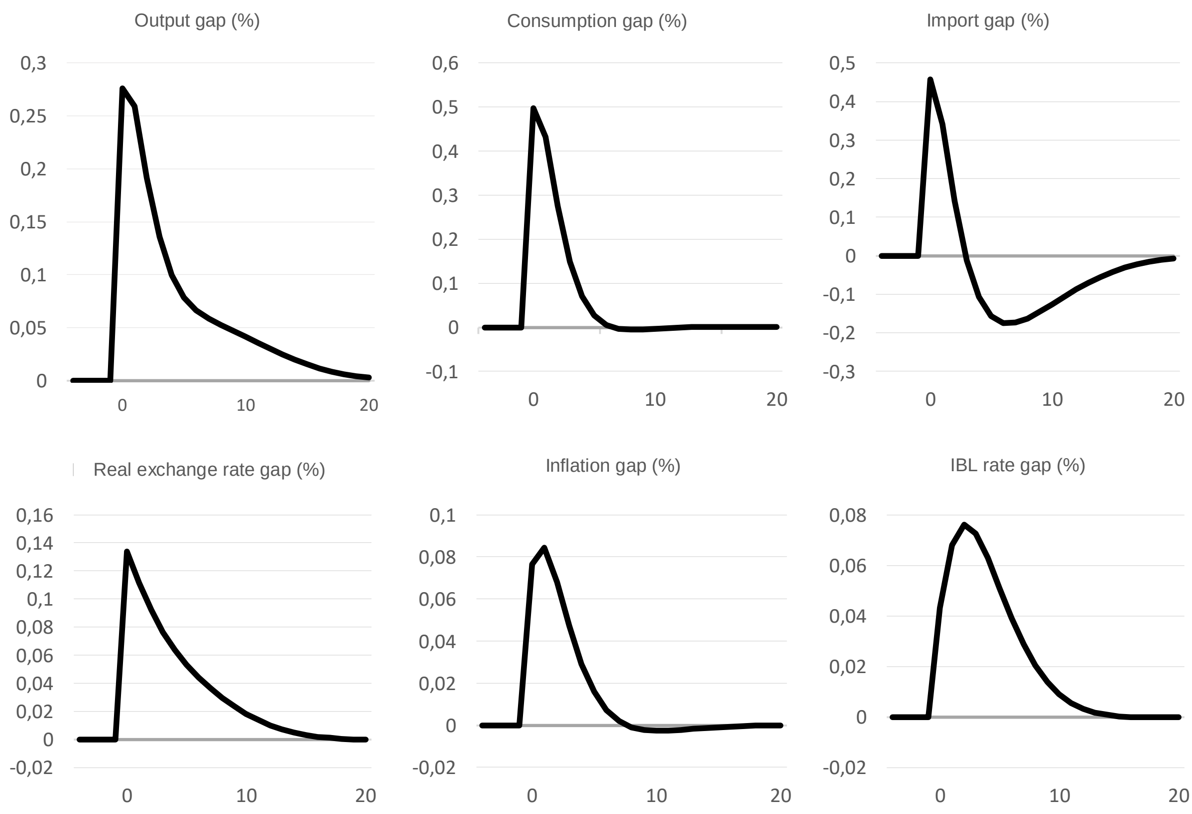
Consumer utility function parameters	
β: discount coefficient	0.999
η: coefficient of the Frisch elasticity of labour supply	1.04
Price formation parameters	
φ^{HD}: Calvo coefficient of domestic goods sold domestically	0.4
φ^{HF}: Calvo coefficient for domestic goods sold abroad	0.6
φ^{F}: Calvo coefficient of domestic goods sold domestically	0.4
γ^{HD}: price indexation of domestic goods sold domestically	0.4
γ^{HF}: price indexation of domestic goods sold abroad	0.4
γ^{F}: price indexation of foreign goods sold domestically	0.4
θ^{HD} : elasticity of substitution between domestic intermediate goods sold domestically	6
θ^{HF}: elasticity of substitution between domestic intermediate substitute goods sold abroad	6
θ^{F} : elasticity of substitution between domestic intermediate substitute goods imported from abroad	6
Monetary policy parameters	
ψ^{PI}: coefficient of monetary policy response to inflation deviation	2.5
ψ^{R}: monetary policy smoothing ratio	0.75
Risk premium parameters	
ρ^{AY}: the elasticity of the risk premium relative to the net position in foreign assets to output	0.0155
ρ^{Oil}: the elasticity of the risk premium for oil price	−0.0057
Stationary states	
$\overline{g_y}$: GDP growth rate	$1{,}015^{0.25}$
Shock parameters	
ρ^{C}: preference shock persistence	0.5
σ^{C}: standard deviation of preference shock	6.7
ρ^{a}: technology shock persistence	0.6
σ^{a}: standard deviation of technology shock	0.3
ρ^{mp}: monetary policy shock persistence	0.2
σ^{mp}: standard deviation of monetary policy shock	1.5
ρ^{pi}: cost shock persistence	0.3
σ^{pi}: standard deviation of cost shock	3
$\rho^{pi\ bar}$: cost trend shock persistence	0.9
$\sigma^{pi\ bar}$: standard deviation of the cost shock trend	0.3

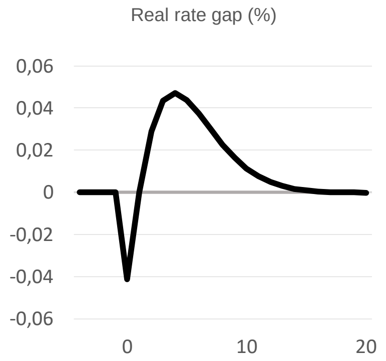
ρ^{ex}: UIP shock persistence	0.5
σ^{ex}: standard deviation of UIP shock	0
$\rho^{ex\ bar}$: UIP trend shock persistence	2
$\sigma^{ex\ bar}$: standard deviation of UIP shock	3
ρ^{x}: export shock persistence	0.5
σ^{x}: standard deviation of export shock	9.5
$\sigma^{x\ bar}$: standard deviation of the export shock trend	1.5
ρ^{iF}: FX rate shock persistence	0.9
σ^{iF}: standard deviation of FX shock	0.4
$\rho^{iF\ bar}$: FX rate trend shock persistence	0.95
$\sigma^{iF\ bar}$: standard deviation of the FX shock trend	0.2
ρ^{piF}: foreign inflation shock persistence	0.3
σ^{piF}: standard deviation of foreign inflation shock	1.7
$\rho^{piF\ bar}$: foreign inflation trend shock persistence	0.8
$\sigma^{piF\ bar}$: standard deviation of foreign inflation shock trend	0.2
ρ^{poil}: oil price shock persistence	0.6
σ^{poil}: standard deviation of oil price shock	16
$\rho^{poil\ bar}$: oil price trend shock persistence	0
$\sigma^{poil\ bar}$: standard deviation of oil price shock trend	22

3.2. ANALYSIS OF THE TRANSMISSION MECHANISM

To analyse the properties of the model, I plot the impulse response functions of the main variables to the main shocks of the model.

Chart 3.1. Impulse response functions, preference shock





As seen in Chart 3.1, a positive preference shock leads to an increase in consumption. Since the consumer basket includes both domestic and imported goods, the demand for goods in both categories is growing. Imports are growing following the increase in consumer demand for them. An increase in demand leads to higher prices and a positive inflation gap. This, in turn, encourages the central bank to raise interest rates. Since the real interest rate falls in the first periods after the shock, the real exchange rate weakens. Then, as the real rate rises, the exchange rate strengthens and returns to the equilibrium state.

Chart 3.2. Impulse response functions, cost-pushshock

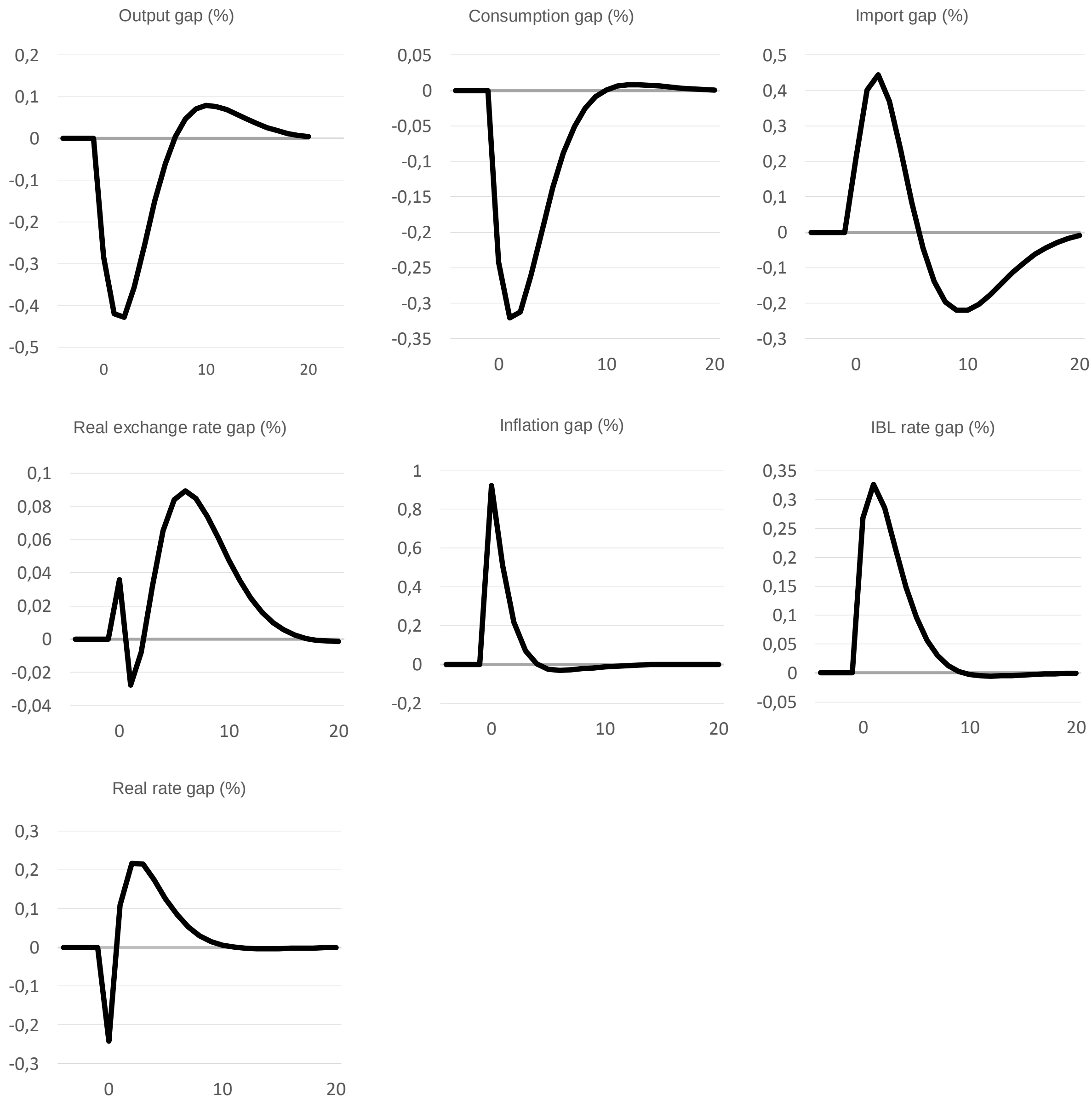


Chart 3.2 shows the responses of the model's main variables to a positive 1% cost shock. A positive cost shock drives up the price of domestic goods through the Phillips curve. To stabilise inflation, the central bank raises the interest rate. An increase in the key rate leads to a redistribution of the utility of consumers between the current and future periods in favour of the future and, consequently, to a drop in consumption of the current period. Since the consumer basket includes both domestic and imported goods, the demand for goods in both categories is decreasing. There are two opposite effects on the real exchange rate. An increase in the interest rate through interest rate parity affects the exchange rate in the direction of its strengthening. Nevertheless, increased demand for foreign goods (that is, an increase in demand for foreign currency) affects the exchange rate in the direction of weakening. Imports also depend on another two phenomena. First, an appreciation makes foreign goods relatively cheaper and opens up a positive import gap. The decline in consumer demand then pushes the import gap into negative territory.

Chart 3.3. Impulse response functions, monetary policy shock

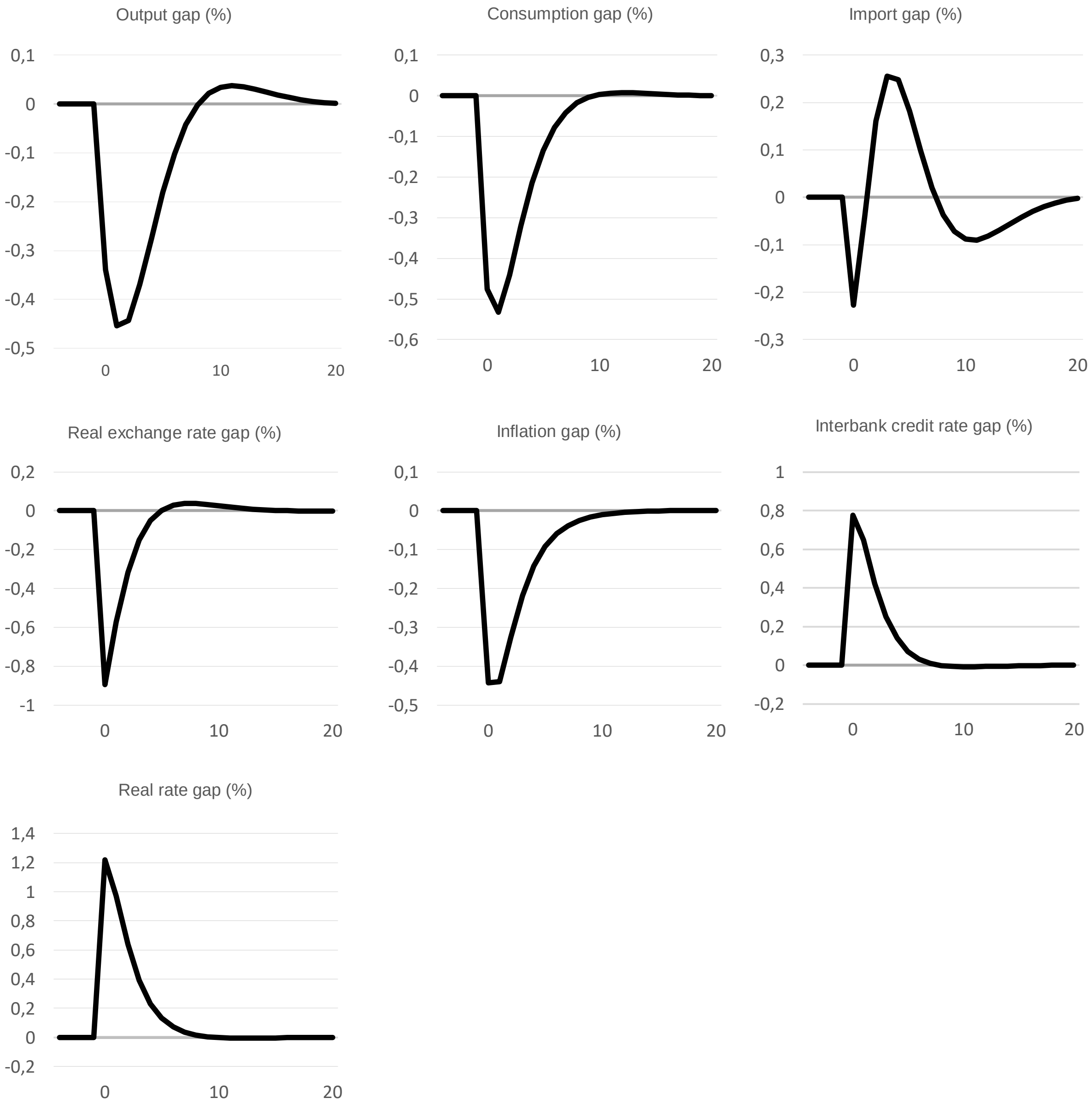


Chart 3.3 shows the responses of the model's main variables to a 1% positive monetary policy shock (the rate rises). An increase in the key rate leads to a redistribution of the utility of consumers between the current and future periods in favour of the future and, consequently, to a drop in consumption of the current period. Since the consumer basket includes both domestic and imported goods, the demand for goods in both categories is decreasing. Decreased demand leads to lower prices and a negative inflation gap. In addition, raising the rate in line with uncovered interest-rate parity leads to real exchange rate appreciation. As a result, imported goods become relatively cheaper, which also has a disinflationary effect.

The response to an oil price shock, as expected, depends on the presence or absence of a fiscal rule in the model.

Chart 3.4. Impulse response functions, oil price shock

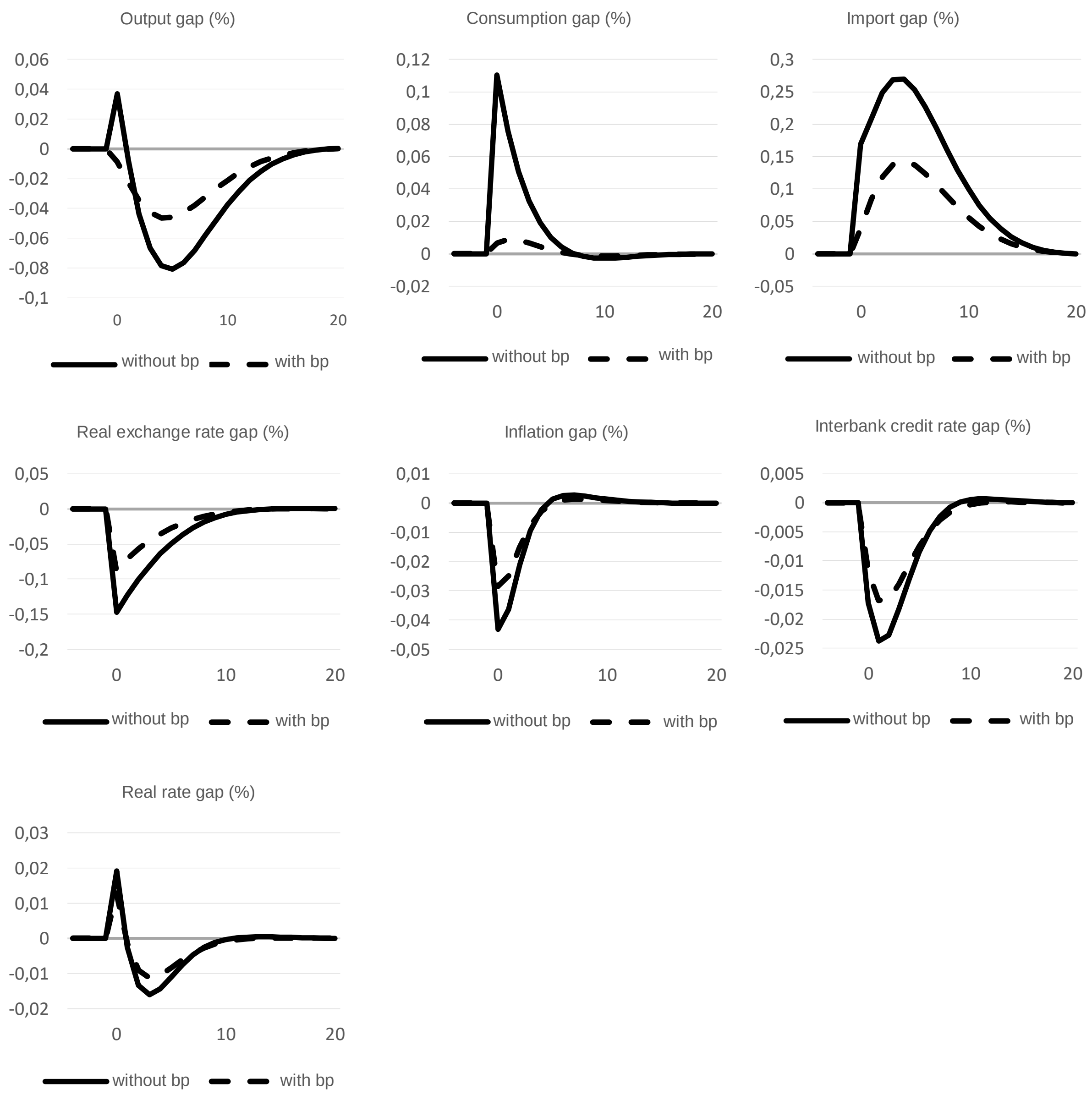


Chart 3.4 shows the responses of the main variables of the model to a 1% positive oil price shock (oil price is rising). An increase in the price of oil leads to a decrease in the risk premium and, accordingly, a strengthening of the exchange rate. With a stronger exchange rate, foreign goods become relatively cheaper. This leads to the opening of a negative inflation gap, and imports increase. The Central Bank cuts the key rate to bring inflation back to the target. Reducing the interest rate leads to a redistribution of consumer income in favour of consumption instead of saving, thus, consumption increases. Output is declining because consumption is growing less than imports. In the situation with the fiscal rule, the responses have the same direction as in the absence of it, but the impact of the shock on the model variables is lower, since the effect of the rule partially offsets the effects of the exchange rate.

Based on the analysis of impulse responses, we can say that the response of variables to model shocks reflects the mechanisms characteristic of neo-Keynesian DSGE models of a small open economy (for example, (Medina, 2007)).

3.3. Checking if the model fits the data

To assess the model adequacy and its compliance with the data, simulations are performed using a linear[4] model with a target of 4% per 100,000 periods. Simulations are made using the built-in function of the IRIS[5] package for Matlab. All shocks of the model are used for the simulation, except for the monetary policy shock. This is because, I assume, the central bank uses its key rate to respond to other shocks, but does not create shocks with its policies. The shocks for the simulations are taken from a normal distribution.

Table 3.2 compares the standard deviations and autocorrelation coefficients of observed data for the period 2003 Q2 to 2021 Q2 and simulated data.

Table 3.2. Comparison of characteristics of observed and simulated data

	Standard deviation		Ratio of autocorrelation AR(1)	
	Model	Data	Model	Data
Consumption gap (%)	5.99	5.87	0.69	0.80
Export gap (%)	3.05	3.33	0.32	0.77
Interbank credit rate gap (%)	2.28	2.59	0.87	0.88
Inflation gap (%)	4.41	3.62	0.43	0.48
Gap of the real ruble/dollar exchange rate (%)	11.52	13.87	0.45	0.92
Oil price gap (%)	20.69	18.56	0.61	0.75
External interest rate gap (%)	0.89	1.03	0.89	0.96
External inflation gap (%)	1.79	1.39	0.33	0.25

As can be seen from the table, the values of the considered characteristics of the simulated and actual data are close, and therefore, the resulting model reflects the actual dynamics of the variables quite well.

Table 3.3 provides a similar comparison for the fiscal rule model.

Table 3.3. comparison of characteristics of observed and simulated data for a model with a fiscal rule

	Standard deviation		Ratio of autocorrelation AR(1)	
	Model	Data	Model	Data

[4] Both the model with ZLB and the model without ZLB are loglinearised around the steady state. In this case, the model c ZLB does not become linear, since the ZLB condition itself is non-linear. Therefore, hereinafter, for brevity, the model without ZLB is called linear, and the model with ZLB is called nonlinear.
[5] IRIS is a package for macroeconomic modeling and forecasting in Matlab.

Consumption gap (%)	5.47	5.87	0.70	0.80
Export gap (%)	3.04	3.33	0.32	0.77
Interbank credit rate gap (%)	2.22	2.59	0.87	0.88
Inflation gap (%)	4.35	3.61	0.42	0.48
Gap of the real ruble/dollar exchange rate (%)	11.06	13.86	0.43	0.92
Oil price gap (%)	20.69	18.56	0.61	0.75
External interest rate gap (%)	0.89	1.03	0.89	0.96
External inflation gap (%)	1.78	1.39	0.33	0.25

As can be seen from the table, for this version of the model, the simulated characteristics are also close to those calculated from the data. It should also be noted that the standard deviations of the variables are less than or equal to those calculated for the model without the rule. This illustrates the operation of the fiscal rule, this mechanism reduces the volatility of variables.

It is worth noting that in the range I used from Q2 2003 to Q2 2021, several significant events occurred in the Russian economy: the adopted inflation targeting at the end of 2014, several revisions of the parameters of the fiscal rule, and periods when the rule was canceled, crises in 2008, 2014 and 2020. However, these periods are too short to give reliable estimates. For example, the period after the adopted inflation targeting includes only 32 points on the quarterly data. In addition, the purpose of this experiment is only to test the possibility of obtaining variables with characteristics close to those of real data using the model. The results show that these characteristics are quite close, and thus the conclusions drawn from the model can be considered relevant for the Russian economy.

4. Optimal inflation target level

This section is devoted to a discussion of the results of simulations on the constructed model. Simulations are made for 12,500[6] periods using the built-in function of the IRIS package for Matlab. The paper considered values from 0.5% per annum to 4% per annum with a step of 0.1.

Based on the data received, I calculate the probabilities of being at the ZLB and find the optimal target level based on the structural loss function. I also study the dependence of the optimal inflation target level on the real neutral interest rate.

4.1. Probability of being at the ZLB

The probabilities of being at the ZLB obtained from the simulated data for the model with the fiscal rule and without the fiscal rule are shown in Charts 4.1 and 4.2.

For the base model without a fiscal rule, the probability of being in the negative area of interest rates with an inflation target of 4% is about 1%.

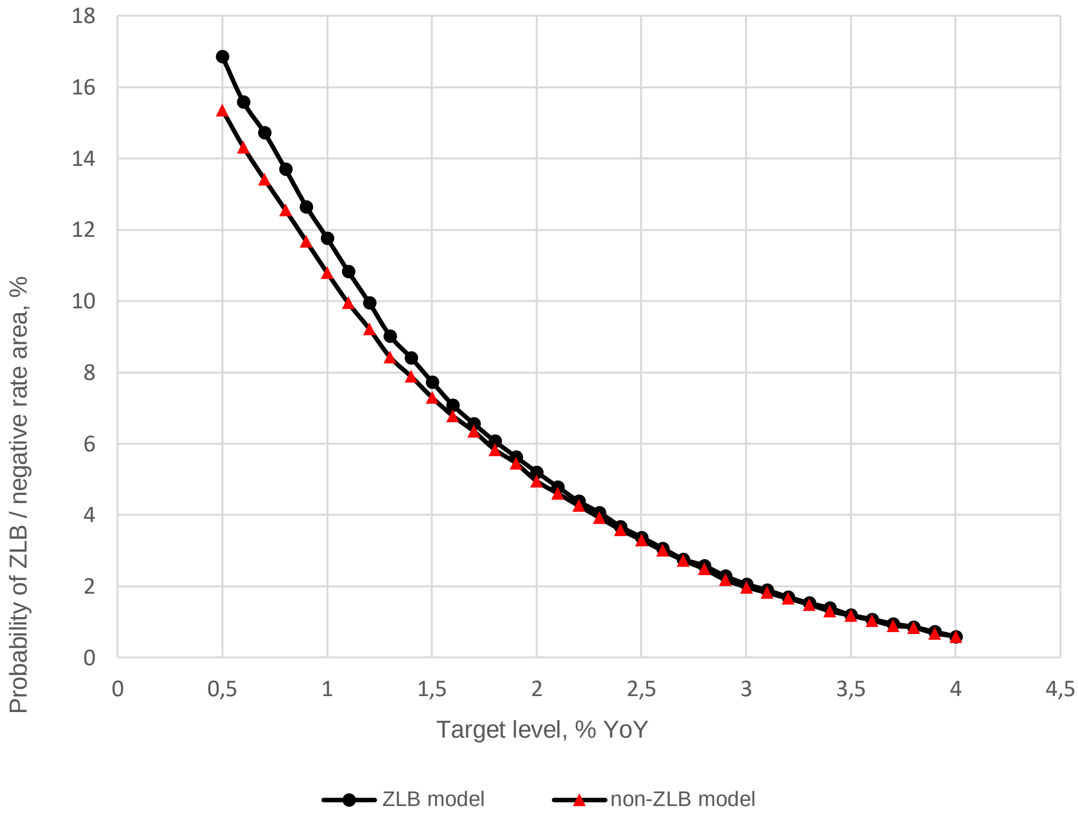
[6] This is due to a trade-off between accuracy and the amount of time required for calculations.

The probability was calculated with the following formula:

$$prob^{ZLB} = 100 * \frac{\sum_{i=0}^{N} i_t^{ZLB} \leq 0}{N},$$

where $prob^{ZLB}$ is the probability of being at the ZLB / in the negative area of interest rates, N is the number of simulation periods.

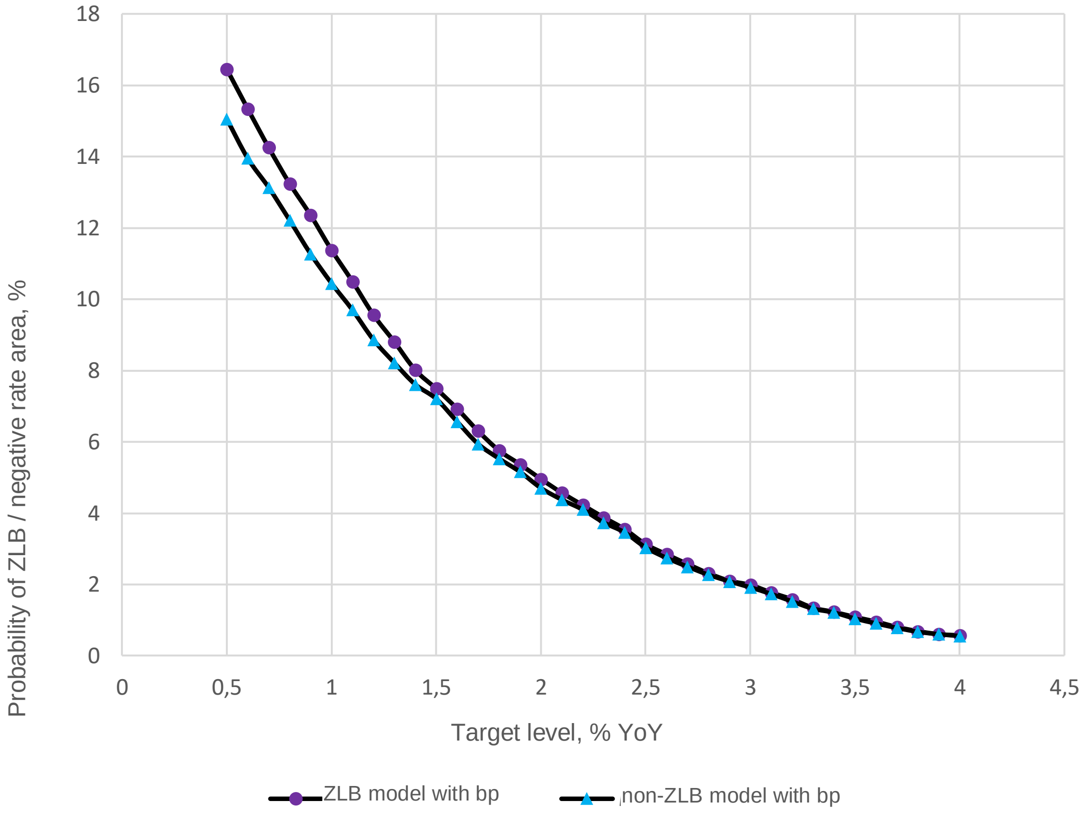
Chart 4.1. ZLB probability in the base model



The probability of ZLB increases as the level of the inflation target decreases. This is due to the fact that the nominal interest rate is the sum of the real interest rate and equilibrium inflation. With a constant real interest rate, as the inflation target decreases, the nominal interest rate also decreases and approaches the zero lower bound. With a sufficiently low nominal rate, even small shocks lead to a zero lower bound, and thus the probability of being at the ZLB increases as the inflation target is lowered.

Moreover, in a situation where the central bank cannot lower the rate below zero, it needs more time to stabilise the economy than in a situation where there is no such restriction.

Chart 4.2. Probability of ZLB by model with fiscal rule



If there is a fiscal rule in the model, the ratio of the model with ZLB / model without ZLB and the negative relationship between the probability of ZLB and the target level, as described above, remain unchanged. However, for each level of the target, this probability becomes lower, since some of the shocks are leveled by the rule.

4.2. Optimal level choice criteria

As discussed above, to investigate the optimal inflation target level, we need to use a structural loss function. I use the function with micro-foundations as in Woodford (2001). This function is a second-order approximation of the consumer utility function. This kind of function has several important advantages. First, it is a structural function that naturally takes into account the mechanisms and parameters of the model. Secondly, it reflects the welfare of consumers, that is, the inflation target is chosen based not on the abstract task of the central bank, but based on the utility of households. Third, this function is considered in deviations from the natural level of variables (that is, variables in an economy without rigidities) and, thus, allows taking into account the costs of inflation considering the rigidities existing in a particular economy. It follows from the last property that the value of this function is negative, since in an economy without rigidities, nominal variables do not affect real variables, and thus consumers do not bear the costs of high inflation. Thus, the function reflects the loss to society in terms of the deviation from an economy with flexible prices.

For my model specification, this function looks like:

$$\log(C_t - \eta * C_{t-1} * e^{-\zeta_{z,t}}) - \frac{1}{1+\sigma_L} * \int_0^1 (N_t(h))^{1+\sigma_L} dh =$$

$$\frac{1}{1-\eta} * \left[\frac{C_t - C^n}{C_t} - \eta * \frac{C_{t-1} - C^n}{C_t} - \frac{1}{2} * \frac{1}{1-\eta} * \left(\frac{C_t - C^n}{C_t}\right)^2 + \frac{1}{1-\eta} * \left(\frac{C_t - C^n}{C_t}\right) * \left(\frac{C_{t-1} - C^n}{C_t}\right) - \frac{1}{2} * \frac{\eta^2}{1-\eta} * \left(\frac{C_{t-1} - C^n}{C_t}\right)^2 + \zeta_{c,t} * \frac{C_t - C^n}{C_t} - \eta * \zeta_{c,t} * \frac{C_{t-1} - C^n}{C_t} - \frac{\eta}{1-\eta} * \zeta_{z,t} * \frac{C_t - C^n}{C_t} + \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\frac{C_{t-1} - C^n}{C_t}\right)\right] \quad \text{Consumption} \tag{4.2.1}$$

$$-\int_0^1 \left((N^n)^{1+\sigma_L} * \frac{N_t(h) - N^n}{N^n} + \frac{1}{2} * \sigma_L * (N^n)^{1+\sigma_L} * \left(\frac{N_t(h) - N^n}{N^n}\right)^2\right) dh \quad \text{Labour} \tag{4.2.2}$$

,

где $C - consumption, N - labour, C^n - natural\ level\ of\ consumption,$

$N^n - natural\ level\ of\ labor, h - labor\ type\ index, \eta - habits\ in\ concumptin,$

$\zeta_c - consumption\ shock, \zeta_z - productivity\ shock,$

$\sigma_L - Frish\ elasticity.$

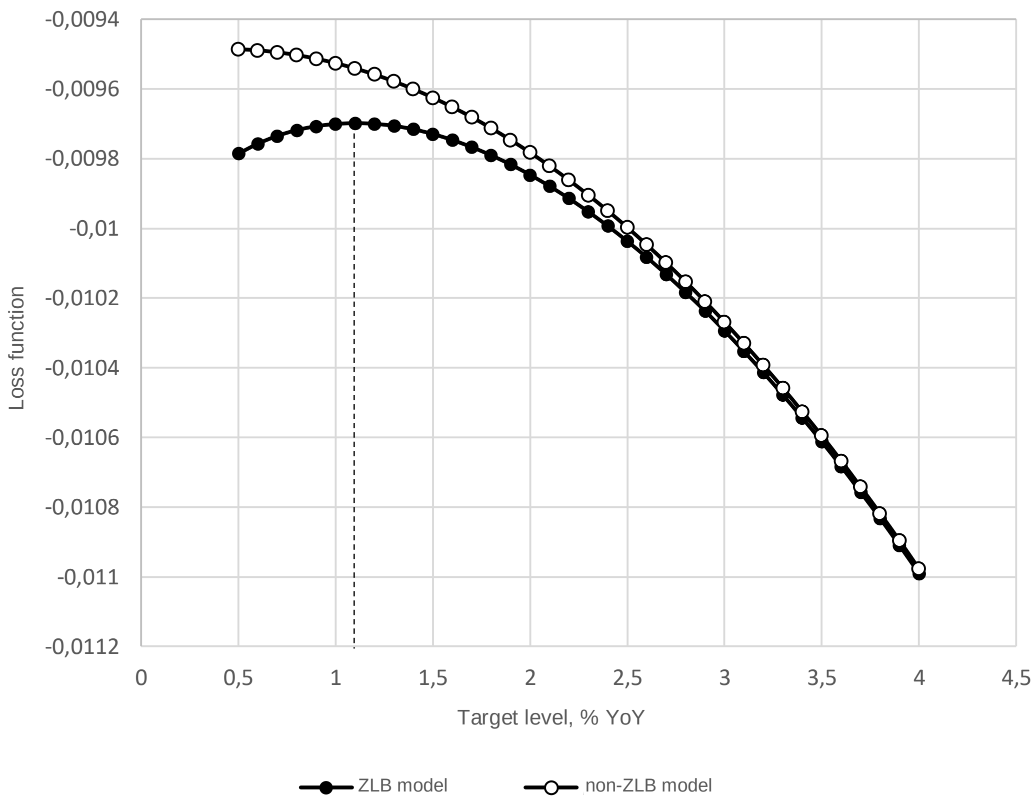
4.3. CHOICE OF OPTIMAL TARGET LEVEL AND ZLB PROBABILITY

The choice of the optimal inflation target level is based on the loss function given in Section 4.2. For each target level, based on the simulated variables, the value of the loss function is calculated, then its average value for all periods is calculated, that is, the unconditional mathematical expectation is calculated. However, I assume that the loss function has the same value for each period. This premise is due to the fact that the number of simulation periods is not a direct analogy of the time scale, but rather a repetition of the experiment in order to bring the sample mean closer to the actual one.

By calculating the loss function in this way, we get that, as a result, each target level corresponds to a certain value of the loss function. The optimal target means a target that corresponds to the smallest (modulo) value of the loss function.

I run simulations for targets from 0.5% to 4% in increments of 0.1. As described above, for each such target, the corresponding value of the loss function is obtained. Such pairs are calculated both for the model with ZLB (non-linear) and for the model without ZLB (linear). The results are shown in Chart 4.3.

Chart 4.3. Loss function for the base model



I would like to draw attention to a few points. First, the values of the loss function for each target level are negative for both the linear and non-linear models. As discussed above, this function is given in terms of deviations from variables in an economy without rigidities and reflects the loss to the economy from price dispersion. Second, for the linear model, the loss function increases (in absolute value) as the target level decreases, if there is no zero lower bound, the lower inflation is, the better consumers are in terms of their welfare. Third, if a zero lower bound on interest rates is added (the ZLB model), there is a trade-off between losses from too high inflation and a zero bound on interest rates. Moreover, at sufficiently high levels of the target, the first effect prevails, and as the level of the target decreases, the second begins to predominate. The optimum in terms of the smallest (modulo) value of the consumer loss function is achieved at a target level of 1.1%.

Chart 4.4. Loss function for a model with a fiscal rule

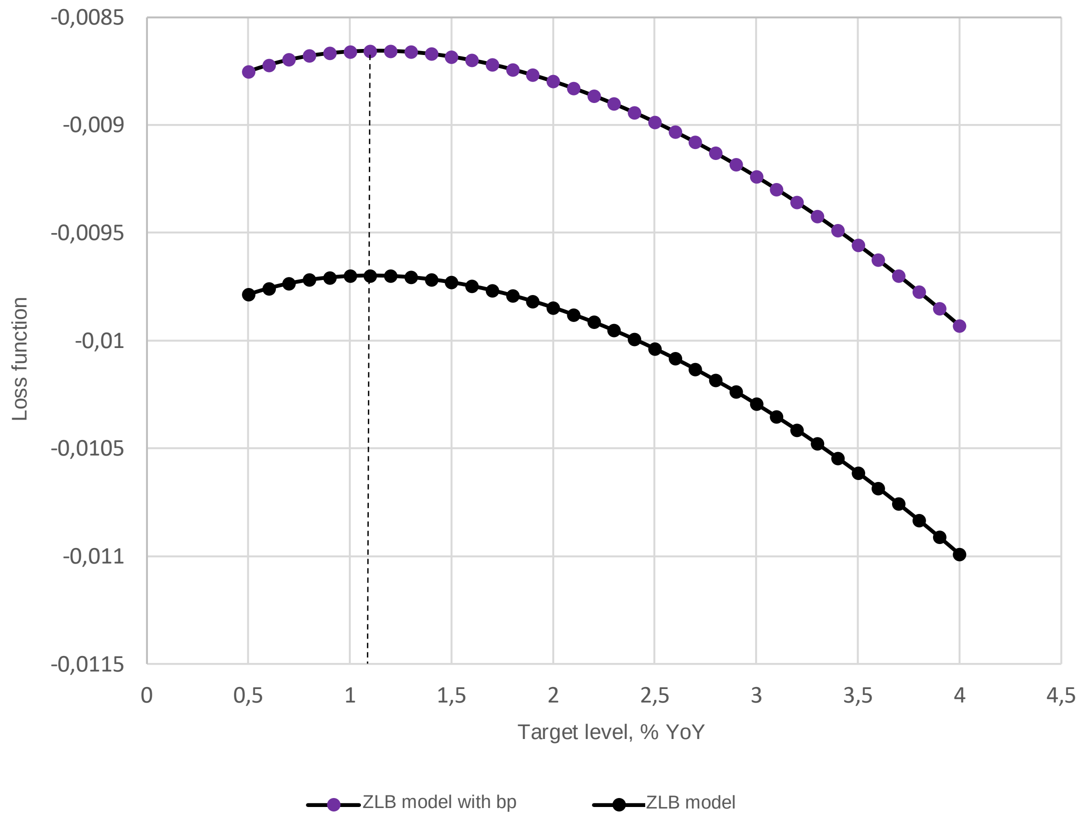


Chart 4.4 shows the loss function for the model with the fiscal rule (bp) and for the base model. If there is a fiscal rule in the model, the optimal target level remains the same of 1.1%. It should be noted that, in general, for a model with a fiscal rule, the modulo loss function is less than for an economy without such a rule at each corresponding target level, since the fiscal rule eliminates some of the shocks, and other things being equal, the ZLB probability is to decrease. Probably, when considering a smaller step of the chosen optimal rates for a model with a fiscal rule, a lower target would be chosen. But given that the choice of the target even with an accuracy of 0.1 is of academic rather than practical interest, and also taking into account the duration of the calculations, it was decided not to conduct such experiments.

4.4. Optimal inflation and real interest rate

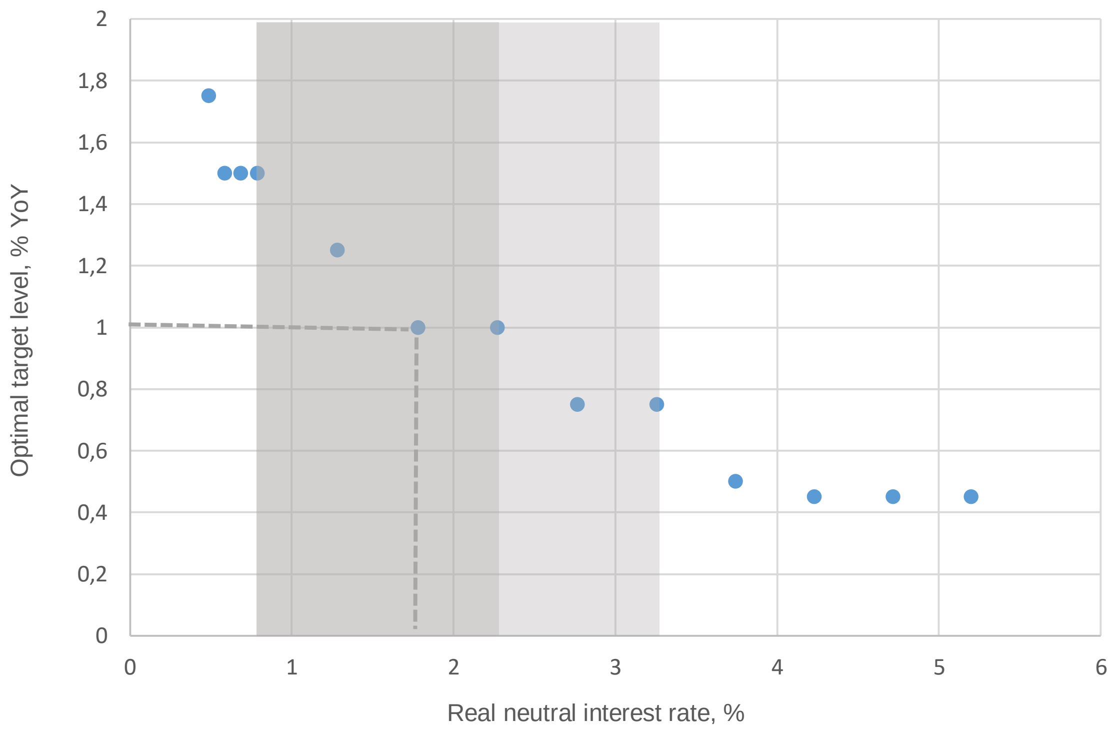
When studying the issue of choosing the optimal target level, it is necessary to take into account the value of the equilibrium real interest rate. As discussed in section 8, I assume this value is 1.78% in base calibration.

To study the influence of different values of the real interest rate on the choice of the optimal target in my model, I run simulations (the parameters are described at the beginning of this section), varying the real rate using the economic growth parameter (I assume growth from 0.2% to 5%, which corresponds to the real rate from 0.48% to 5.2%), and for such real rates I choose the optimal target level from 0.5% to 4% in increments of 0.25. The optimal target level is chosen in the same way as before, based on the consumer loss function.

Chart 4.5 points mean pairs (r^*, π^*), where r^* is the real neutral interest rate, π^* is the optimal target level. I have concluded that with an increase in the real rate in the economy, the optimal target level decreases.

The light grey area in Chart 4.5 shows the area of 1-3% - the current consensus estimate of the real neutral rate of analysts and researchers for Russia. Dark gray indicates the range of 1-2%, which is given in the reports on monetary policy by the Bank of Russia. As seen in Chart 4.5, for a wider range, the optimum lies in the range [0.75; 1.5], and for a narrower range - in the range [1; 1.5].

Chart 4.5. Optimal target level and real neutral interest rate



4.5. Robustness to changing model parameters

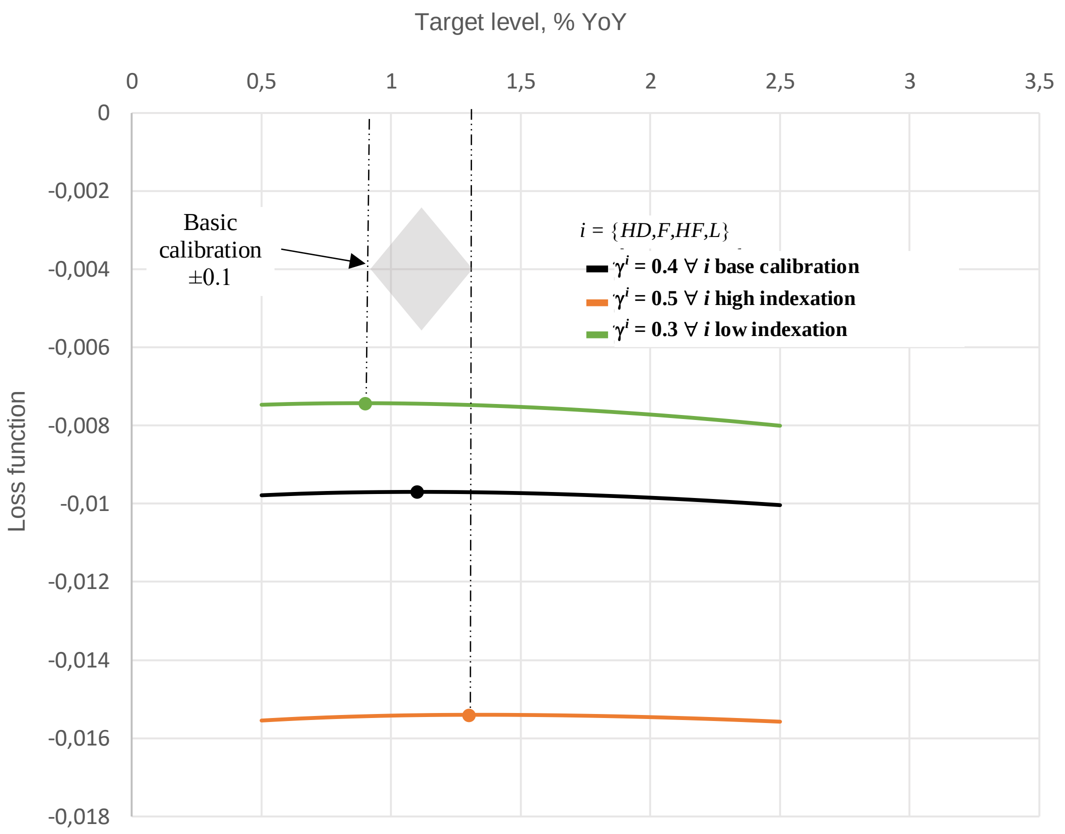
To check the robustness of the results, I consider the loss functions when changing some model parameters, such as the degree of indexing and the Calvo coefficient. These parameters are chosen because estimating their true values from macro data is the most difficult and they tend to be best estimated from micro data. However, for Russia, there are very few studies evaluating these coefficients. In addition, rigidities in the economy directly create a mechanism for the impact of inflation on real variables, and in this regard, it is important to understand how the conclusions of the model about the optimal target change when the coefficients change.

I build loss functions for coefficient deviations on the basis of a base calibration of ±0.1. For coefficients lying in the range $(0,1)$, such a change is at least 10% of the original value.

Alternative calibration - degree of indexation

The first set of coefficients I consider is the degree of indexing. As seen in Chart 4.6, under the assumption that the degree of indexation lies in the range $[0{,}3;\ 0{,}5]$, the optimal inflation rate lies in the range $[0{,}9;\ 1{,}3]$. It is important that for all the studied deviations of the coefficients, the logic predicted by the theory is preserved, and the closer the degree of indexation to perfect is (since it is equal to one), the higher the target level is chosen as optimal. The logic of this relationship is as follows: at a very low degree of indexation, firms practically do not change prices taking into account inflation, the price dispersion increases, having a negative impact on consumers. Thus, in terms of consumer welfare, lower inflation is optimal, despite losses from the ZLB.

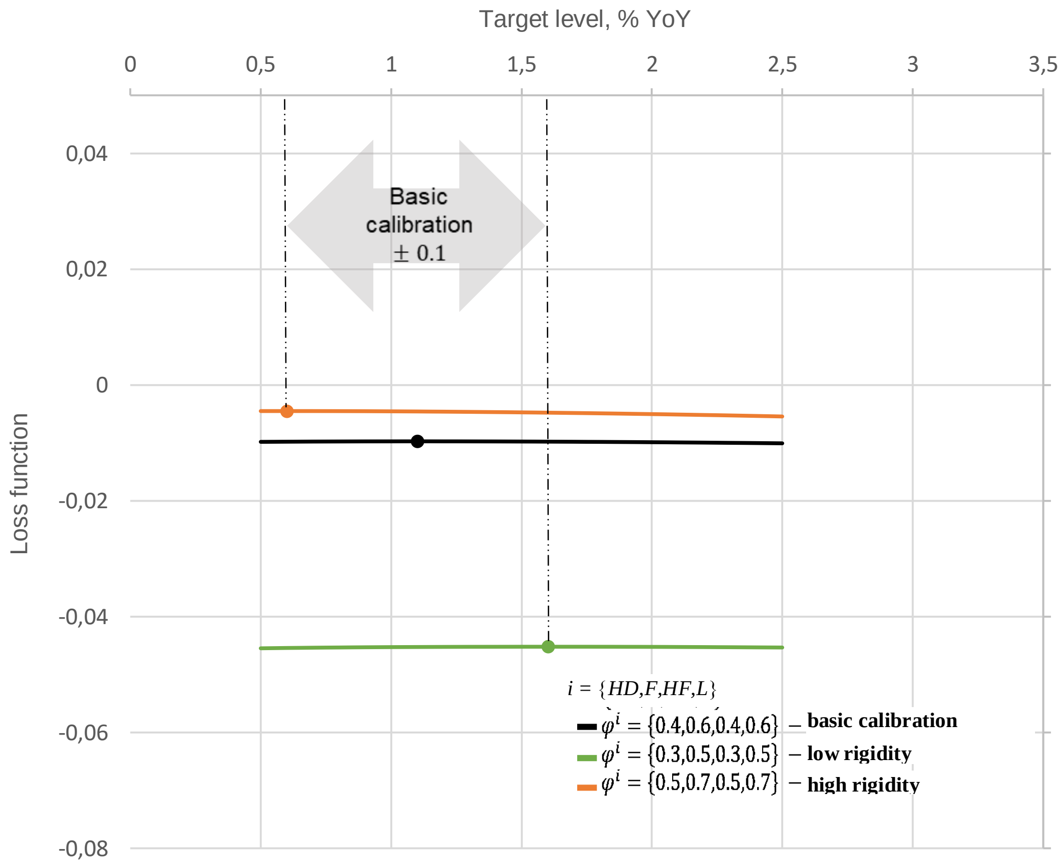
Chart 4.6. Loss function. Alternative calibration - degree of indexation



Alternative calibration - Calvo coefficient

The second set of coefficients I am considering are the Calvo coefficients. As seen in Chart 4.7, assuming that the coefficient deviates from the base calibration by ±0.1, the optimal inflation rate lies in the range $[0{,}6;\ 1{,}6]$. As in the previous paragraph, it is important that for all the studied coefficient deviations, the logic predicted by the theory is preserved - the higher the rigidity, the lower the inflation rate is chosen. The logic of this relationship is similar to the logic of the previous paragraph. If prices are very tight, i.e. the Calvo ratio is high, then price dispersion increases and a lower target becomes preferable.

Chart 4.7. Loss function. Alternative calibration - Calvo coefficient

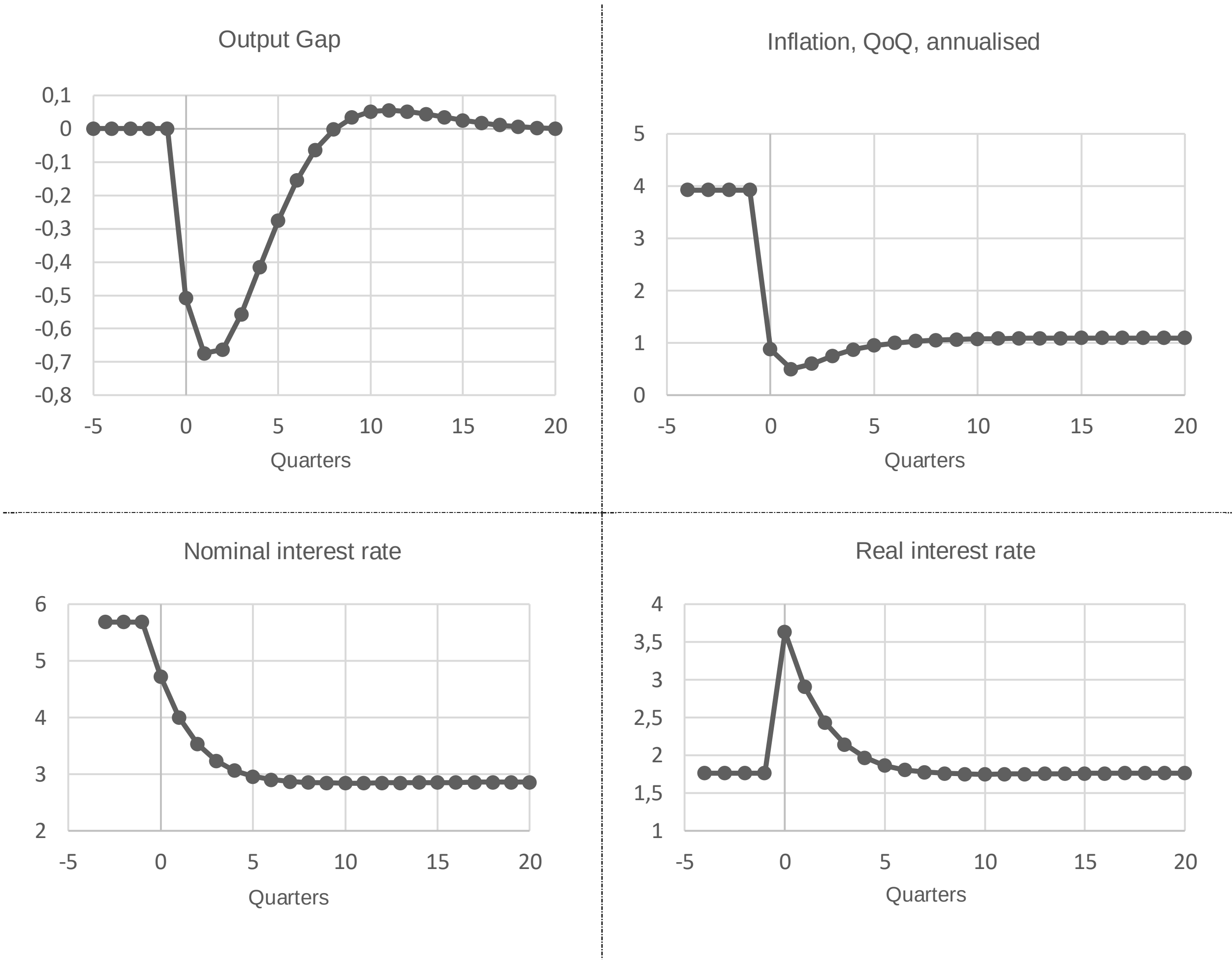


4.6. THE COSTS OF DISINFLATION

The key issue in this section is the cost of moving to a new (lower) target level. First, I examine the dynamics of the transition from the current 4% inflation rate targeted by the Bank of Russia to the 1.1% target that I have found optimal for the Russian economy.

I investigate the adoption of a new target based on the impulse responses of the variables of the model constructed in this paper to the shock of the initial conditions. The resulting paths are shown in Chart 4.8.

Chart 4.8. Impulse responses of variables when moving to a new target



As seen in Chart 4.8, our economy needs about 20 quarters to adjust to a new equilibrium and return output to potential levels. Similar results are obtained by Ascari and Ropele (2012) for the US economy. Meanwhile, the cumulative losses in quarterly output for the Russian economy are about 3%.

As discussed in Ascari and Ropele (2012), the sacrifice ratio SR is commonly used as a loss indicator due to lower target:

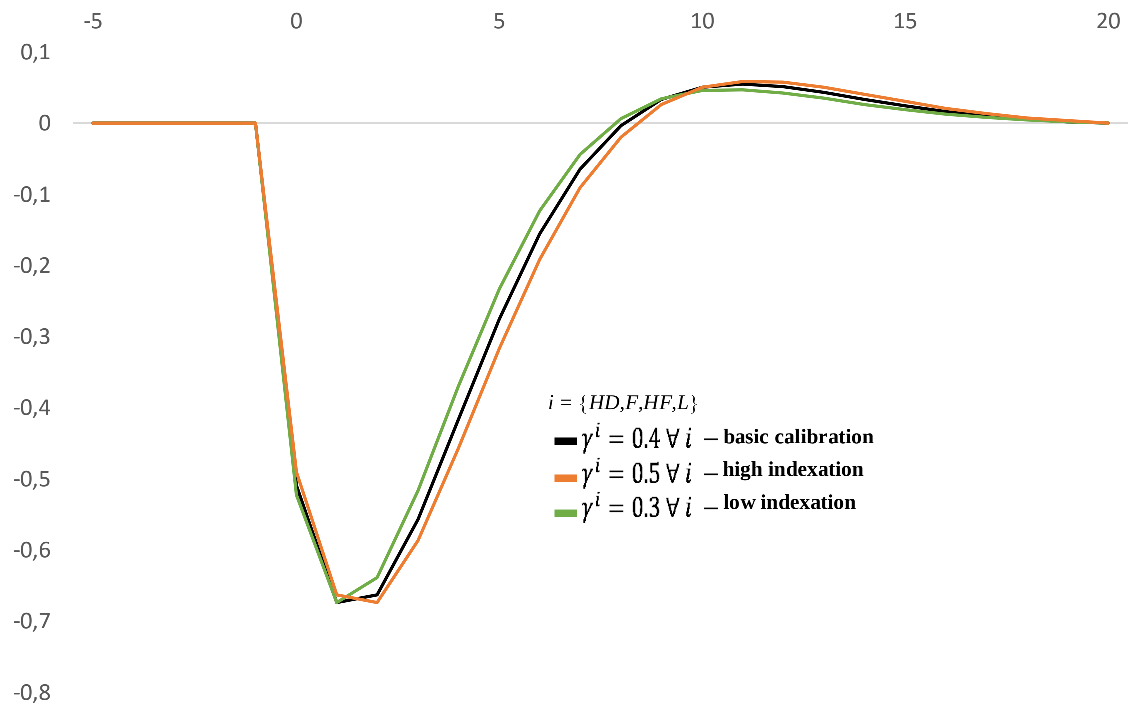
$$SR = -\frac{\sum_{t=0}^{T}(\hat{y}_t)}{\pi_{high}^{*} - \pi_{low}^{*}},$$

where $\hat{y}_t$ is the deviation of output from the equilibrium value, *T* is the number of periods for which the output gap closes.

I calculate this ratio based on the above impulse responses. For my model, this coefficient is 1.03.

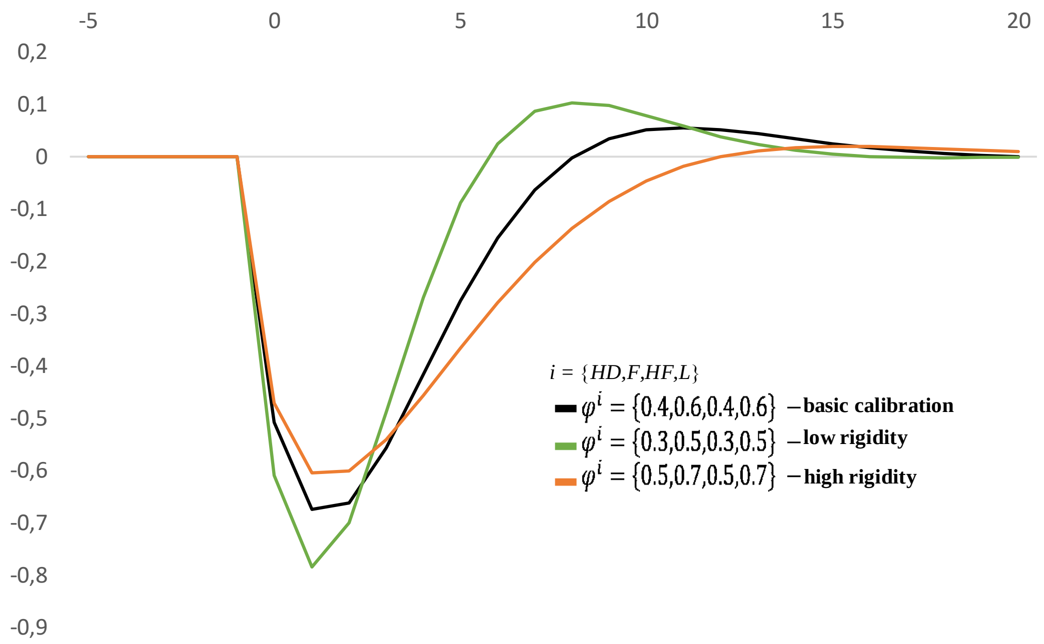
To test the robustness of the results, I calculate the sacrifice ratio for alternative calibrations. I use the same set of parameters as in the previous paragraph. The impulse responses of the output gap for various calibration options are shown in Charts 4.9 and 4.10. I conclude that for different calibrations the sacrifice ratio lies in the range $[0{,}84; 1{,}27]$.

Chart 4.9. Output gap impulse responses for different degrees of indexation



As seen in Chart 4.9, the paths of the return of the output gap to equilibrium are quite close at different degrees of indexation. The sacrifice ratio for varying the degree of indexation lies in the range [0.98; 1.08].

Chart 4.10. Output gap impulse responses for different Calvo coefficients



For deviations in the Calvo ratio, the trajectories of closing the output gap differ somewhat more (Chart 4.10). At lower rigidity, the output falls more strongly, though its recovery is faster; at higher stiffness, the reverse situation is observed. As a result, the sacrifice ratio is in the $[0{,}84; 1{,}27]$ range for this deviation.

As discussed in Ascari and Ropele (2012), the SR ratio typically ranges from 0.5 to 3. Thus, the coefficient 1.03 obtained by me for the basic calibration lies in this range. The range for SR $[0{,}84; 1{,}27]$ obtained for alternative calibrations is also in this range and lies closer to its lower bound.

The SR coefficient obtained for Russia means that the output costs of reducing the target level for the Russian economy are small compared to the economies of the US, euro area or UK, which may be an additional argument in favour of lower target. However, this result should be approached with caution, since some coefficients from this range were calculated not on the basis of structural models, but on the basis of econometric models, which could play a role in the discrepancy between estimates. In addition, as discussed above, the issue of adopted new equilibrium involves many practical issues and requires further study.

5. Conclusions

In this paper, I explore the issue of choosing the optimal inflation target level, taking into account the trade-off between the costs of high inflation and the increasing probability of facing the zero lower bound (ZLB) issued with a lower inflation target. I have concluded that 1.1% for the base model is the optimal target level for the Russian economy. This corresponds to a probability of being at the ZLB of about 11%.

In addition, examining the dependence of the optimal target level on the real interest rate, I have concluded that this dependence is negative, that is, ceteris paribus, a higher real rate allows you to set a lower inflation target level, and each percentage point of the rate increase allows you to lower the target by about 0.5 percentage points.

I also describe the adoption of a new inflation target. To do this, the paper calculates the sacrifice ratio, which is the cumulative decline in output divided by the difference between the old and new inflation targets. For my model, this coefficient is 1.03, which is closer to the lower bound of similar indicators for the US, euro area are and UK, meaning that for Russia, lower target is associated with relatively small GDP costs. It is also worth noting that the sacrifice ratio does not take into account the positive effects that a decrease in the target level entails, for example, a decrease in price volatility. Thus, positive effects can offset some of the losses. On the other hand, the standard neo-Keynesian DSGE model is based on the logic of rational expectations, and this premise is also fulfilled in my model. If this assumption is weakened, the losses from its adoption may be greater, as expectations will not immediately adapt to new conditions, and the central bank may need to reduce (or even raise) the nominal interest rate more slowly. The final benefits/costs of moving to a lower target depend on many factors (including the benefits/costs of which economic agents we are considering: consumers or firms; the period we analyse, whether it is short, medium or long term; types of agent expectations that may be rational, adaptive, learning, and so on) and require further study.

Thus, based on the study, I have come to the conclusion that, first, with the current target of 4%, the probability of facing the ZLB issue for the Russian economy is quite low and amounts to about 1%. This is consistent with historical data, since the Russian economy has never actually faced such problem. Second, the optimal inflation target for the Russian economy in terms of consumer welfare is 1.1%. Moreover, the target level negatively depends on the real interest rate. The range of the real rate of 1% to 3% corresponds to the optimal level of 0.75% to 1.5%. In addition, the optimal target level depends on model parameters such as Calvo rigidity and indexation degree. The optimal target value amid the above uncertainty lies in the range from 0.6% to 1.6%, which is lower than the current target of the Bank of Russia.

Loss of output during the adoption of a new level of the target, calculated on the basis of the sacrifice ratio, is closer to the lower bound of the range calculated for the economies of the US, Europe and the UK, which is an additional argument in favour of lowering the inflation target.

When interpreting the results, it should be noted that if there is some consensus in the academic literature regarding the very fact of society's losses from high inflation and the mechanism according to which this occurs, then for the costs of low inflation and the mechanism(s) for spreading these costs, there are quite a lot of opinions.

First, instead of the zero lower bound issue, the Effective Lower Bound (ELB) issue can be considered. The presence of ELB in the economy can lead to the fact that when a certain level (greater than zero) of the key rate is reached, its further reduction will not lead to a stimulating effect, and in some cases may even have a deterrent effect[7]. Thus, some positive ELB in the economy may require to set an inflation target higher than in the case when the ZLB issue is considered. Although, in line with the considerations described above, ELB could theoretically be relevant for the Russian economy (as well as for other emerging market countries). However, there are difficulties that prevent it from being used instead of ZLB as a lower inflation target. First of all, ELB (unlike ZLB) is probably not a constant. This indicator may depend on the development of financial institutions, the risks that have developed in the economy, the expectations of economic agents, their confidence in the policy being pursued, the historical level of rates in the economy (for example, if consumers are used to low rates, then ELB may decrease), and so on. In addition, the chosen target level itself can influence the ELB value. Today, for the Russian economy, not only are there no estimates of factors affecting ELB, but even point estimates of the ELB value. Thus, its use as a lower bound on the target value requires additional studies of this mechanism.

Second, my model assumes that non-oil output is homogeneous, that is, it does not take into account the influence of relative prices on the choice of the optimal target level. Including several sectors in the model is likely to cause additional costs from low inflation. However, such an extension of the model significantly complicates its structure and may complicate the interpretation of the results.

There are several other mechanisms to model the costs of low inflation. For example, Abbritti et al. (2021) include labour market friction, endogenous productivity, and downward wage rigidity (DWR) in the neo-Keynesian DSGE model. It leads to asymmetry, which creates the prerequisites for setting a higher target than that which is recognised as optimal in models without such prerequisites.

Dierks (2017) shows that more detailed modeling of the financial sector (and related non-linearities) than is accepted in standard neo-Keynesian DSGE models leads to a higher optimal target level. A detailed list of papers focused on the optimal target level is given in Dierks (2019).

In general, the optimal level of the inflation target is likely to remain a key topic of economic research in the foreseeable future. This is due to its practical significance for inflation targeting central banks as well as the poor development of the topic, which may suggest further research.

It is also worth noting that there are several practical aspects that may be relevant when moving to a new target level outside the scope of this study. For example, I do not focus on the following questions: the moment of adoption of a new target, what should be the economic conditions and the moment of the economic cycle, whether its adoption should be carried out at once or in several stages, how the central bank should conduct an information policy when adopting a new target.

Finally, I deliberately do not include capital restrictions and other changes that have been taking place in the Russian economy since the end of February 2022 in the model. This is due to the fact that this paper focuses on the choice of the optimal target, which is a fundamental matter, and such choice must be made based on the long-term equilibrium structure of the economy. As with any model of the DSGE type, the conclusions of my model depend on its structure. Now, in a period of significant restructuring of economic ties, the equilibrium that the economy of Russia and other world economies will come to remains uncertain. And when the dust settles and it becomes clear which of the current changes will remain temporary shocks and which ones will turn into a new reality, we will inevitably return to this subject.

[7] This can happen if, in the wake of a rate cut, the following financial stability risks occur: following the dollarisation of deposits and the outflow of capital, the national currency weakens, the probability of defaults on foreign exchange obligations of individuals and legal entities goes up, the burden on the capital of financial institutions increases, and, as a result, the number of loan offers decreases.

REFERENCES

Abbritti, Mirko, Consolo, Agostino and Weber, Sebastian. 2021. *Endogenous Growth, Downward Wage Rigidity and Optimal Inflation*, Working Paper Series 2635, European Central Bank.

Andrade P., Jordi Galí, Hervé Le Bihan, Julien Matheron. *The Optimal Inflation Target and the Natural Rate of Interest.* 2019.

Ascari, G. and Ropele, T. *Sacrifice Ratio in a Medium-Scale New Keynesian Model.* s.l. : Journal of Money, Credit and Banking, Vol. 44, No. 2-3 (March-April 2012), 2012.

Ball, L. *What Determines the Sacrifice Ratio?* s.l. : Monetary Policy, pp. 155–82. Chicago : University of Chicago Press, 1994b.

Cecchetti, Stephen G., and Robert W. Rich. *Structural Estimates of the U.S. Sacrifice Ratio.* s.l. : Journal of Business and Economic Statistics, 19, 416–27, 2001.

Céspedes L.F., García-Cicco J. , Saravia D. *Monetary Policy at the Zero lower bound: the chilean experience.* 2014.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. *Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.* s.l. : Journal of Political Economy, 113, 1–45, 2005.

Chung H., Laforte D., Reifschneider D., Willams J. C. *Have We Underestimated the Likelihood and Severity of Zero Lower Bound Events?* s.l. : Journal of Money, Credit and Banking, 44, 47–82, 2012.

Coibion, Gorodnichenko, & Wieland (2012) и Andrade, Gali, Bihan, & Matheron. *The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?* 2019.

Durand, Jean-Jacques, Maryline Huchet-Bourdon, and Julien Licheron. *Sacrifice Ratio Dispersion within the Euro Zone: What Can Be Learned about Implementing a Single Monetary Policy?* s.l. : International Review of Applied Economics, 22, 601–21, 2008.

Gordon, Robert J., and Stephen King. *The Output Cost of Disinflation in Traditional and Vector Autoregressive Models.* s.l. : Brookings Papers on Economic Activity, 1, 205–42, 1982.

IMF. *Country report.* 2021.

International Monetary Fund, Russian Federation. *Staff Report for the 2019 Article IV Consultation.* 2019.

Isakov A., Latypov R. *The Ibsen Manoeuvre: Yet Another R* Estimate.* s.l. : VTB Capital Research Alert, (Very) Technical Brief series, 2019.

Kiley M., Roberts J. M. *Monetary Policy in a Low Interest RateWorld.* s.l. : Brookings Papers on Economic Activity, 48, 317–396, 2017.

Lawrence J. Christiano, Martin S. Eichenbaum, and Mathias Trabandt. *On DSGE Models.* 2018.

Leif B., Husebo T. A., Maih J., Olsen K., Ostnor M. *Finding NEMO: Documentation of the Norwegian economy model.* 2005.

Lindé, J. *DSGE models: still usefull in policy analysis?* 2018.

Medina, Soto. *The chilean business cycles through the lens of a stochastic general equilibrium model.* 2007.

Mikael Apel, Hanna Armelius and Carl Andreas. *The level of the inflation target. 2017.*

S., Bernanke B., T., Kiley M. and J.M., Roberts. *Monetary Policy Strategies for a Low-Rate.* s.l. : AEA Papers and Proceedings, 109, 421–426, 2019.

Smets, Frank, and Raf Wouters. *An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area.* s.l. : Journal of the European Economic Association, 1, 1123–75, 2003.

—.*Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach.* s.l. : American Economic Review, 97, 586–606, 2007.

Woodford. *Interest and Prices.* s.l. : Princeton, NJ: Princeton University Press, 2003.

WorldBank. *Russian economic report.* 2021.

Андреев М.Ю., Полбин А.В. *Optimal monetary policy rules for a resource-rich economy and Zero Lower Bound. 2021*

Дробышевский С., Трунин П., Синельникова-Мурылева Е., Макеева Н., Гребенкина А. *Optimal inflation in Russia: theory and practice.* s.l. : Экономическая политика Т.15 №4 с.8-29, 2020.

Крепцев Д., Поршаков А., Селезнев С., Синяков А. Equilibrium interest rate: estimates for Russia. 2016.

ЦБРФ. *Monetary Policy Report No. 2(38). May 2022.*

—. *Monetary Policy Report. July 2022.*

—. *Monetary Policy Guidelines for 2022–2024. 2021.*

ЦБРФ, ДИП. *Drivers of price inertia: survey evidence. 2017.*

APPENDICES

I OPTIMISATION PROBLEMS

Producers of intermediate domestic goods

Intermediate goods are produced by firms under monopolistic competition in accordance with the production function:

$$Y_t^{HD}(f) = Z_t L_t(f),$$

where Z_t is the stochastic performance trend and

$$Z_t = Z_{t-1} e^{\zeta_t^a}.$$

Intermediate goods are produced with nominal à la Calvo price rigidities. This means that firms are ϕ^i probability to face an inability to change prices, $i \in \{HD, HF\}$, HD are domestic goods sold domestically, HF is domestic goods sold abroad.

If the firm cannot optimise its price in period *t*, then it sets it according to the following rule:

$$P_t^i(f) = \left(\Pi_{t-1}^i\right)^{\gamma_i} P_{t-1}^i(f),$$

$$where\ i \in \{HD, HF\}, \Pi_t^i \equiv \frac{P_t^i}{P_{t-1}^i},$$

$$\Pi^i - steady - state\ value\ of\ \ inflation\ , \gamma_i - degree\ of\ indexation\ \text{и}\ 0 \leq \ \gamma_i < 1.$$

If the firm can revise its price for domestic goods sold domestically in period *t*, then it chooses it based on the profit maximisation condition:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi^{HD}) \Lambda_{t+s} \left(\frac{V_{t,t+s}^{HD} P_t^{HD*}(f)}{P_{t+s}} Y_{t,t+s}^{HD} - \frac{W_{t+s}}{P_{t+s}} \frac{Y_{t,t+s}^{HD}}{Z_{t,t+s}} \right),$$

where Λ_t – is the marginal utility of consumers and $Y_{t,t+s}^i(f)$ is the demand for the products of a monopolist who fixed the price in period *t*, in period *t* + *s*, which has the form:

$$Y_{t,t+s}^i(f) = \left(\frac{V_{t,t+s}^i P_t^{i*}}{P_{t+s}} \right)^{-\epsilon_i} Y_{t+s}^i,$$

where V_t^i is the cumulative effect of price indexation on inflation in previous periods:

$$V_{t,t+s}^i = \prod_{j=t}^{t+s-1} \left(\Pi_j\right)^{\gamma_i}.$$

The first order condition for this problem is:

$$\Sigma_{s=0}^{\infty} (\beta \phi^{HD}) \Lambda_{t+s} \left(\frac{\left(V_{t,T}^{HD} P_t^{HD*}(f)\right)^{1-\theta^{HD}}}{P_{t+s}} \left(\frac{1}{P_{t+s}^{HD}}\right)^{-\theta^{HD}} Y_{t+s}^{HD} - \frac{\theta^{HD}}{\theta^{HD}-1} \frac{W_{t+s}}{P_{t+s}} \left(\frac{V_{t,t+s}^{HD} P_t^{HD*}(f)}{P_{t+s}^{HD}}\right)^{-\theta^{HD}} \frac{Y_{t+s}^{HD}}{Z_{t+s}} \right) = 0.$$

Transforming, we get:

$$\frac{P_t^{HD*}(f)}{P_t} \frac{P_t}{P_t^{HD}} = \frac{\theta^{HD}}{\theta^{HD}-1} \frac{K_t^{HD}}{F_t^{HD}},$$

where

$$K_t^{HD} = \Lambda_{z,t}\frac{W_{z,t}}{P_t}Y_{z,t}^{HD} + \beta\phi^{HD}\mathbb{E}_t\left(\frac{(\Pi_t^{HD})^{\gamma^{HD}}}{\Pi_{t+1}^{HD}}\right)^{-\theta^{HD}} K_{t+1}^{HD},$$

$$F_t^{HD} = \Lambda_{z,t}Y_{z,t}^{HD} + \beta\phi^{HD}\mathbb{E}_t\left(\frac{1}{\Pi_{t+1}^{HD}}\right)^{-\theta^{HD}}\frac{1}{\Pi_{t+1}}\left((\Pi_t^{HD})^{\gamma^{HD}}\right)^{1-\theta^{HD}}\frac{P_t^{HD}}{P_t}F_{t+1}^{HD},$$

$$\Pi_t^{HD} \equiv \frac{P_t^{HD}}{P_{t-1}^{HD}}, \Pi_t \equiv \frac{P_t}{P_{t-1}}.$$

In addition, by transforming

$$P_t^{HD^{1-\theta^{HD}}} = \int_0^1 P_t^{HD}(f)^{1-\theta^{HD}}df = (1-\phi^{HD}) * P_t^{HD*^{1-\theta^{HD}}} + \int_0^1 \left(\Pi_{t-1}^{HD^{\gamma^{HD}}}P_{t-1}^{HD}(f)\right)^{1-\theta^{HD}} df,$$

we see that

$$\left(\frac{P_t^{HD*}}{P_t}\right)^{1-\theta^{HD}}\left(\frac{P_t}{P_t^{HD}}\right)^{1-\theta^{HD}} = \frac{1-\phi^{HD}\left(\frac{\Pi_{t-1}^{HD^{\gamma^{HD}}}}{\Pi_t^{HD}}\right)^{1-\theta^{HD}}}{1-\phi^{HD}}.$$

Similarly, the price is chosen for domestic goods sold abroad:

$$\mathbb{E}_t\sum_{s=0}^{\infty}(\beta\phi^{HF})\,\Lambda_{t+s}\left(\frac{V_{t,t+s}^{HF}P_t^{HF*}(f)}{P_{t+s}}Y_{t+s}^{HF} - \frac{1}{\mathcal{E}_{t+s}}\frac{W_{t+s}}{P_{t+s}}\frac{Y_{t+s}^{HF}}{Z_{t+s}}\right).$$

The first order condition for this problem is:

$$\sum_{s=0}^{\infty}(\beta\phi^{HF})\,\Lambda_{t+s}\left(\frac{\left(V_{t,T}^{HD}P_t^{HF*}(f)\right)^{1-\theta^{HF}}}{P_{t+s}}\left(\frac{1}{P_{t+s}^{HF}}\right)^{-\theta^{HD}}Y_{t+s}^{HF} - \frac{\theta^{HF}}{\theta^{HF}-1}\frac{1}{\mathcal{E}_{t+s}}\frac{W_{t+s}}{P_{t+s}}\left(\frac{V_{t,t+s}^{HF}P_t^{HF*}(f)}{P_{t+s}}\right)^{-\theta^{HF}}\frac{Y_{t+s}^{HF}}{Z_{t+s}}\right) = 0.$$

Transforming, we get

$$\frac{P_t^{HF*}(f)}{P_t}\frac{P_t}{P_t^{HF}} = \frac{\theta^{HF}}{\theta^{HF}-1}\frac{K_t^{HF}}{F_t^{HF}},$$

where

$$K_t^{HF} = \Lambda_{z,t}\frac{1}{\mathcal{E}_{t+s}}\frac{W_{z,t}}{P_t}Y_{z,t}^{HF} + \beta\phi^{F}\mathbb{E}_t\left(\frac{(\Pi_t^{HF})^{\gamma^{HF}}}{\Pi_{t+1}^{HF}}\right)^{-\theta^{HF}} K_{t+1}^{HF},$$

$$F_t^{HF} = \Lambda_{z,t}Y_{z,t}^{HF} + \beta\phi^{HF}\mathbb{E}_t\left(\frac{1}{\Pi_{t+1}^{HF}}\right)^{-\theta^{HF}}\frac{P_t^{HF}}{\mathrm{P}_t}\left((\Pi_t^{HF})^{\gamma^{HF}}\right)^{1-\theta^{HF}}F_{t+1}^{HF},$$

$$where\ \Pi_t^{HF} \equiv \frac{P_t^{HF}}{P_{t-1}^{HF}}.$$

Transforming,

$$P_t^{HF^{1-\theta^{HF}}} = \int_0^1 P_t^{HF}(f)^{1-\theta^{HF}}df = (1-\phi^{HF}) * P_t^{HF*^{1-\theta^{HF}}} + \int_0^1 \left(\Pi_{t-1}^{HF^{\gamma^{HF}}}P_{t-1}^{HF}(f)\right)^{1-\theta^{HF}} df,$$

we see that

$$\left(\frac{P_t^{HF*}}{P_t}\right)^{1-\theta^{HF}} \left(\frac{P_t}{P_t^{HF}}\right)^{1-\theta^{HF}} = \frac{1-\phi^{HF}\left(\frac{\Pi_{t-1}^{HF}{}^{\gamma^{HF}}}{\Pi_t^{HF}}\right)^{1-\theta^{HF}}}{1-\phi^{HF}}.$$

Producers of intermediate imported goods

Intermediate goods are produced by firms under monopolistic competition from foreign goods.

Just like domestic goods, foreign goods are produced with nominal à la Calvo price rigidities. This means that firms are ϕ^F probability to be unable to change prices.

If the firm can revise its price for domestic goods sold domestically in period *t*, then it chooses it based on the profit maximisation condition:

$$\mathbb{E}\sum_{s=0}^{\infty}(\beta\phi^F)\,\Lambda_{t+s}\left(\frac{V_{t,t+s}^F P_t^{F*}}{P_{t+s}}Y_{t,t+s}^F - \mathcal{E}_{t+s}\frac{P_t^{For}}{P_{t+s}}\frac{Y_{t,t+s}^F}{Z_{t,t+s}}\right),$$

where P_t^{For} is the price of intermediate goods abroad, P_t^{F*} is the effective price of importers.

If the firm cannot optimise its price in period *t*, then it sets it according to the following rule:

$$P_t^F(f) = (\Pi_{t-1}^F)^{\gamma_F} P_{t-1}^F(f),$$

$$\Pi_t^F \equiv \frac{P_t^F}{P_{t-1}^F}, \Pi^F - inflation\ stady\ state, 0 \le \gamma_F < 1.$$

As in the case of producers of domestic goods, price indexation for importers is imperfect.

The first order condition for this problem is:

$$\sum_{s=0}^{\infty}(\beta\phi^F)\,\Lambda_{t+s}\left(\frac{\left(V_{t,T}^F P_t^{F*}(f)\right)^{1-\epsilon^F}}{P_{t+s}}\left(\frac{1}{P_{t+s}^F}\right)^{-\epsilon^F} Y_{t+s}^F - \frac{\epsilon^F}{\epsilon^F-1}\mathcal{E}_{t+s}\frac{P_t^{For}}{P_{t+s}}\left(\frac{V_{t,t+s}^F P_t^{F*}(f)}{P_{t+s}}\right)^{-\epsilon^F}\frac{Y_{t+s}^F}{Z_{t+s}}\right) = 0.$$

Transforming, we get

$$\frac{P_t^{F*}(f)}{P_t}\frac{P_t}{P_t^F} = \frac{\epsilon^F}{\epsilon^F - 1}\frac{K_t^F}{F_t^F},$$

where

$$F_t^F = \Lambda_{z,t}Y_{z,t}^F + \beta\phi^F\mathbb{E}_t\left(\frac{(\Pi_t^F)^{\gamma^F}}{\Pi_{t+1}^F}\right)^{-\epsilon^F} K_{t+1}^F,$$

$$K_t^F = \Lambda_{z,t}\mathcal{E}_t\frac{P_t^{For}}{P_t}Y_{z,t}^F + \beta\phi^F\mathbb{E}_t\left(\frac{1}{\Pi_{t+1}^F}\right)^{-\epsilon^F}\frac{P_t^F}{P_t}\left((\Pi_t^F)^{\gamma^F}\right)^{1-\epsilon^F} F_{t+1}^F,$$

$$\Pi_t^F \equiv \frac{P_t^F}{P_{t-1}^F}.$$

Transforming,

$$P_t^{F\,1-\theta^F} = \int_0^1 P_t^F(f)^{1-\epsilon^F}df = (1-\phi^F) * P_t^{F*\,1-\epsilon^F} + \int_0^1\left(\Pi_{t-1}^F{}^{\gamma^F}P_{t-1}^F(f)\right)^{1-\epsilon^F} df,$$

we will ascertain that

$$\left(\frac{P_t^{F*}}{P_t}\right)^{1-\epsilon^F}\left(\frac{P_t}{P_t^F}\right)^{1-\epsilon^F}=\frac{1-\phi^F\left(\frac{\Pi_{t-1}^F{}^{\gamma^F}}{\Pi_t^F}\right)^{1-\epsilon^F}}{1-\phi^F}.$$

II LOSS FUNCTION

First of all, let us derive some definitions.

Let us define the deviation from the equilibrium state in the model with rigid prices as:

$$\frac{X_t-X}{X}=\widehat{x_t}+\frac{1}{2}\widehat{x_t^2}+\mathcal{O}(\|\zeta\|^3).$$

Let us define the deviation from the equilibrium state in the model with flexible prices as:

$$\frac{X_t-X^n}{X^n}=\widetilde{x_t}+\frac{1}{2}\widetilde{x_t^2}+\mathcal{O}(\|\zeta\|^3).$$

Two ratios for further use according to the Taylor expansion:

$$g(x)=g(x^*)+\frac{g'(x^*)}{1!}(x-x^*)+\frac{g''(x^*)}{2!}(x-x^*)^2+\cdots. \tag{II.1}$$

Let's take the mathematical expectation of the left and right parts:

$$E\big(g(x)\big)=E\big(g(x^*)\big)+E\left(\frac{g'(x^*)}{1!}(x-x^*)\right)+E\left(\frac{g''(x^*)}{2!}(x-x^*)^2\right)+\mathcal{O}(\|x\|^3).$$

Given that $x^*=E(x)$ and $E\left(\big(x-E(x)\big)^2\right)=V(x)$ by definition (where $V(x)$ is the variance of a random variable), we obtain that

$$E\big(g(x)\big)=g(E(x))+\frac{1}{2}g''\big(E(x)\big)V(x)+\mathcal{O}(\|x\|^3). \tag{II.2}$$

Now take the variance from both sides of the equation (II.1):

$$V\big(g(x)\big)=V\big(g(x^*)\big)+V\left(\frac{g'(x^*)}{1!}(x-x^*)\right)+V\left(\frac{g''(x^*)}{2!}(x-x^*)^2\right)+\mathcal{O}(\|x\|^3).$$

Transforming, we get

$$V\big(g(x)\big)=\big(g'(E(x))\big)^2V(x)+\mathcal{O}(\|x\|^3). \tag{II.3}$$

Now let's go directly to the loss function.

The value of the welfare function is used as a criterion for choosing the optimal inflation target level. This function is an approximation (Taylor expansion) of the second order of the consumer's utility function and has the form:

$$\log(C_t - \eta * C_{t-1} * e^{-\zeta_{z,t}}) - \frac{1}{1+\sigma_L} * \int_0^1 (N_t(h))^{1+\sigma_L} dh =$$

Consumption

$$\frac{1}{1-\eta} * \Bigg[\frac{C_t - C^n}{C^n} - \eta * \frac{C_{t-1} - C^n}{C^n} - \frac{1}{2} * \frac{1}{1-\eta} * \left(\frac{C_t - C^n}{C^n}\right)^2 + \frac{\eta}{1-\eta} * \left(\frac{C_t - C^n}{C^n}\right) * \left(\frac{C_{t-1} - C^n}{C^n}\right) - \frac{1}{2} * \frac{\eta^2}{1-\eta} * \left(\frac{C_{t-1} - C^n}{C^n}\right)^2 + \zeta_{c,t} * \frac{C_t - C^n}{C^n} - \eta * \zeta_{c,t} * \frac{C_{t-1} - C^n}{C^n} - \frac{\eta}{1-\eta} * \zeta_{z,t} * \frac{C_t - C^n}{C^n} + \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\frac{C_{t-1} - C^n}{C^n}\right)\Bigg] \tag{II.4}$$

Labour

$$-\int_0^1 \left((N^n)^{1+\sigma_L} * \frac{N_t(h) - N^n}{N^n} + \frac{1}{2} * \sigma_L * (N^n)^{1+\sigma_L} * \left(\frac{N_t(h) - N^n}{N^n}\right)^2\right) dh \tag{II.5}$$

where C consumption, N labour, C^n natural level of consumption,

N^n natural level of labour, h labour type, η consumption habits,

ζ_c consumption shock, ζ_z producivity shock, σ_L Frisch elasticity of labour supply.

Let's first transform the consumption part. Given that

$$\frac{C_t - C^n}{C^n} = \frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1,$$

let us rewrite (9.2.1) as

$$\frac{1}{1-\eta} * \left[\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1 - \eta * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \frac{1}{2} * \frac{1}{1-\eta} * \left(\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right)^2 + \frac{\eta}{1-\eta} * \left(\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \frac{1}{2} * \frac{\eta^2}{1-\eta} * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right)^2 + \zeta_{c,t} * \left(\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \eta * \zeta_{c,t} * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) + \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right)\right].$$

Using

$$\frac{C_t - C}{C} = \hat{c}_t + \frac{1}{2}\hat{c}_t^2 + \mathcal{O}(\|\zeta\|^3),$$

let us rewrite as

$$\frac{1}{1-\eta} * \Bigg[\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n} - \eta * \left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n} - \frac{1}{2} * \frac{1}{1-\eta} * \left(\hat{c}_t^2\left(\frac{C}{C^n}\right)^2 + 2\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n} * \left(\frac{C}{C^n} - 1\right)\right) + \frac{\eta}{1-\eta} * \left(\frac{C}{C^n}^2 \hat{c}_t\hat{c}_{t-1} + \left(\frac{C}{C^n} - 1\right)\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) + \left(\frac{C}{C^n} - 1\right)\left(\hat{c}_{t-1} + \frac{1}{2}\hat{c}_{t-1}^2\right)\right) - \frac{1}{2} * \frac{\eta^2}{1-\eta} * \left(\hat{c}_{t-1}^2\left(\frac{C}{C^n}\right)^2 + 2\left(\hat{c}_{t-1} + \frac{1}{2}\hat{c}_{t-1}^2\right)\frac{C}{C^n}\left(\frac{C}{C^n} - 1\right)\right) + \zeta_{c,t} * \left(\hat{c}_t\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \eta * \zeta_{c,t} * \left(\hat{c}_{t-1}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\hat{c}_t\frac{C}{C^n} + \frac{C}{C^n} - 1\right) + \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\hat{c}_{t-1}\frac{C}{C^n} + \frac{C}{C^n} - 1\right)\Bigg] + t.i.p. + \mathcal{O}(\|\zeta\|^3), \tag{II.6}$$

where t.i.p. means terms independent of policy.

Now let's transform the labour part.

Given that

$$\frac{N_t - N^n}{N^n} = \frac{N_t - N}{N}\frac{N}{N^n} + \frac{N}{N^n} - 1,$$

let us transform (9.2.2):

$$\int_0^1 \left((N^n)^{1+\sigma_L} * \left(\frac{N_t(h) - N}{N}\frac{N}{N^n} + \frac{N}{N^n} - 1\right) + \frac{1}{2} * \sigma_L * (N^n)^{1+\sigma_L} * \left(\frac{N_t(h) - N}{N}\frac{N}{N^n} + \frac{N}{N^n} - 1\right)^2\right) dh.$$

Using

$$\frac{N_t - N}{N} = \hat{n}_t + \frac{1}{2}\hat{n}_t^2 + \mathcal{O}(\|\zeta\|^3)$$

the integrand has the form:

$$\frac{1}{1+\sigma_L}(N_t(h))^{1+\sigma_L} = (N^n)^{1+\sigma_L} * \left(\hat{n}_t\frac{N}{N^n}\left(1 + \sigma_L\left(\frac{N}{N^n} - 1\right)\right) + \hat{n}_t^2\frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\right) + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).$$

Integrating by type of labour:

$$\frac{1}{1+\sigma_L}\int_0^1 (N_t(h))^{1+\sigma_L} dh = (N^n)^{1+\sigma_L}\left[\mathbb{E}_h(\hat{n}_t(h))\frac{N}{N^n}\left(1 + \sigma_L\left(\frac{N}{N^n} - 1\right)\right) + \mathbb{E}_h(\hat{n}_t^2(h))\frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\right] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).$$

Since the variance can be written as

$$\mathbb{V}_h(\hat{n}_t(h)) = \mathbb{E}_h(\hat{n}_t(h)^2) - \mathbb{E}_h(\hat{n}_t(h))^2,$$

then

$$\frac{1}{1+\sigma_L}\int_0^1 \left(N_t(h)\right)^{1+\sigma_L} dh$$
$$= (N^n)^{1+\sigma_L}\Bigg[\mathbb{E}_h(\hat{n}_t(h))\frac{N}{N^n}\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)+\frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)$$
$$* (\mathbb{V}_h(\hat{n}_t(h))+\mathbb{E}_h\hat{n}_t(h))^2)\Bigg] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3). \tag{II.7}$$

Aggregation of labour and output

Take the logarithm (7.5.1) and get:

$$\frac{\epsilon_L - 1}{\epsilon_L}\hat{n}_t = \log\left(\int_0^1 \left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}} dh\right).$$

Using (II.2), we transform as

$$\hat{n}_t = \mathbb{E}_h\left(\hat{n}_t(h)\right) + \frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\mathbb{E}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right)^{-2}\mathbb{V}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right) + \mathcal{O}(\|\zeta\|^3). \tag{II.8}$$

Using the logarithm property:

$$\mathbb{V}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right) = \mathbb{V}_h\left(\exp\left((1-\epsilon_L^{-1})\log\left(\frac{N_t(h)}{N^n}\right)\right)\right).$$

Then, using (II.3), we transform it as

$$\mathbb{V}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right) = (1-\epsilon_L^{-1})^2\exp\left((1-\epsilon_L^{-1})\mathbb{E}_h\left(\hat{n}_t(h)\right)\right)^2\mathbb{V}_h\left(\hat{n}_t(h)\right) + \mathcal{O}(\|\zeta\|^3).$$

Using the logarithm property again

$$\mathbb{E}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right) = \mathbb{E}_h\left(\exp\left((1-\epsilon_L^{-1})\hat{n}_t(h)\right)\right)$$

and using (II.3), we transform it as

$$\mathbb{E}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right) = \exp\left((1-\epsilon_L^{-1})\mathbb{E}_h\left(\hat{n}_t(h)\right)\right)\left(1+\frac{1}{2}(1-\epsilon_L^{-1})^2\mathbb{V}_h\left(\hat{n}_t(h)\right)\right) + \mathcal{O}(\|\zeta\|^3).$$

Then (II.8) takes the form:

$$\hat{n}_t = \mathbb{E}_h\big(\hat{n}_t(h)\big) + \frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\left(\exp\Big((1-\epsilon_L^{-1})\mathbb{E}_h\big(\hat{n}_t(h)\big)\Big)\left(1 + \frac{1}{2}(1-\epsilon_L^{-1})^2\mathbb{V}_h\big(\hat{n}_t(h)\big)\right)\right)^{-2} (1 - \epsilon_L^{-1})^2 \exp\Big((1-\epsilon_L^{-1})\mathbb{E}_h\big(\hat{n}_t(h)\big)\Big)^2 \mathbb{V}_h\big(\hat{n}_t(h)\big) + \mathcal{O}(\|\zeta\|^3)$$
$$= \mathbb{E}_h\big(\hat{n}_t(h)\big) + \frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\left(\left(1 + \frac{1}{2}(1-\epsilon_L^{-1})^2\mathbb{V}_h\big(\hat{n}_t(h)\big)\right)\right)^{-2} (1-\epsilon_L^{-1})^2\mathbb{V}_h\big(\hat{n}_t(h)\big) + \mathcal{O}(\|\zeta\|^3)$$
$$= \mathbb{E}_h\big(\hat{n}_t(h)\big) + \frac{\frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}(1-\epsilon_L^{-1})^2\mathbb{V}_h\big(\hat{n}_t(h)\big)}{\left(\left(1 + \frac{1}{2}(1-\epsilon_L^{-1})^2\mathbb{V}_h\big(\hat{n}_t(h)\big)\right)\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let us define $\mathbb{V}_h\big(\tilde{n}_t(h)\big) \equiv \Delta_{h,t}$, then (II.8):

$$\hat{n}_t = \mathbb{E}_h\big(\hat{n}_t(h)\big) + \frac{\frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\Delta_{h,t}}{\left(\left(1 + \frac{1}{2}(1-\epsilon_L^{-1})^2\Delta_{h,t}\right)\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let's expand in a Taylor series:

$$\frac{\frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\Delta_{h,t}}{\left(\left(1 + \frac{1}{2}(1-\epsilon_L^{-1})^2\Delta_{h,t}\right)\right)^2} = \frac{\frac{1-\epsilon_L{}^{-1}}{2}\Delta_n}{\left(1 + \frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n\right)^2} + $$
$$+ \frac{1-\epsilon_L{}^{-1}}{2}\frac{1 - \frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n}{\left(1 + \frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n\right)^3}(\Delta_{h,t} - \Delta_n).$$

Then we finally get

$$\begin{aligned}\hat{n}_t = \mathbb{E}_h\big(\hat{n}_t(h)\big) &+ \frac{\frac{1-\epsilon_L{}^{-1}}{2}\Delta_n}{\left(1 + \frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n\right)^2} + \\ &+ \frac{1-\epsilon_L{}^{-1}}{2}\frac{1 - \frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n}{\left(1 + \frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n\right)^3}(\Delta_{h,t} - \Delta_n) \\ &+ \mathcal{O}(\|\zeta\|^3).\end{aligned} \tag{II.9}$$

Similarly for output:

$$\hat{y}_t^{HD} = \mathbb{E}_f\big(\hat{y}_t^{HD}(f)\big) + \frac{\frac{1}{2}\frac{\epsilon_{HD}}{\epsilon_{HD} - 1}\Delta_{y^{HD},t}}{\left(\left(1 + \frac{1}{2}(1-\epsilon_{HD}^{-1})^2\Delta_{y^{HD},t}\right)\right)^2} + \mathcal{O}(\|\zeta\|^3)$$

$$\hat{y}_t^{HF} = \mathbb{E}_f\big(\hat{y}_t^{HF}(f)\big) + \frac{\frac{1}{2}\frac{\epsilon_{HF}}{\epsilon_{HF} - 1}\Delta_{y^{HF},t}}{\left(\left(1 + \frac{1}{2}(1-\epsilon_{HF}^{-1})^2\Delta_{y^{HF},t}\right)\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let's expand in a Taylor series:

$$\frac{\frac{1}{2}\frac{\epsilon_{HD}-1}{\epsilon_{HD}}\Delta_{y,t}}{\left(\left(1+\frac{1}{2}(1-\epsilon_{HD}^{-1})^2\Delta_{y,t}\right)\right)^2}=\frac{\frac{1-\epsilon_{HD}{}^{-1}}{2}\Delta_{y^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}\right)^2}+$$

$$+\frac{1-\epsilon_{HD}{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}\right)^3}(\Delta_{y^{HD},t}-\Delta_{y^{HD}})$$

$$\frac{\frac{1}{2}\frac{\epsilon_{HF}-1}{\epsilon_{HF}}\Delta_{y^{HF},t}}{\left(\left(1+\frac{1}{2}(1-\epsilon_{HF}^{-1})^2\Delta_{y^{HF},t}\right)\right)^2}=\frac{\frac{1-\epsilon_{HF}{}^{-1}}{2}\Delta_{y^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}\right)^2}+$$

$$+\frac{1-\epsilon_{HF}{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}\right)^3}(\Delta_{y^{HF},t}-\Delta_{y^{HF}}).$$

Given that $\widehat{yh}_t=\hat{y}_t^{HD}+\hat{y}_t^{HF}$, and that $\mathbb{E}_f(\hat{y}_t^{HD}+\hat{y}_t^{HF})=\mathbb{E}_f(\hat{y}_t^{HD})+\mathbb{E}_f(\hat{y}_t^{HF})=\mathbb{E}_f\left(\widehat{yh}_t(f)\right)$,

we see that

$$\begin{aligned}\widehat{yh}_t=\mathbb{E}_f\left(\widehat{yh}_t(f)\right)+\frac{\frac{1-\epsilon_{HD}{}^{-1}}{2}\Delta_{y^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}\right)^2}+\frac{1-\epsilon_{HD}{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}\right)^3}(\Delta_{y^{HD},t}\\ -\Delta_{y^{HD}})+\frac{\frac{1-\epsilon_{HF}{}^{-1}}{2}\Delta_{y^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}\right)^2}\\ ++\frac{1-\epsilon_{HF}{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}\right)^3}(\Delta_{y^{HF},t}-\Delta_{y^{HF}})\\ +\mathcal{O}(\|\zeta\|^3).\end{aligned} \tag{II.10}$$

Now, note that (7.5.2) suggests that

$$\frac{N_t}{N}=\int_0^1\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}df+\int_0^1\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}df.$$

Given that, take the logarithm (7.5.2) and see

$$\hat{n}_t=log\left(\int_0^1\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}df+\int_0^1\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}df\right)=log\left(\mathbb{E}_f\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right)+\mathbb{E}_f\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right)\right).$$

Using (II.2) and (II.3) we see that

$$\hat{n}_t=\mathbb{E}_f\left(\widehat{yh}_t(f)\right)+\frac{\frac{1}{2}\mathbb{V}_f\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right)}{\left(\mathbb{E}_f\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right)\right)^2}+\frac{\frac{1}{2}\mathbb{V}_f\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right)}{\left(\mathbb{E}_f\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right)\right)^2}+\mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{V}_f\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right) = \exp\left(\mathbb{E}_f\big(\hat{y}_t^{HD}(f)\big)\right)^2 \mathbb{V}_f\big(\hat{y}_t^{HD}(f)\big) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{V}_f\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right) = \exp\left(\mathbb{E}_f\big(\hat{y}_t^{HF}(f)\big)\right)^2 \mathbb{V}_f\big(\hat{y}_t^{HF}(f)\big) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{E}_h\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right) = \exp\left(\mathbb{E}_h\big(\hat{y}_t^{HD}(f)\big)\right)\left(1 + \frac{1}{2}\mathbb{V}_f\big(\hat{y}_t^{HD}(f)\big)\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{E}_h\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right) = \exp\left(\mathbb{E}_h\big(\hat{y}_t^{HF}(f)\big)\right)\left(1 + \frac{1}{2}\mathbb{V}_f\big(\hat{y}_t^{HF}(f)\big)\right) + \mathcal{O}(\|\zeta\|^3).$$

Then

$$\hat{n}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{\frac{1}{2}\mathbb{V}_f\big(\hat{y}_t^{HD}(f)\big)}{\left(1 + \frac{1}{2}\mathbb{V}_f\big(\hat{y}_t^{HD}(f)\big)\right)^2} + \frac{\frac{1}{2}\mathbb{V}_f\big(\hat{y}_t^{HF}(f)\big)}{\left(1 + \frac{1}{2}\mathbb{V}_f\big(\hat{y}_t^{HF}(f)\big)\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

By defining $\mathbb{V}_f\left(\hat{y}_{t_t}^{HD}(f)\right) \equiv \Delta_{y^{HD,t}}$ and $\mathbb{V}_f\left(\hat{y}_{t_t}^{HF}(f)\right) \equiv \Delta_{y^{HF,t}}$,

then

$$\hat{n}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{\frac{1}{2}\Delta_{y^{HD,t}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD,t}}\right)^2} + \frac{\frac{1}{2}\Delta_{y^{HF,t}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF,t}}\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let's expand in a Taylor series:

$$\frac{\Delta_{y^{HD,t}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD,t}}\right)^2} = \frac{\Delta_{y^{HD}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD}}\right)^2} + \frac{1 - \frac{1}{2}\Delta_{y^{HD}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD}}\right)^3}\left(\Delta_{y^{HD,t}} - \Delta_{y^{HD}}\right),$$

$$\frac{\Delta_{y^{HF,t}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF,t}}\right)^2} = \frac{\Delta_{y^{HF}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF}}\right)^2} + \frac{1 - \frac{1}{2}\Delta_{y^{HF}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF}}\right)^3}\left(\Delta_{y^{HF,t}} - \Delta_{y^{HF}}\right).$$

Then we finally get

$$\begin{aligned}\hat{n}_t = {} & \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{\Delta_{y^{HD}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD}}\right)^2} + \frac{1 - \frac{1}{2}\Delta_{y^{HD}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD}}\right)^3}\left(\Delta_{y^{HD,t}} - \Delta_{y^{HD}}\right), \\ & \frac{\Delta_{y^{HF}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF}}\right)^2} + \frac{1 - \frac{1}{2}\Delta_{y^{HF}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF}}\right)^3}\left(\Delta_{y^{HF,t}} - \Delta_{y^{HF}}\right) + \mathcal{O}(\|\zeta\|^3).\end{aligned} \tag{II.11}$$

Aggregation of prices and wages

Let us recall the equations for the level of prices and wages from Section 2:

$$P_t^{HD} = \left(\int_0^1 P_t^{HD}(f)^{\epsilon_{HD}-1} df\right)^{\frac{1}{\epsilon_{HD}-1}}, \qquad (7.2.2)$$

$$P_t^{HF} = \left(\int_0^1 P_t^{HF}(f)^{\epsilon_{HF}-1} df\right)^{\frac{1}{\epsilon_{HF}-1}}, \qquad (7.2.2)$$

$$W_t = \left(\int_0^1 W_t(h)^{1-\epsilon_L} dh\right)^{1/(1-\epsilon_L)}. \tag{7.5.3}$$

Then, using (II.2), we rewrite it as

$$p_t^{HD} = \mathbb{E}_f\left(p_t^{HD}(f)\right) + \frac{\frac{1}{2}\frac{1}{1-\epsilon_{HD}}\mathbb{V}_f\left(P_t^{HD}(f)^{1-\epsilon_{HD}}\right)}{\left(\mathbb{E}_f\left(P_t^{HD}(f)^{\epsilon_{HD}-1}\right)\right)^2} + \mathcal{O}(\|\zeta\|^3),$$

$$p_t^{HF} = \mathbb{E}_f\left(p_t^{HF}(f)\right) + \frac{\frac{1}{2}\frac{1}{1-\epsilon_{HF}}\mathbb{V}_f\left(P_t^{HF}(f)^{1-\epsilon_{HF}}\right)}{\left(\mathbb{E}_f\left(P_t^{HF}(f)^{\epsilon_{HF}-1}\right)\right)^2} + \mathcal{O}(\|\zeta\|^3),$$

$$w_t = \mathbb{E}_h\left(w_t(h)\right) + \frac{\frac{1}{2}\frac{1}{1-\epsilon_L}\mathbb{V}_f\left(W_t(h)^{1-\epsilon_L}\right)}{\left(\mathbb{E}_h\left(W_t(h)^{1-\epsilon_L}\right)\right)^2}.$$

Using (II.3), we will get that

$$\mathbb{V}_f\left(P_t^{HD}(f)^{1-\epsilon_{HD}}\right) = \mathbb{V}_f\left(\exp\left((1-\epsilon_{HD})p_t^{HD}(f)\right)\right) = (1-\epsilon_{HD})^2 \exp\left((1-\epsilon_{HD})\mathbb{E}_f\left(p_t^{HD}(f)\right)\right)^2 \mathbb{V}_f\left(p_t^{HD}(f)\right),$$

$$\mathbb{V}_f\left(P_t^{HF}(f)^{1-\epsilon_{HF}}\right) = \mathbb{V}_f\left(\exp\left((1-\epsilon_{HF})p_t^{HF}(f)\right)\right) = (1-\epsilon_{HF})^2 \exp\left((1-\epsilon_{HF})\mathbb{E}_f\left(p_t^{HF}(f)\right)\right)^2 \mathbb{V}_f\left(p_t^{HF}(f)\right),$$

$$\mathbb{V}_h\left(W_t(h)^{1-\epsilon_L}\right) = \mathbb{V}_h\left(\exp\left((1-\epsilon_L)w_t(h)\right)\right) = (1-\epsilon_L)^2 \exp\left((1-\epsilon_L)\mathbb{E}_h\left(w_t(h)\right)\right)^2 \mathbb{V}_h\left(w_t(h)\right).$$

Let us define:

$$\bar{\boldsymbol{p}}_{\boldsymbol{t}}^{\boldsymbol{HD}} = \mathbb{E}_f\left(p_t^{HD}(f)\right), \bar{\boldsymbol{p}}_{\boldsymbol{t}}^{HF} = \mathbb{E}_f\left(p_t^{HF}(f)\right), \bar{\boldsymbol{w}}_{\boldsymbol{t}} = \mathbb{E}_h\left(w_t(h)\right),$$

$$\Delta_{\boldsymbol{p}^{\boldsymbol{HD}},\boldsymbol{t}} = \mathbb{V}_f\left(p_t^{HD}(f)\right), \Delta_{\boldsymbol{p}^{HF},\boldsymbol{t}} = \mathbb{V}_f\left(p_t^{HF}(f)\right), \Delta_{\boldsymbol{w},\boldsymbol{t}} = \mathbb{V}_h\left(w_t(h)\right).$$

Then

$$\mathbb{V}_f\left(P_t^{HD}(f)^{1-\epsilon_{HD}}\right) = (1-\epsilon_{HD})^2 \exp\left((1-\epsilon_{HD})\bar{p}_t^{HD}\right)^2 \Delta_{p^{HD},t,}$$

$$\mathbb{V}_f\left(P_t^{HF}(f)^{1-\epsilon_{HF}}\right) = (1-\epsilon_{HF})^2 \exp\left((1-\epsilon_{HF})\bar{p}_t^{HF}\right)^2 \Delta_{p^{HF},t},$$

$$\mathbb{V}_h\left(W_t(h)^{1-\epsilon_L}\right) = (1-\epsilon_L)^2 \exp\left((1-\epsilon_L)\bar{w}_t\right)^2 \Delta_{w,t}.$$

Using (II.2), we will get

$$\mathbb{E}_f\left(P_t^{HD}(f)^{1-\epsilon_{HD}}\right) = \mathbb{E}_f\left(\exp\left((1-\epsilon_{HD})p_t^{HD}(f)\right)\right) = \exp\left((1-\epsilon_{HD})\bar{p}_t^{HD}\right)\left(1 + \frac{1}{2}(1-\epsilon_{HD})^2 \Delta_{p^{HD},t}\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{E}_f\left(P_t^{HF}(f)^{1-\epsilon_{HF}}\right) = \mathbb{E}_f\left(\exp\left((1-\epsilon_{HF})p_t^{HF}(f)\right)\right) = \exp\left((1-\epsilon_{HF})\bar{p}_t^{HF}\right)\left(1 + \frac{1}{2}(1-\epsilon_{HF})^2 \Delta_{p^{HF},t}\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{E}_h\left(W_t(h)^{1-\epsilon_L}\right) = \mathbb{E}_h\left(\exp\left((1-\epsilon_L)w_t(h)\right)\right) = \exp\left((1-\epsilon_L)\bar{w}_t\right)\left(1 + \frac{1}{2}(1-\epsilon_L)^2 \Delta_{w,t}\right) + \mathcal{O}(\|\zeta\|^3).$$

Then we finally get

$$p_t^{HD} = \bar{p}_t^{HD} + \frac{\frac{1}{2}(1-\epsilon_{HD})\Delta_{p^{HD},t}}{\left(1 + \frac{1}{2}(1-\epsilon_{HD})^2 \Delta_{p^{HD},t}\right)^2},$$

$$p_t^{HF} = \bar{p}_t^{HF} + \frac{\frac{1}{2}(1-\epsilon_{HF})\Delta_{p^{HF},t}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF},t}\right)^2},$$

$$w_t = \bar{w}_t + \frac{\frac{1}{2}(1-\epsilon_L)\Delta_{w,t}}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_{w,t}\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let's expand in a Taylor series:

$$\frac{\frac{1}{2}(1-\epsilon_{HD})\Delta_{p^{HD},t}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD},t}\right)^2} = \frac{\frac{1-\epsilon_{HD}}{2}\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^2} +$$

$$+\frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\Delta_{p^{HD},t}-\Delta_{p^{HD}}),$$

$$\frac{\frac{1}{2}(1-\epsilon_{HF})\Delta_{p^{HF},t}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF},t}\right)^2} = \frac{\frac{1-\epsilon_{HF}}{2}\Delta_{p^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}\right)^2} +$$

$$+\frac{1-\epsilon_{HF}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}\right)^3}(\Delta_{p^{HF},t}-\Delta_{p^{HF}}),$$

$$\frac{\frac{1}{2}(1-\epsilon_L)\Delta_{w,t}}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_{w,t}\right)^2} = \frac{\frac{1-\epsilon_L}{2}\Delta_w}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_w\right)^2} +$$

$$+\frac{1-\epsilon_L}{2}\frac{1-\frac{1}{2}(1-\epsilon_L)^2\Delta_w}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_w\right)^3}(\Delta_{w,t}-\Delta_w).$$

Then, as a result, for wages and prices we get

$$p_t^{HD} = \bar{p}_t^{HD} + \frac{\frac{1-\epsilon_{HD}}{2}\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^2} + \\ +\frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\Delta_{p^{HD},t}-\Delta_{p^{HD}}) + \mathcal{O}(\|\zeta\|^3), \tag{II.12}$$

$$p_t^{HF} = \bar{p}_t^{HF} + \frac{\frac{1-\epsilon_{HF}}{2}\Delta_{p^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}\right)^2} + \\ +\frac{1-\epsilon_{HF}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}\right)^3}(\Delta_{p^{HF},t}-\Delta_{p^{HF}}) + \mathcal{O}(\|\zeta\|^3), \tag{II.13}$$

$$w_t = \bar{w}_t + \frac{\frac{1-\epsilon_L}{2}\Delta_w}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_w\right)^2} + \tag{II.14}$$

$$+\frac{1-\epsilon_L}{2}\frac{1-\frac{1}{2}(1-\epsilon_L)^2\Delta_w}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_w\right)^3}(\Delta_{w,t}-\Delta_w)+\mathcal{O}(\|\zeta\|^3).$$

We loglinearise the demand functions:

$$\hat{y}^{HD}(f)=-\epsilon_{HD}(p_t^{HD}(f)-p_t^{HD})+\hat{y}^{HD}{}_t,$$

$$\hat{y}^{HF}(f)=-\epsilon_{HF}(p_t^{HF}(f)-p_t^{HF})+\hat{y}^{HF}{}_t,$$

$$\tilde{n}_t(h)=-\epsilon_L(w_t(h)-w_t)+\tilde{n}_t.$$

Take the variance from both parts and as a result we get

$$\Delta_{y^{HD},t}=\epsilon_{HD}^2\Delta_{p^{HD},t}, \tag{II.15}$$
$$\Delta_{y^{HF},t}=\epsilon_{HF}^2\Delta_{p^{HF},t}, \tag{II.16}$$
$$\Delta_{l,t}=\epsilon_L^2\Delta_{w,t}. \tag{II.17}$$

Price and wage dispersion

For convenience, we write the variance as

$$\Delta_{p^{HD},t}=\mathbb{V}_f(p_t^{HD}(f)-\bar{p}_{t-1}^{HD}),$$

$$\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD}=\phi_{HD}\gamma_{HD}\pi_{t-1}^{HD}+(1-\phi_{HD})(p^{*HD}-\bar{p}_{t-1}^{HD}).$$

The standard formula for dispersion is:

$$\Delta_{p^{HD},t}=\mathbb{E}_f((p_t^{HD}(f)-\bar{p}_{t-1}^{HD})^2)-\left(\mathbb{E}_f(p_t^{HD}(f)-\bar{p}_{t-1}^{HD})\right)^2.$$

Using this, we get that

$$\Delta_{p^{HD},t}=\mathbb{E}_f((p_{t-1}^{HD}(f)-\bar{p}_{t-1}^{HD}+\gamma_{HD}\pi_{t-1}^{HD})^2)+(1-\phi_{HD})(p^{*HD}-\bar{p}_{t-1}^{HD})^2-(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD})^2. \tag{II.18}$$

As

$$\begin{aligned}(1-\phi_{HD})&(p^{*HD}-\bar{p}_{t-1}^{HD})^2-(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD})^2\\ &=(1-\phi_{HD})\left(\frac{1}{1-\phi_{HD}}(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD})-\frac{\phi_{HD}}{1-\phi_{HD}}\gamma_{HD}\pi_t^{HD}\right)^2-(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD})^2\\ &=\frac{\phi_{HD}}{1-\phi_{HD}}(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD}-\gamma_{HD}\pi_t^{HD})^2-\phi_{HD}(\gamma_{HD}\pi_t^{HD})^2\end{aligned}$$

and

$$\phi_{HD}\mathbb{E}_f((p_{t-1}^{HD}(f)-\bar{p}_{t-1}^{HD}+\gamma_{HD}\pi_{t-1}^{HD})^2)=\phi_{HD}\mathbb{E}_f((p_{t-1}^{HD}(f)-\bar{p}_{t-1}^{HD}+\gamma_{HD}\pi_{t-1}^{HD})^2)-\phi_{HD}(\gamma_{HD}\pi_{t-1}^{HD})^2,$$

then (II.18) is rewritten as

$$\Delta_{p^{HD},t}=\phi_{HD}\mathbb{E}_f((p_{t-1}^{HD}(f)-\bar{p}_{t-1}^{HD})^2)+\frac{\phi_{HD}}{1-\phi_{HD}}(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD}-\gamma_{HD}\pi_t^{HD})^2=$$

$$\Delta_{p^{HD},t}=\phi_{HD}\Delta_{p^{HD},t-1}+\frac{\phi_{HD}}{1-\phi_{HD}}(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD}-\gamma_{HD}\pi_t^{HD})^2.$$

From

$$p_t^{HD} = \bar{p}_t^{HD} + \frac{\frac{1-\epsilon_{HD}}{2}\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^2} +$$

$$+\frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\Delta_{p^{HD},t}-\Delta_{p^{HD}}) + \mathcal{O}(\|\zeta\|^3)$$

follows that

$$\bar{p}_t^{HD} - \bar{p}_{t-1}^{HD} = \pi_t^{HD} - \frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}\left(\Delta_{p^{HD},t}-\Delta_{p^{HD},t-1}\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\Delta_{p^{HD},t} = \phi_{HD}\Delta_{p,t-1} + \frac{\phi_{HD}}{1-\phi_{HD}}\left[\pi_t^{HD} - \frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\Delta_{p^{HD},t}-\Delta_{p^{HD},t-1}) - \gamma_{HD}\pi_{t-1}^{HD}\right]^2 + \mathcal{O}(\|\zeta\|^3).$$

Then the equilibrium value of the dispersion is equal to

$$\Delta_{p^{HD}} = \frac{(1-\gamma_{HD})^2\phi_{HD}}{(1-\phi_{HD})^2}(\pi^{HD})^2,$$

$$\Delta_{p^{HD},t} = \phi_{HD}\Delta_{p,t-1,}$$

$$+\frac{\phi_{HD}}{1-\phi_{HD}}\left[(1-\gamma_{HD})\pi^{HD} + \hat{\pi}_t^{HD} - \gamma_{HD}\hat{\pi}_{t-1}^{HD} - \frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\hat{\pi}_t^{HD} - \hat{\pi}_{t-1}^{HD})\right]^2 + \mathcal{O}(\|\zeta\|^3).$$

Similarly

$$\Delta_{w,t} = \alpha_w\Delta_{w,t-1} + \frac{\alpha_w}{1-\alpha_w}\left[(1-\gamma_z)\mu_z + (1-\gamma_w)\pi + \hat{\pi}_t - \gamma_w\hat{\pi}_{t-1} - \frac{1-\epsilon_L}{2}\frac{1-\frac{1}{2}(1-\epsilon_L)^2\Delta_{p^w}}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_{p^w}\right)^3}(\Delta_{p^w,t} - \Delta_{p^w,t-1})\right]^2 + \mathcal{O}(\|\zeta\|^3).$$

Considering that the stationary state $\Delta_{p^{HD}}$ is of the second order, the previously derived expressions can be simplified:

$$p_t^{HD} = \bar{p}_t^{HD} + +\frac{1-\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \mathcal{O}(\|\zeta\|^3), \quad \text{(II.12)}$$

$$p_t^{HF} = \bar{p}_t^{HF} + +\frac{1-\epsilon_{HF}}{2}\Delta_{p^{HF},t} + \mathcal{O}(\|\zeta\|^3), \quad \text{(II.13)}$$

$$w_t = \bar{w}_t + + \frac{1-\epsilon_L}{2}\Delta_{w,t} + \mathcal{O}(\|\zeta\|^3), \tag{II.14}$$

$$\hat{n}_t = \mathbb{E}_h\big(\hat{n}_t(h)\big) + \frac{1-\epsilon_L{}^{-1}}{2}\Delta_{n,t} + \mathcal{O}(\|\zeta\|^3), \tag{II.19}$$

$$\widehat{yh}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{1-\epsilon_{HD}{}^{-1}}{2}\Delta_{y^{HD},t} + \frac{1-\epsilon_{HF}{}^{-1}}{2}\Delta_{y^{HF},t} + \mathcal{O}(\|\zeta\|^3), \tag{II.20}$$

$$\hat{n}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{1}{2}\left(\Delta_{y^{HD},t} + \Delta_{y^{HF},t}\right) + \mathcal{O}(\|\zeta\|^3). \tag{II.21}$$

Loss function

Let us transform the loss function given at the beginning of the section. Let us do this separately for the consumption and labour parts.

Labour

Let us recall that

$$\begin{aligned}\frac{1}{1+\sigma_L}\int_0^1 \big(N_t(h)\big)^{1+\sigma_L} dh \\ &= (N^n)^{1+\sigma_L}\left[\mathbb{E}_h(\hat{n}_t(h))\frac{N}{N^n}\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right) + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)\right. \\ &\left. * \left(\mathbb{V}_h(\hat{n}_t(h)) + \mathbb{E}_h\hat{n}_t(h))^2\right)\right] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).\end{aligned} \tag{II.22}$$

Using

$$\hat{n}_t = \mathbb{E}_h\big(\hat{n}_t(h)\big) + \frac{1-\epsilon_L{}^{-1}}{2}\Delta_{w,t} + \mathcal{O}(\|\zeta\|^3), \tag{II.19}$$

we will ascertain that

$$\begin{aligned}\frac{1}{1+\sigma_L}\int_0^1 \big(N_t(h)\big)^{1+\sigma_L} dh \\ &= (N^n)^{1+\sigma_L}\left[\frac{N}{N^n}\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)\hat{n}_t + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)\hat{n}_t^2\right. \\ &\left. + \left(1+\sigma_L\left(\frac{N}{N^n}-1\right) + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)\right) * \Delta_{w,t}\right] + \text{t.i.p.} \quad + \mathcal{O}(\|\zeta\|^3),\end{aligned}$$

$$(N^n)^{1+\sigma_L}\left[-\frac{1}{2}\frac{\epsilon_L-1}{\epsilon_L}\Delta_{h,t} + \frac{1}{2}\Delta_{h,t} + \frac{1}{2}\sigma_L\Delta_{h,t}\right] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).$$

Let us take $\widehat{yh}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right)$

from

$$\widehat{yh}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{1-\epsilon_{HD}{}^{-1}}{2}\Delta_{y^{HD},t} + \frac{1-\epsilon_{HF}{}^{-1}}{2}\Delta_{y^{HF},t} + \mathcal{O}(\|\zeta\|^3) \tag{II.23}$$

and put it to

$$\hat{n}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{1}{2}\left(\Delta_{y^{HD},t} + \Delta_{y^{HF},t}\right) + \mathcal{O}(\|\zeta\|^3). \tag{II.24}$$

We will ascertain

$$\hat{n}_t = \widehat{yh}_t + \frac{{\epsilon_{HD}}^{-1}}{2}\Delta_{y^{HD},t} + \frac{{\epsilon_{HF}}^{-1}}{2}\Delta_{y^{HF},t} + \mathcal{O}(\|\zeta\|^3). \tag{II.25}$$

Given that (II.25) and $\Delta_{y^{HD},t} = \epsilon_{HD}^2\Delta_{p^{HD},t}$ (II.15), $\Delta_{y^{HF},t} = \epsilon_{HF}^2\Delta_{p^{HF},t}$ (II.16) and $\Delta_{l,t} = \epsilon_L^2\Delta_{w,t}$ (II.17), the loss function for labour has the form:

$$\begin{aligned}
&\frac{1}{1+\sigma_L}\int_0^1 \left(N_t(h)\right)^{1+\sigma_L} dh \\
&= (N^n)^{1+\sigma_L}\Bigg[\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)\left(\widehat{yh}_t + \frac{\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \frac{\epsilon_{HF}}{2}\Delta_{p^{HF},t}\right) \\
&+ \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\left(\widehat{yh}_t + \frac{\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \frac{\epsilon_{HF}}{2}\Delta_{p^{HF},t}\right)^2 \\
&+ \left(1+\sigma_L\left(\frac{N}{N^n}-1\right) + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\right) * \epsilon_L^2\Delta_{w,t}\Bigg] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).
\end{aligned} \tag{II.26}$$

Combining (II.26) and (II.6), we get the final loss function:

$$\begin{aligned}
&\log(C_t - \eta * C_{t-1} * e^{-\zeta_{z,t}}) - \frac{1}{1+\sigma_L} * \int_0^1 (N_t(h))^{1+\sigma_L} dh \\
&= \frac{1}{1-\eta} \\
&* \Bigg[\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n} - \eta * \left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n} - \frac{1}{2} * \frac{1}{1-\eta} \\
&* \left(\hat{c}_t^2\left(\frac{C}{C^n}\right)^2 + 2\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n} * \left(\frac{C}{C^n} - 1\right)\right) + \frac{\eta}{1-\eta} \\
&* \left(\frac{C}{C^n}^2 \hat{c}_t\hat{c}_{t-1} + \left(\frac{C}{C^n} - 1\right)\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) + \left(\frac{C}{C^n} - 1\right)\left(\hat{c}_{t-1} + \frac{1}{2}\hat{c}_{t-1}^2\right)\right) - \frac{1}{2} * \frac{\eta^2}{1-\eta} \\
&* \left(\hat{c}_{t-1}^2\left(\frac{C}{C^n}\right)^2 + 2\left(\hat{c}_{t-1} + \frac{1}{2}\hat{c}_{t-1}^2\right)\frac{C}{C^n}\left(\frac{C}{C^n} - 1\right)\right) + \zeta_{c,t} * \left(\hat{c}_t\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \eta * \zeta_{c,t} \\
&* \left(\hat{c}_{t-1}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\hat{c}_t\frac{C}{C^n} + \frac{C}{C^n} - 1\right) + \frac{\eta}{1-\eta} * \zeta_{z,t} \\
&* \left(\hat{c}_{t-1}\frac{C}{C^n} + \frac{C}{C^n} - 1\right)\Bigg] - (N^n)^{1+\sigma_L}\Bigg[\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)\left(\widehat{yh}_t + \frac{\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \frac{\epsilon_{HF}}{2}\Delta_{p^{HF},t}\right) \\
&+ \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\left(\widehat{yh}_t + \frac{\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \frac{\epsilon_{HF}}{2}\Delta_{p^{HF},t}\right)^2 \\
&+ \left(1+\sigma_L\left(\frac{N}{N^n}-1\right) + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\right) * \epsilon_L^2\Delta_{w,t}\Bigg] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).
\end{aligned}$$

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Abstract

In this paper, I explore the optimal inflation target level in the New Keynesian DSGE-model with imperfect price indexation for non-zero trend inflation and a zero lower bound on interest rates. In addition, I study the impact of the real interest rate on the choice of the optimal inflation target and discuss the costs of adopting a new target level. As a criterion for determining the optimal target level, I use a structural, consumer utility-based loss function. My model is calibrated for the Russian economy but may also be relevant for other resource-rich emerging market countries. I have found out that the optimal inflation target level in this setting of the problem is below the current target of the Bank of Russia of 4%, and this conclusion is robust to the model parameters. In addition, I have ascertained a stable negative relationship between the real interest rate and optimal inflation rate.

Keywords: monetary policy, inflation targeting, interest rate zero lower bound, ZLB, equilibrium real interest rate, optimal inflation target level, DSGE, structural models.

JEL-classification: E12, E31, E52, E58, C68.

CONTENTS

6. INTRODUCTION

Inflation targeting has proven itself over the past decades, prompting central banks in both advanced economies and emerging market countries to adopt to this regime. There is an extensive body of research describing inflation targeting in comparison to other monetary policy regimes. Meanwhile, the main issue of inflation targeting, the inflation target choice, has not been studied thoroughly enough.

On the one hand, if the target is chosen too high, the economy bears the cost of high inflation. But setting a target too low can also be problematic. A lot of inflation-targeting central banks have recently faced the issue of Zero Lower Bound (ZLB). The ZLB issue means a situation where, in response to a shock(s), the central bank needs to lower its key rate, but the optimal rate is in the negative area and cannot actually be set. In this situation, the central bank temporarily loses the opportunity to stabilise the economy with the help of its main tool. This issue was initially faced by advanced economies, but in the context of recent significant and prolonged shocks leading to a decline in consumption and production, this topic is becoming relevant for emerging market countries as well. Although the Russian economy has not experienced the issue so far, the possibility of hitting the ZLB should be taken into account when choosing a long-term inflation target.

Thus, the choice of inflation target is a key matter of inflation targeting and is made taking into account the trade-off between the risk of facing the ZLB issue (and losses from the inability to stabilise the economy) and the costs of economic agents from high inflation.

In the course of the research, I was looking for answers to several questions for the Russian economy. What is the probability of facing the ZLB issue depending on inflation targeting? What is the relationship between the probability of being at the ZLB and the probability of being in the negative area of interest rates[8]? What is the optimal level of inflation target in terms of consumer welfare? How does such optimal level depend on the real neutral interest rate? What is the loss of output when moving to a new target level?

I answer the questions posed on the basis of the DSGE model. My DSGE model is New Keynesian in nature and, compared to standard models of this type (Smets and Wouters, 2003, 2007, Christiano et al., 2005), includes several features that are important for understanding the functioning of the ZLB mechanism, choosing the optimal inflation target, and the relevance of the results to the Russian economy.

First, as shown in the Bank of Russia Analytical Note (2017), most firms in Russia prefer to change prices not all the time, but once in a certain period. In addition to inflation, firms are guided by production costs, the structure of contracts and other factors. Thus, part of the prices in the economy does not change or changes partially for some time. In modeling terms, this means that there is price rigidity and imperfect indexation. The imperfect indexation of producer prices causes distortions in relative prices, thus creating costs from high inflation.

Second, the zero lower bound on interest rates is a natural constraint on inflation targeting. Although emerging market countries have not often encountered this issue, for example, the experience of Chile in 2008-2010 demonstrates the issue in practice (Céspedes et al., 2014).

Together, these two mechanisms create a trade-off between the losses from high inflation (by targeting too high) and the losses from ZLB (by choosing too low inflation target, which increases the probability of ZLB).

Accounting for the peculiarities of the Russian economy is ensured by including the oil sector and calibrating the parameters.

The academic literature describing the issue of choosing the optimal level of inflation target amid a zero lower bound using the general equilibrium approach is very limited. For the US economy, such DSGE

[8] Here and below, when I talk about the probability of being at the ZLB, I mean the probability calculated from the model that includes the ZLB cap, and when I talk about the probability of being in the negative area of interest rates, I mean the probability from the model that does not include such a cap.

models are built, for example, by Andrade et al. (2019) and Coibion et al. (2012). However, these are models of a closed economy. Thus, the key difference between my paper and the existing ones is that it considers the relationship of the domestic economy with the external sector when choosing an inflation target.

A number of papers explore the probabilities of being at the ZLB for the US economy, such as Chung et al. (2012). Kiley and Roberts (2017) and Bernanke et al. (2019) show that choosing a higher inflation target reduces the probability of being close to the ZLB. To my knowledge, there are no studies examining the choice of the optimal inflation target given the ZLB for the Russian economy or other emerging market countries. For the Russian economy, the issue of a zero lower bound is described by Andreev and Polbin (2021), in which ZLB probabilities are calculated. The authors find that the probability of ZLB at the optimal level is in the range from 6.0% to 20.1%. As an optimal level criterion, a semi-structural rule is used, which assumes the minimisation of inflation, key rate, and output dispersions. For the current target of 4%, the authors get a ZLB probability of 0.3%, which is lower than what I have in my calculations. This is probably due to the fact that the authors include only two shocks in their work, while my model includes 14 shocks.

In my research, I find a negative relationship between the chosen inflation target and the probability of being at the ZLB. So, with a target inflation of 4%, the ZLB probability is about 1%, and with a target of 0.5%, about 17%. At the same time, if there is no ZLB in the model, this probability will slightly decrease for each similar target level. This is due to the fact that in a situation where the central bank cannot lower the rate below zero, it needs more time to stabilise the economy than in a situation where there is no such restriction.

To talk about optimal level, we first need to define what is meant by this term. The literature on the optimal inflation target often assumes that the optimal level will be the one that minimises the squared inflation and output deviations from their natural levels à la Woodford (2003), meaning that a problem of the form is solved: $\psi^{PI} * \hat{\pi}^2{}_t + \psi^Y \hat{y}^2{}_t \rightarrow min.$ Whereas in non-structural and semi-structural models, the parameters ψ^{PI} and ψ^Y are usually calibrated or estimated, that is, in general, the loss function doesn't have micro-foundations. For structural models of a closed economy with full price indexation[9], a function of this kind can be derived from the utility function of consumers. In such case, structural coefficients will already be obtained before the inflation gap and the output gap. When imperfect indexing is added to the model, the loss function will take on a more complex form (see, for example, Andrade et al.; 2019, Coibion et al., 2012). When the model is extended to an open economy, this relationship will become even more complex, primarily because the assumption that consumption equals output is no longer satisfied. The derivation of the loss function for such a formulation of the model is given in this paper.

Based on the constructed base model and the loss function above, I find that the optimal inflation target for the Russian economy is 1.1%. This target level corresponds to an 11% probability of being at the ZLB.

Another important matter in choosing the optimal inflation target level is understanding the value of the real neutral interest rate. Given that the nominal interest rate is the sum of the real rate and inflation, the higher the real rate, the lower target can be chosen, ceteris paribus, without changing the probability of being at the ZLB. Meanwhile, the choice of the calibrated level of the real neutral interest rate for the model is not obvious, since this value is unobservable.

According to existing works, the real interest rate for the Russian economy is likely to be in the range of 1% to 3%. For example, Kreptsev et al. (2016) – 1%–3.2%, IMF (2019) – 1%–3%, Isakov and Latypov (2019) – 1.5%–2.5%. The Monetary Policy Report of the Bank of Russia (October 2022) suggests a range of 1% to 2% for the long-term real neutral interest rate. While the Monetary Policy Report of the Bank of Russia (May 2022) has noted that the Central Bank of the Russian Federation expects a real rate increase in the near future due to uncertainty in the economy. I calibrate the equilibrium rate based on fundamental factors and set it at 1.78% per annum.

To my knowledge, Andrade et al. (2019) is the only paper to explicitly examine the relationship between the real rate and optimal target level. The authors show that a decrease in the real rate by 1pp should be

[9] Imperfect indexation means that some firms cannot fully adjust prices for inflation of the previous period or for the equilibrium inflation rate.

offset by an increase in the inflation target by approximately the same amount. Coibion et al. (2012) focuses on the choice of optimal inflation given ZLB but assume that the real interest rate is constant.

In my paper, I present the model-calculated optimal inflation target for different levels of the real rate. I have concluded that, first, as the theory predicts, a higher real rate corresponds to a lower optimal inflation target, and the decreased real rate by 1pp requires an increased inflation target by about 0.5pp; second, for each target level, the lower the probability of ZLB is, the higher the real interest rate.

When adopting a new inflation target, the strategy of such adoption, its duration and the amount of output losses are also important to focus on. There are a number of studies showing that the adopted lower inflation is accompanied by an output fall, such as Ball (1994b), Cecchetti and Rich (2001), Gordon and King (1982).

An indicator commonly used in the literature to measure the negative impact on output from disinflation is the sacrifice ratio (SR) (such as Ascari and Ropele, 2012). This ratio is the cumulative percentage loss of the output gap (the difference between the current value of output and its trend) divided by the difference between the old and new negative inflation targets. Thus, the coefficient shows the loss of output relative to the size of the target change. This indicator depends on the number of periods it takes for the economy to move to a new equilibrium, the magnitude of the change in target, as well as the strength of the transmission mechanism. According to empirical studies (such as Gordon and King, 1982; Cecchetti and Rich, 2001; Durand et al., 2008), this coefficient ranges from 0.5 to 3.

Ascari and Ropele (2012) is focused on the output effect of a permanent decline in inflation based on a medium-scale New Keynesian model such as Smets and Wouters (2003, 2007), Christiano et al. (2005). The authors obtain a coefficient value from 0.95 to 1.13, depending on the current and new target (three options for reducing the inflation target are being studied, from 4% to 2%, from 6% to 2%, from 8% to 2%), and the central bank policy rigidity parameter (1.5 and 3).

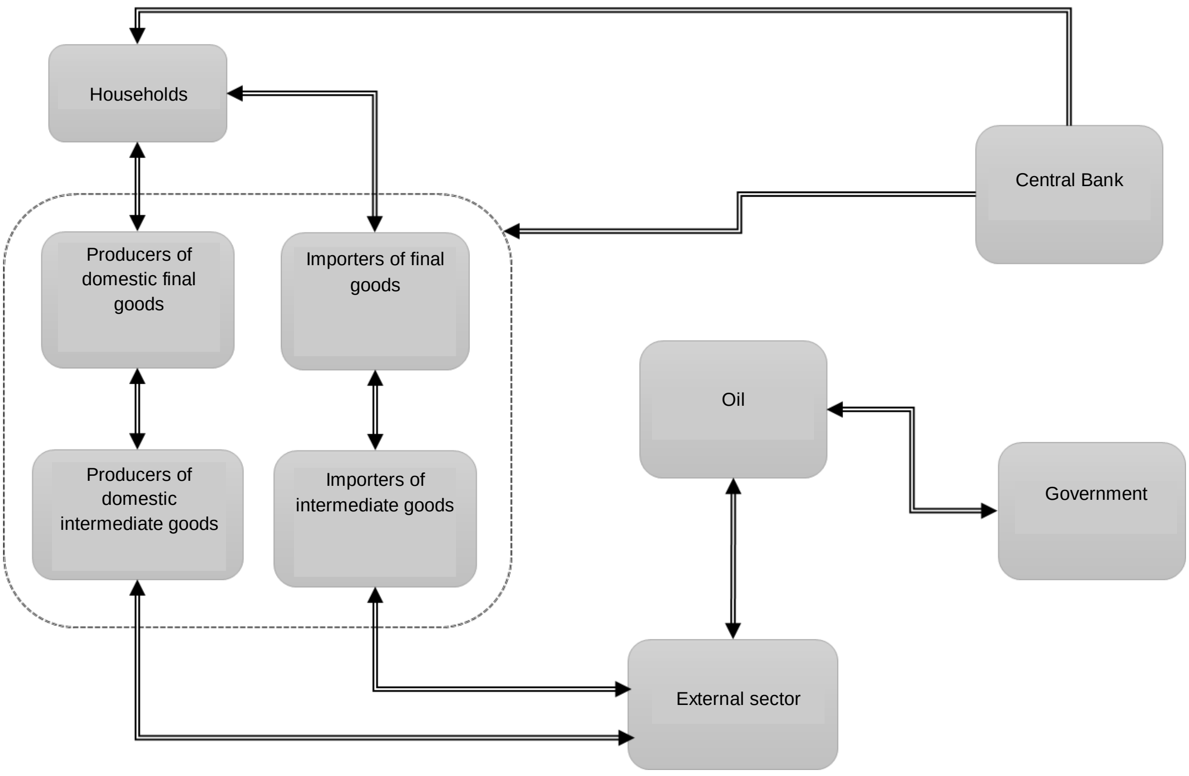
In my work, I find that the SR ratio is 1.03. Thus, for Russia this coefficient is closer to the lower bound of the range discussed in Ascari and Ropele (2012) [0.5; 3], which contains this coefficient for other works. It can be explained by the Russian economy structure. The range is based on calculations for European countries and the US. The fact that I'm getting a fairly low ratio suggests that lowering the inflation target comes at a lower cost than the average for other countries.

The rest of the paper is structured as follows. The second part describes the model. The third discusses the calibration of the model, its properties, and the fit of the model to the data. The fourth part of the paper describes the theoretical basis for choosing the optimal inflation target and related estimates. The fifth part gives conclusions.

7. MODEL

My DSGE model is similar to Smets and Wouters (2003, 2007). I also draw on the Medina and Soto (2007) model, which takes into account the characteristics of a resource-based economy. My model is a medium scale DSGE- model for a small open economy, calibrated for the Russian economy. In addition to the sectors of households, businesses and the state that are standard for DSGE models, this model includes the sector of natural resources (oil). To simplify the structure of the model and facilitate the interpretation of the results, capital was excluded from the model. This model is New Keynesian in nature and includes nominal rigidities. In addition, an important feature of my model is the imperfect indexation of prices, which ensures that there is a distortion in relative prices (that is, the costs of high inflation), and a zero lower bound on the interest rate. Figure 2.1 shows the model structure.

Chart 2.1. Model chart



7.1. Households

The consumer sector is modeled as a continuum of households $h \in [0;1]$. Households buy consumer goods, providing labour to firms. Each household offers a certain type of labour service to the producers of intermediate products. Households live indefinitely and maximise their utility function of the form:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(e^{\zeta_{t+s}^C} \log(C_{t+s} - \eta C_{t+s-1}) - \frac{1}{1+\sigma^L} \int_0^1 L_{t+s}(h)^{1+\sigma^L} dh \right),$$

where utility depends positively on consumption C_t and negatively on the number of hours worked $L_t(h)$. The parameter η characterises consumption habits. The parameter σ^L is the inverse Frisch elasticity of labour supply. ζ_{t+s}^C is a preference shock, which is a first-order autoregressive process:

$$\zeta_t^C = \rho_C \zeta_{t-1}^C + \varsigma_t^C,$$

where $\varsigma_t^C \sim i.i.d$ is innovation with mean zero.

Households consume a composite good in which the share ν_C is domestic goods and $(1-\nu_C)$ is foreign goods:

$$C_t = \left(\nu_C^{\frac{1}{\eta^C}} * (C_t^H)^{\frac{\eta^C-1}{\eta^C}} + (1-\nu_C)^{\frac{1}{\eta^C}} * (C_t^F)^{\frac{\eta^C-1}{\eta^C}} \right)^{\frac{\eta^C}{\eta^C-1}},$$

where C_t^H is consumption of domestic goods, C_t^F is consumption of foreign goods, η^C is elasticity between domestic and foreign goods.

I assume that there are two types of households, non-Ricardian (constituting a proportion λ of all households) and Ricardian (constituting a proportion $(1-\lambda)$), of those differing in access to financial assets.

Ricardian households can buy one-period bonds B_t with payments in the next period in rubles at the rate i_t and in foreign currency B_t^F at the rate of i_t^{rrF}. In addition, they receive dividends D_t, paid by monopoly firms. Thus, they maximise their utility subject to the following budget constraint:

$$P_t C_t + \frac{1}{i_t} B_t + \frac{1}{i_t^{rrF}} \mathcal{E}_t B_t^F \le \int_0^1 W_t(h) L_t(h) dh + B_{t-1} + \mathcal{E}_t B_{t-1}^F + D_t,$$

where P_t is the price level in the economy, $W_t(h)$ is the nominal wage of a type *h* household.

The bond rate in foreign currency i_t^{rrF} is risky and depends on the risk-free rate i_t^F and the risk premium θ:

$$i_t^{rrF} = i_t^F * \theta_t,$$

where θ_t is the risk premium, defined as:

$$\theta = \left(\frac{B_t^F}{P_t^Y Y_t}\right)^{\rho^{AY}} * \left(\frac{P_t^{Oil}}{P_t^Y}\right)^{\rho^{Oil}},$$

where ρ^{AY} is the elasticity of the risk premium with respect to the net position in foreign assets to output, ρ^{Oil} is the elasticity of the risk premium with respect to the oil price.

Thus, as a result of solving the optimisation problem, the following relations are obtained:

$$\frac{e^{\xi_{c,t}}}{C_t - \eta C_{t-1}} - \beta\eta \frac{e^{\xi_{c,t+1}}}{C_{t+1} - \eta C_t} = \Lambda_t$$ ⟵ equation for the Lagrange multiplier

$$\Lambda_t = \beta \frac{\Lambda_{t+1}}{\Pi_{t+1}} i_t$$ ⟵ Euler equation

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$

$$i_t = i_t^F \theta_t \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$$ ⟵ uncovered interest rate parity (UIP)

Non-Ricardian households spend all their labour income on consumption. In addition, they receive oil revenues (paid by the state as transfers), which are also spent on consumption:

$$P_t C_t \le \int_0^1 W_t(h) L_t(h) dh + \chi * P_t^{OilRef} O_t,$$

where χ is the state share in oil revenues, P_t^{OilRef} is the base price of oil, and O_t is the physical volume of oil.

7.2. Producers of final domestic and imported goods

Final goods are produced under conditions of perfect competition from intermediate goods. The production function is the Dixit–Stiglitz function:

$$Y_t^i = \left(\int_0^1 Y_t^i(f)^{\frac{\epsilon_i - 1}{\epsilon_i}} df\right)^{\frac{\epsilon_i}{\epsilon_i - 1}}, \tag{7.2.1}$$

where Y_t^i is the output of producers of final goods, $Y_t^i(f)$ is the output f-ro of a producer of intermediate goods, ϵ_i is the elasticity of substitutes between two intermediate goods and $i \in \{HD, F, HF\}$, HD are domestic goods sold domestically, $F$is foreign goods sold domestically, HF is domestic goods sold abroad.

The prices of individual firms $P_t^i(f)$ are aggregated into a general price index using the Dixit-Stiglitz function:

$$P_t^i = \left(\int_0^1 P_t^i(f)^{\epsilon_i - 1} df\right)^{\frac{1}{\epsilon_i - 1}}. \tag{7.2.2}$$

A representative firm maximises profit of the form:

$$P_t^i Y_t^i - \int_0^1 P_t^i(f) Y_t^i(f) = P_t^i \left(\int_0^1 Y_t^i(f)^{\frac{\epsilon_i - 1}{\epsilon_i}} df\right)^{\frac{\epsilon_i}{\epsilon_i - 1}} - \int_0^1 P_t^i(f) Y_t^i(f),$$

where P_t^i is the price of the final product, $P_t^i(f)$ is the price of the intermediate product of the fth producer.

From the condition of equality of profit to zero, we obtain the demand for intermediate products:

$$Y_t^i(f) = \left(\frac{P_t^i}{P_t}\right)^{-\epsilon_i} Y_t^i. \tag{7.2.3}$$

7.3. PRODUCERS OF INTERMEDIATE DOMESTIC GOODS

Intermediate goods are produced by firms under monopolistic competition in accordance with the production function:

$$Y_t^{HD}(f) = Z_t L_t(f),$$

where Z_t is the stochastic performance trend and

$$Z_t = Z_{t-1} e^{\zeta_t^a}.$$

Some of the goods are sold as raw materials to producers of final domestic products, and some are sold as non-primary exports.

Intermediate goods are produced with nominal à la Calvo price rigidities. This means that firms are ϕ^i probability to face an inability to optimise prices, $i \in \{HD, HF\}$, HD are domestic goods sold domestically, HF is domestic goods sold abroad.

If the firm cannot optimise its price in period *t*, then it sets it according to the following rule:

$$P_t^i(f) = \left(\Pi_{t-1}^i\right)^{\gamma_i} P_{t-1}^i(f),$$

$$where\ i \in \{HD, HF\}, \Pi_t^i \equiv \frac{P_t^i}{P_{t-1}^i},$$

$\Pi^i - steady\ state\ value\ of\ inflation, \gamma_i - degree\ of\ price\ indexation\ parameter\ and\ \ 0 \leq \gamma_i < 1.$

Essential in the context of choosing the optimal inflation target is the imperfect indexation in the model, that is, the fact that the coefficient γ_i is calibrated strictly less than one. Imperfect indexation means that some firms cannot fully adjust prices from the previous period to past or equilibrium inflation. The imperfect indexing mechanism allows modeling the relationship between price dispersion and trend inflation and results in high inflation costs.

If the firm can revise its price for domestic goods sold domestically in period *t*, then it chooses it based on the profit maximisation condition:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\phi^{HD})\, \Lambda_{t+s} \left(\frac{V_{t,t+s}^{HD} P_t^{HD*}(f)}{P_{t+s}} Y_{t,t+s}^{HD} - \frac{W_{t+s}}{P_{t+s}} \frac{Y_{t,t+s}^{HD}}{Z_{t,t+s}} \right),$$

where Λ_t – is the marginal utility of consumers and $Y_{t,t+s}^i(f)$ is the demand for the products of a monopolist who fixed the price in period *t*, in period *t* + *s*, which has the form:

$$Y_{t,t+s}^i(f) = \left(\frac{V_{t,t+s}^i P_t^{i*}}{P_{t+s}} \right)^{-\epsilon_i} Y_{t+s}^i,$$

where V_t^i is the cumulative effect of price indexation on inflation in previous periods:

$$V_{t,t+s}^i = \prod_{j=t}^{t+s-1} (\Pi_j)^{\gamma_i}.$$

Λ_t in the firms problem is due to the fact that monopolistic firms are owned by consumers, and consumers receive the profit they earn as dividends.

The first order condition for this problem is[10]:

$$\sum_{s=0}^{\infty} (\beta\phi^{HD})\, \Lambda_{t+s} \left(\frac{\left(V_{t,T}^{HD} P_t^{HD*}(f)\right)^{1-\epsilon^{HD}}}{P_{t+s}} \left(\frac{1}{P_{t+s}^{HD}}\right)^{-\epsilon^{HD}} Y_{t+s}^{HD} - \frac{\epsilon^{HD}}{\epsilon^{HD}-1} \frac{W_{t+s}}{P_{t+s}} \left(\frac{V_{t,t+s}^{HD} P_t^{HD*}(f)}{P_{t+s}^{HD}} \right)^{-\epsilon^{HD}} \frac{Y_{t+s}^{HD}}{Z_{t+s}} \right) = 0.$$

Similarly, the price is chosen for domestic goods sold abroad:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\phi^{HF})\, \Lambda_{t+s} \left(\frac{V_{t,t+s}^{HF} P_t^{HF*}(f)}{P_{t+s}} Y_{t+s}^{HF} - \frac{1}{\mathcal{E}_{t+s}} \frac{W_{t+s}}{P_{t+s}} \frac{Y_{t+s}^{HF}}{Z_{t+s}} \right).$$

The first order condition for this problem is:

$$\sum_{s=0}^{\infty} (\beta\phi^{HF})\, \Lambda_{t+s} \left(\frac{\left(V_{t,T}^{HF} P_t^{HF*}(f)\right)^{1-\epsilon^{HF}}}{P_{t+s}} \left(\frac{1}{P_{t+s}^{HF}}\right)^{-\epsilon^{HF}} Y_{t+s}^{HF} - \frac{\epsilon^{HF}}{\epsilon^{HF}-1} \frac{1}{\mathcal{E}_{t+s}} \frac{W_{t+s}}{P_{t+s}} \left(\frac{V_{t,t+s}^{HF} P_t^{HF*}(f)}{P_{t+s}} \right)^{-\epsilon^{HF}} \frac{Y_{t+s}^{HF}}{Z_{t+s}} \right) = 0.$$

7.4. PRODUCERS OF INTERMEDIATE IMPORTED GOODS

Intermediate goods are produced by firms under monopolistic competition from foreign goods.

Just like domestic goods, foreign goods are produced with nominal à la Calvo price rigidities. This means that firms are ϕ^F probability to be unable to change prices.

[10] A complete derivation of the model equations is contained in Appendix I.

If the firm can revise its price for domestic goods sold domestically in period *t*, then it chooses it based on the profit maximisation condition:

$$\mathbb{E}\sum_{s=0}^{\infty}(\beta\phi^F)\,\Lambda_{t+s}\left(\frac{V^F_{t,t+s}P^{F*}_t}{P_{t+s}}Y^F_{t,t+s}-\mathcal{E}_{t+s}\frac{P^{For}_t}{P_{t+s}}\frac{Y^F_{t,t+s}}{Z_{t,t+s}}\right),$$

where P^{For}_t is the price of intermediate goods abroad, P^{F*}_t is the effective price of importers.

If the firm cannot optimise its price in period *t*, then it sets it according to the following rule:

$$P^F_t(f) = (\Pi^F_{t-1})^{\gamma_F}P^F_{t-1}(f),$$

$$\Pi^F_t \equiv \frac{P^F_t}{P^F_{t-1}},\Pi^F - \text{равновесное значение}, 0 \leq \gamma_F < 1.$$

As in the case of producers of domestic goods, price indexation for importers is imperfect.

The first order condition for this problem is:

$$\sum_{s=0}^{\infty}(\beta\phi^F)\,\Lambda_{t+s}\left(\frac{\left(V^F_{t,T}P^{F*}_t(f)\right)^{1-\epsilon^F}}{P_{t+s}}\left(\frac{1}{P^F_{t+s}}\right)^{-\epsilon^F}Y^F_{t+s}-\frac{\epsilon^F}{\epsilon^F-1}\mathcal{E}_{t+s}\frac{P^{For}_t}{P_{t+s}}\left(\frac{V^F_{t,t+s}P^{F*}_t(f)}{P_{t+s}}\right)^{-\epsilon^F}\frac{Y^F_{t+s}}{Z_{t+s}}\right)=0.$$

7.5. LABOUR SUPPLY

Each household h offers its own specific type of work $N_t(h)$. The labour of individual households is aggregated into the total labour supply N_t using the CES function:

$$N_t = \left(\int_0^1 N_t(h)^{\frac{\epsilon_L-1}{\epsilon_L}}dh\right)^{\epsilon_L/(\epsilon_L-1)}, \tag{7.5.1}$$

where ϵ_L is the elasticity between the types of labour of different firms.

$$N_t = \int_0^1 L_t(f)df, \tag{7.5.2}$$

where $L_t(f)$ is the firm's demand for labour f, L_t is the total demand for labour in the economy, aggregated by firms, $N_t(h)$ is the household's labour supply, and h, N_t are the labour supply aggregated across all households.

$$W_t = \left(\int_0^1 W_t(h)^{1-\epsilon_L}dh\right)^{1/(1-\epsilon_L)}, \tag{7.5.3}$$

where W_t is the nominal total wage, $W_t(h)$ is the wage paid to a household of type h.

I assume the presence of nominal à la Calvo rigidity in wages, that is, with the ϕ^L probability that households cannot optimise wages. In this case, the wage is set according to the following indexation rule:

$$W_t(h) = (\Pi_{t-1})^{\gamma_L}W_{t-1}(h),$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, Π is the equilibrium value of inflation, and the γ_L degree of indexation parameter lies in the range $0 \leq \gamma_L < 1$, that is, the indexation is imperfect.

In the case when households can choose a wage, it is found from the solution of the following optimisation problem:

$$\mathbb{E}\sum_{s=0}^{\infty}(\beta\phi^L)\,\Lambda_{t+s}\left(\frac{V_{t,t+s}^L W_t^*}{W_{t+s}}L_{t,t+s} - \frac{1}{v}L_{t,t+s}^{1+v}\right),$$

where the demand function for labour in a period $t+s$ for a household that changed the price in a period t, is:

$$N_{t,t+s} = \left(\frac{V_{t,t+s}^L W_t^*}{W_{t+s}}\right)^{-\epsilon_L} N_{t+s},$$

where is the cumulative effect on wages from indexation:

$$V_{t,t+s}^L = \prod_{j=t}^{t+s-1}(\Pi_j)^{\gamma_L}.$$

7.6. Oil sector

It is assumed that the oil firm produces a homogeneous commodity and all of it is exported. Oil production O_t depends on production in the previous period and foreign demand Y_t^F:

$$O_t = (O_{t-1})^{\rho^O} * (Y_t^F)^{\alpha^O},$$

where α^O is the elasticity of oil production to foreign demand.

The price of oil is determined exogenously:

$$\widehat{prO}_t = \rho^{pro}\widehat{prO}_{t-1} + \varsigma_t^{pro}.$$

The state receives a share χ of the sale of oil and pays it to households as transfers.

7.7. Base oil price

The fiscal rule is a mechanism for reducing the volatility of a country's income. This policy tool is often used in countries where natural resources make up a significant part of their exports. The meaning of this mechanism is to establish a long-term (base) price for the exported resource. If the actual price is higher than the base price, then the excess is transferred to a special fund for storage. If the actual price is lower than the base price, then the state budget uses the fund to finance the missing part of the planned expenditures. Thus, more stable government spending is balanced against more volatile government commodity revenues.

In Russia, the fiscal rule was originally introduced in 2004 and has since been revised several times, such as in 2008 when the Stabilisation Fund of the Russian Federation was split into the Reserve Fund and the National Wealth Fund, or in 2008 when the rule was changed in connection with the global financial crisis. In addition, the rule was suspended in order to be able to respond more flexibly to the situation in 2015, 2020 and 2022. At the time of this writing, a law has been passed to change the fiscal rule from 2023. The new fiscal rule proposes to understand 8 trillion rubles as basic oil and gas revenues, and income above this value is considered super income.

Considering that at the time of writing the new fiscal rule has not yet entered into force, it is difficult to assess its possible impact on the economy and the optimal level of the key rate as the main issue of my research. However, in order to illustrate that the fiscal rule has an impact on the choice of the optimal target level (by offsetting some of the shocks that affect the economy), I consider two versions of the model, without a fiscal rule and with a fiscal rule. The mechanism associated with the cut-off price, which was used in Russia from 2017 to 2022, is considered as a fiscal rule.

For simplicity, the model does not explicitly describe the reserve fund. It is assumed that each period the government saves/borrows the difference between the base and actual oil revenues in foreign currency $\chi * \left(P_t^{Oil} - P_t^{OilRef}\right) O_t$. In this formulation of the problem, it is implicitly assumed that the fund is inexhaustible. Basic oil revenues $\chi * P_t^{OilRef} O_t$ are paid to non-Ricardian households as dividends.

7.8. External sector

The external sector is modeled exogenously with respect to the domestic economy. The foreign interest rate $\hat{\imath}_t^F$ and foreign inflation $\hat{\pi}_t^F$ are respectively equal to:

$$\hat{\imath}_t^F = \rho^{iF} \hat{\imath}_{t-1}^F + \varepsilon_t^{iF} \sim AR(1),$$

$$\hat{\pi}_t^F = \rho^{piF} \hat{\pi}_{t-1}^F + \varepsilon_t^{piF} \sim AR(1).$$

7.9. Monetary policy and the zero lower bound of interest rates

The central bank policy rule is:

$$i_t^{ZLB} - \bar{\imath}_t = \psi_R (i_{t-1}^{ZLB} - \bar{\imath}_{t-1}) + (1 - \psi_R) \psi_{PI} * \hat{\pi}_{t+1} + \zeta_t^{MP},$$

$$i_t^{ZLB} = \begin{cases} i_t^{ZLB}, if\ i_t^{ZLB} \geq 0 \\ 0, otherwise \end{cases},$$

where $i_t^{ZLB} \equiv \log(i_t^{ZLB})$ and $\bar{\imath}_t$ is the interest rate trend, ζ_t^{MP} is the monetary policy shock, ψ_R is the monetary policy response smoothing factor, ψ_{PI} is the coefficient of monetary policy response to inflation deviation from the target level.

7.10. Markets clearing

Equilibrium in the economy is determined by the following relations.

Demand for the goods of producers of intermediate products is equal to the total supply of these goods:

$$Y_t^H(f) = \left(\frac{P_t^H(f)}{P_t^H}\right)^{-\epsilon_H} Y_t^H + \left(\frac{P_t^{HF}(f)}{P_t^{HF}}\right)^{-\epsilon_{HF}} Y_t^{HF}.$$

The demand for domestic goods is equal to their supply:

$$Y_t^H = C_t^H + \left(1 - \frac{O}{X}\right) Y_t^{HF}.$$

Output represents household consumption expenditure C_t, oil and non-oil exports X_t, imports M_t:

$$Y_t = C_t + X_t - M_t.$$

The balance of payments is as follows:

$$\frac{\mathcal{E}_t B_t^F}{(1+i_t^F)} = \mathcal{E}_{t-1} B_{t-1}^F - (1-\chi) * P_t^{Oil} O_t - \chi * \left(P_t^{Oil} - P_t^{OilRef}\right) O_t + P_t^X X_t - P_t^M M_t,$$

where P_t^{OilRef} is the base oil price.

In the model with the fiscal rule, the impact on the exchange rate occurs through the balance of payments. In the model with the rule $P_t^{Oil} \neq P_t^{OilRef}$, therefore, part of the influence of the oil price on the exchange rate is leveled. In the model without the rule $P_t^{Oil} = P_t^{OilRef}$, therefore, the additional element associated with the base price of oil in the balance of payments is set to zero, and this effect does not occur.

8. Calibration

In this section, I discuss the parameter calibration used in this paper. To calibrate the parameters, I use Russian and foreign empirical works, as well as direct statistical data for Russia. The decision was made not to evaluate the model using Bayesian estimation for several reasons. First, the ability to adequately estimate micro-parameters, such as degrees of price rigidity or degree of price indexation, for individual types of firms using aggregated data series is questionable. Second, in many papers using Bayesian parameter estimation, the choice of priors and standard deviations is not described in detail, which, in fact, makes the distinction between estimation and calibration blurry. In this regard, I prefer to use calibration, while checking the robustness of the estimates in the corresponding section.

8.1. Basic calibration

Basic parameter calibration is given in Table 3.1.

The GDP growth rate is assumed to be 1.5% per year (i.e. $1{,}015^{0.25}$ $per\ quarter,\ \overline{g_y} = 1{,}015^{0.25}$) The mid-term consensus for Russia's potential GDP growth is estimated at 1.5–2%. For example, IMF (2021) - 1.6%, World Bank (2021) - 1.8%.

The value of the discount factor β is set at 0.999, which corresponds to a real interest rate of 1.78% per annum. As discussed above, according to empirical studies, the real rate for Russia lies in the range of 1 to 3%.

I calibrate the elasticity of substitution between domestic and imported goods at 0.9. This value is close to one, since the share of imports in GDP for Russia is stable and has not changed much over the past two decades and is about 20%.

I calibrate the share of imports in consumption at 0.43. I define this indicator as the ratio of the shares of imports to GDP and consumption to GDP, focusing on Rosstat data on the components of GDP from 2014 to 2020.

The degree of utility loss to the consumer from an additional unit of labour (the inverse of Frisch elasticity) ν indicates how much additional compensation workers need in order for them to be willing to provide an additional small unit of labour, and the higher this figure, the more compensation is required. I set this ratio at 1.04.

Estimates of elasticity of substitution between intermediate substitutes, as discussed in Leif et al. (2005), for the US, euro area and UK range from 3 to 11. The higher this value, the closer the market is to perfect competition. I set 6 for domestic goods sold domestically θ^{HD},, 6 for domestic goods sold abroad θ^{HF}, and 6 for foreign goods sold domestically θ^{F},, i.e. corresponding to a moderate level of competition, with a markup of 20%.

Calvo coefficients φ^{HD} are calibrated at the level of 0.4, φ^{HF} – 0.6, φ^{F} – 0.4. To my knowledge, there are no works for the Russian economy that explicitly estimate these coefficients on microeconomic data. I calibrate the Calvo coefficient for goods sold domestically to be lower than for industries where goods are sold abroad. This is due to the fact that I assume that exporters' prices are more rigid due to the influence of external factors. Thus, producers of goods sold domestically change prices on average every 2 months, while exporters change prices on average once every 4.5 months.

As for degree of indexation, γ^{i}, $where\ i \in \{HD, F, HF, L\}$ as discussed above, this parameter should be less than one to model price dispersion between firms. I calibrate this parameter at 0.4, which provides a sufficient level of rigidity in the economy, but still allows prices to be adjusted for previous inflation.

I calibrate the coefficient of monetary policy response to inflation deviation ψ^{PI} at 2.5. The monetary policy smoothing coefficient ψ^{R} is set at 0.75.

Table 3.1. Calibration of model parameters

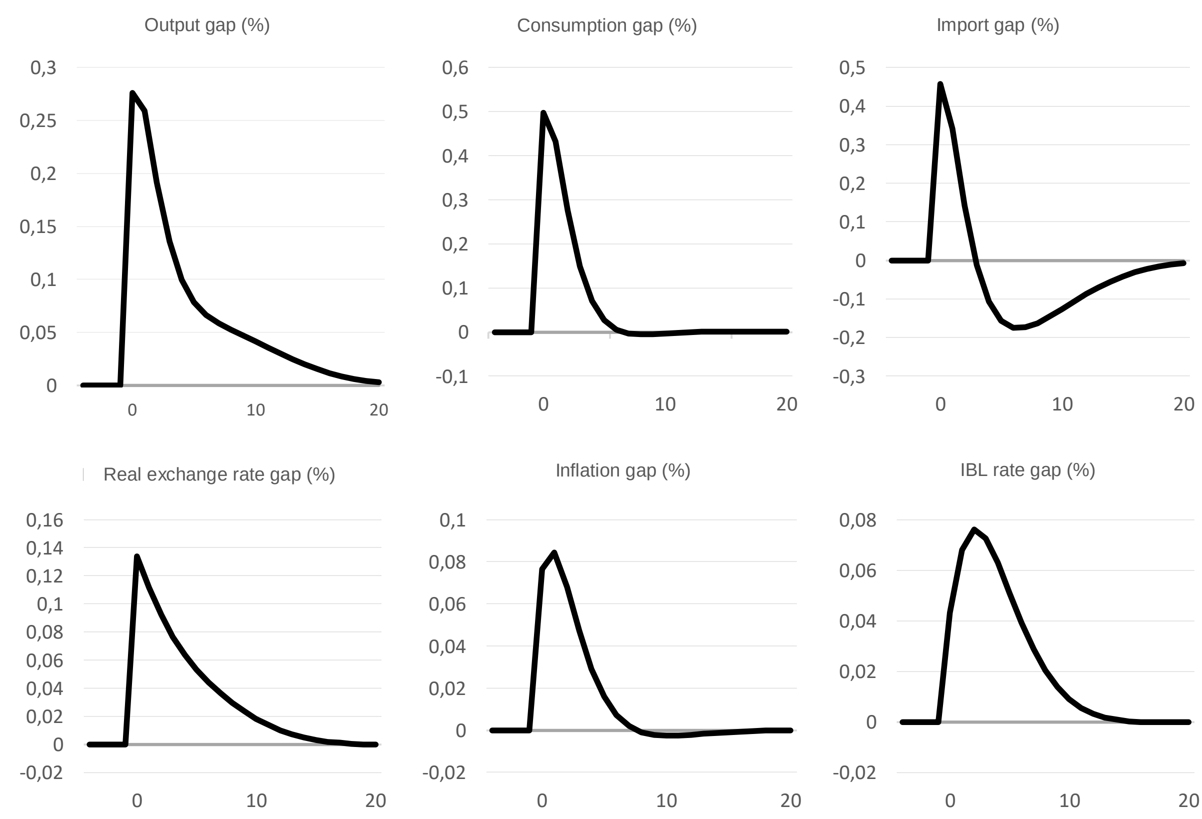
Consumer utility function parameters	
β: discount coefficient	0.999
η: coefficient of the Frisch elasticity of labour supply	1.04
Price formation parameters	
φ^{HD}: Calvo coefficient of domestic goods sold domestically	0.4
φ^{HF}: Calvo coefficient for domestic goods sold abroad	0.6
φ^{F}: Calvo coefficient of domestic goods sold domestically	0.4
γ^{HD}: price indexation of domestic goods sold domestically	0.4
γ^{HF}: price indexation of domestic goods sold abroad	0.4
γ^{F}: price indexation of foreign goods sold domestically	0.4
θ^{HD} : elasticity of substitution between domestic intermediate goods sold domestically	6
θ^{HF}: elasticity of substitution between domestic intermediate substitute goods sold abroad	6
θ^{F} : elasticity of substitution between domestic intermediate substitute goods imported from abroad	6
Monetary policy parameters	
ψ^{PI}: coefficient of monetary policy response to inflation deviation	2.5
ψ^{R}: monetary policy smoothing ratio	0.75
Risk premium parameters	
ρ^{AY}: the elasticity of the risk premium relative to the net position in foreign assets to output	0.0155
ρ^{Oil}: the elasticity of the risk premium for oil price	−0.0057
Stationary states	
$\overline{g_y}$: GDP growth rate	$1{,}015^{0.25}$
Shock parameters	
ρ^{C}: preference shock persistence	0.5
σ^{C}: standard deviation of preference shock	6.7
ρ^{a}: technology shock persistence	0.6
σ^{a}: standard deviation of technology shock	0.3
ρ^{mp}: monetary policy shock persistence	0.2
σ^{mp}: standard deviation of monetary policy shock	1.5
ρ^{pi}: cost shock persistence	0.3
σ^{pi}: standard deviation of cost shock	3
$\rho^{pi\ bar}$: cost trend shock persistence	0.9
$\sigma^{pi\ bar}$: standard deviation of the cost shock trend	0.3

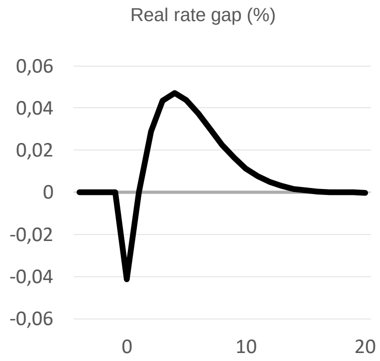
ρ^{ex}: **UIP shock persistence**	0.5
σ^{ex}: **standard deviation of UIP shock**	0
$\rho^{ex\ bar}$: **UIP trend shock persistence**	2
$\sigma^{ex\ bar}$: **standard deviation of UIP shock**	3
ρ^{x}: **export shock persistence**	0.5
σ^{x}: **standard deviation of export shock**	9.5
$\sigma^{x\ bar}$: **standard deviation of the export shock trend**	1.5
ρ^{iF}: **FX rate shock persistence**	0.9
σ^{iF}: **standard deviation of FX shock**	0.4
$\rho^{iF\ bar}$: **FX rate trend shock persistence**	0.95
$\sigma^{iF\ bar}$: **standard deviation of the FX shock trend**	0.2
ρ^{piF}: **foreign inflation shock persistence**	0.3
σ^{piF}: **standard deviation of foreign inflation shock**	1.7
$\rho^{piF\ bar}$: **foreign inflation trend shock persistence**	0.8
$\sigma^{piF\ bar}$: **standard deviation of foreign inflation shock trend**	0.2
ρ^{poil}: **oil price shock persistence**	0.6
σ^{poil}: **standard deviation of oil price shock**	16
$\rho^{poil\ bar}$: **oil price trend shock persistence**	0
$\sigma^{poil\ bar}$: **standard deviation of oil price shock trend**	22

8.2. Analysis of the Transmission Mechanism

To analyse the properties of the model, I plot the impulse response functions of the main variables to the main shocks of the model.

Chart 3.1. Impulse response functions, preference shock





As seen in Chart 3.1, a positive preference shock leads to an increase in consumption. Since the consumer basket includes both domestic and imported goods, the demand for goods in both categories is growing. Imports are growing following the increase in consumer demand for them. An increase in demand leads to higher prices and a positive inflation gap. This, in turn, encourages the central bank to raise interest rates. Since the real interest rate falls in the first periods after the shock, the real exchange rate weakens. Then, as the real rate rises, the exchange rate strengthens and returns to the equilibrium state.

Chart 3.2. Impulse response functions, cost-pushshock

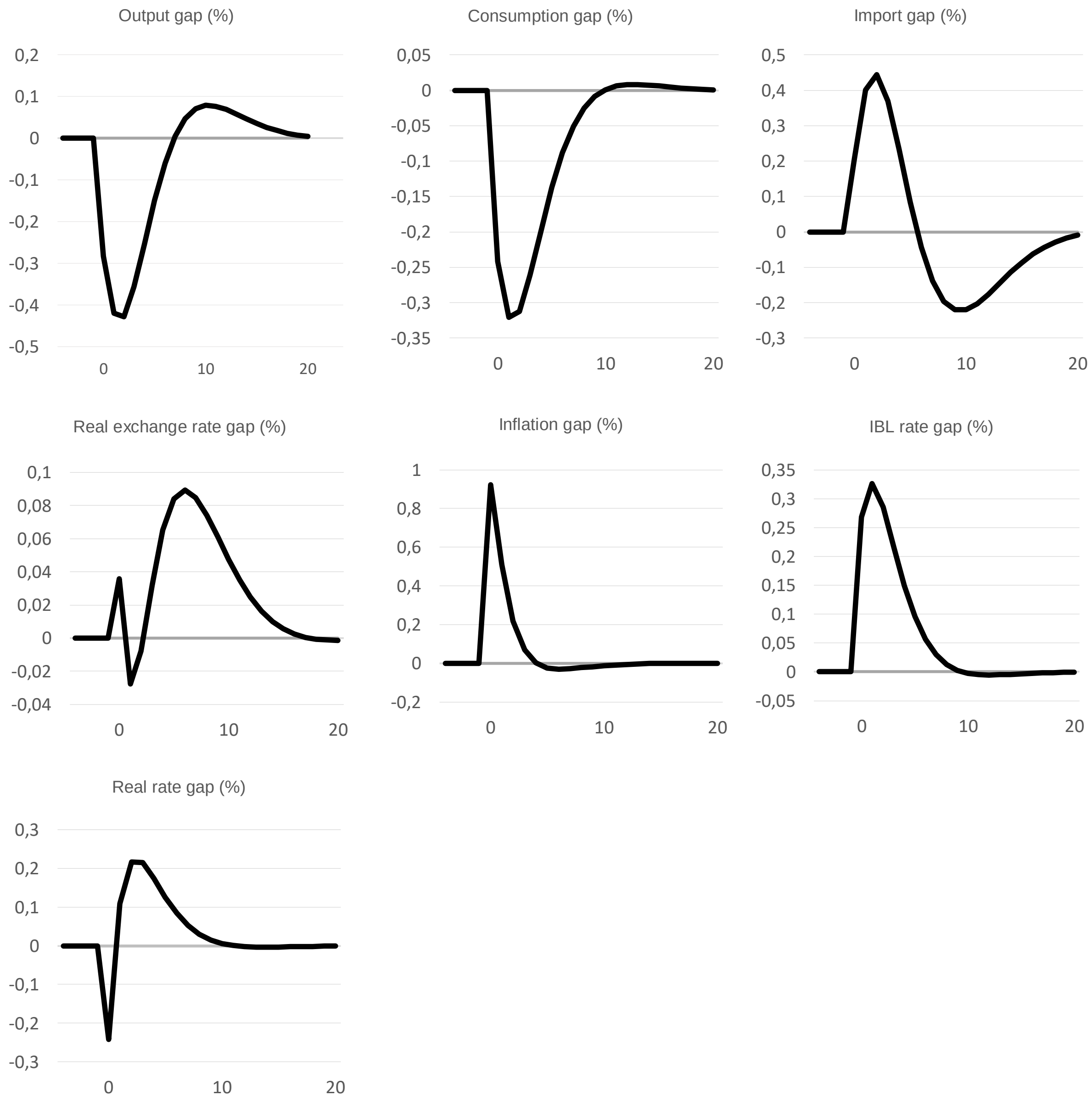


Chart 3.2 shows the responses of the model's main variables to a positive 1% cost shock. A positive cost shock drives up the price of domestic goods through the Phillips curve. To stabilise inflation, the central bank raises the interest rate. An increase in the key rate leads to a redistribution of the utility of consumers between the current and future periods in favour of the future and, consequently, to a drop in consumption of the current period. Since the consumer basket includes both domestic and imported goods, the demand for goods in both categories is decreasing. There are two opposite effects on the real exchange rate. An increase in the interest rate through interest rate parity affects the exchange rate in the direction of its strengthening. Nevertheless, increased demand for foreign goods (that is, an increase in demand for foreign currency) affects the exchange rate in the direction of weakening. Imports also depend on another two phenomena. First, an appreciation makes foreign goods relatively cheaper and opens up a positive import gap. The decline in consumer demand then pushes the import gap into negative territory.

Chart 3.3. Impulse response functions, monetary policy shock

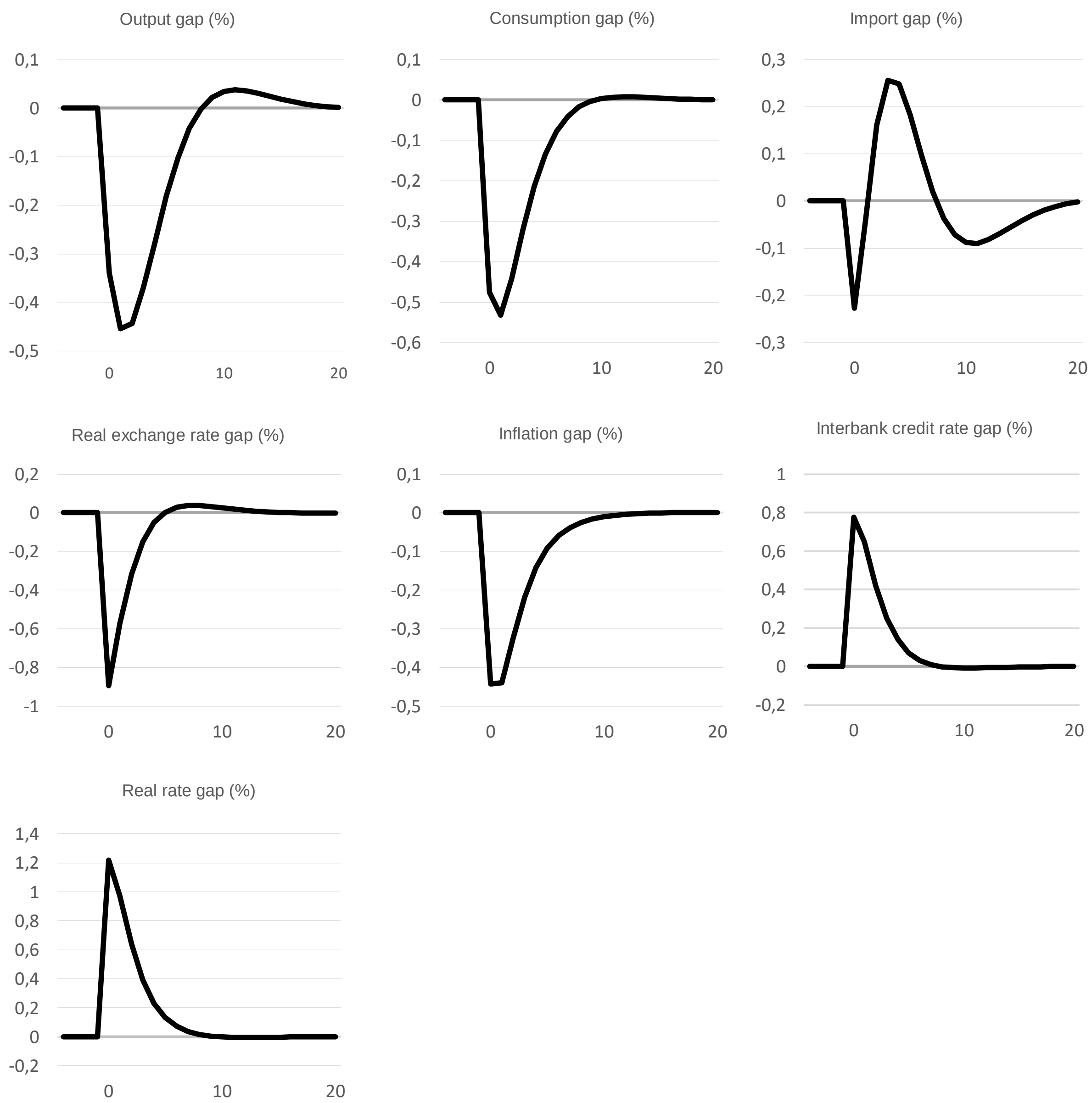


Chart 3.3 shows the responses of the model's main variables to a 1% positive monetary policy shock (the rate rises). An increase in the key rate leads to a redistribution of the utility of consumers between the current and future periods in favour of the future and, consequently, to a drop in consumption of the current period. Since the consumer basket includes both domestic and imported goods, the demand for goods in both categories is decreasing. Decreased demand leads to lower prices and a negative inflation gap. In addition, raising the rate in line with uncovered interest-rate parity leads to real exchange rate appreciation. As a result, imported goods become relatively cheaper, which also has a disinflationary effect.

The response to an oil price shock, as expected, depends on the presence or absence of a fiscal rule in the model.

Chart 3.4. Impulse response functions, oil price shock

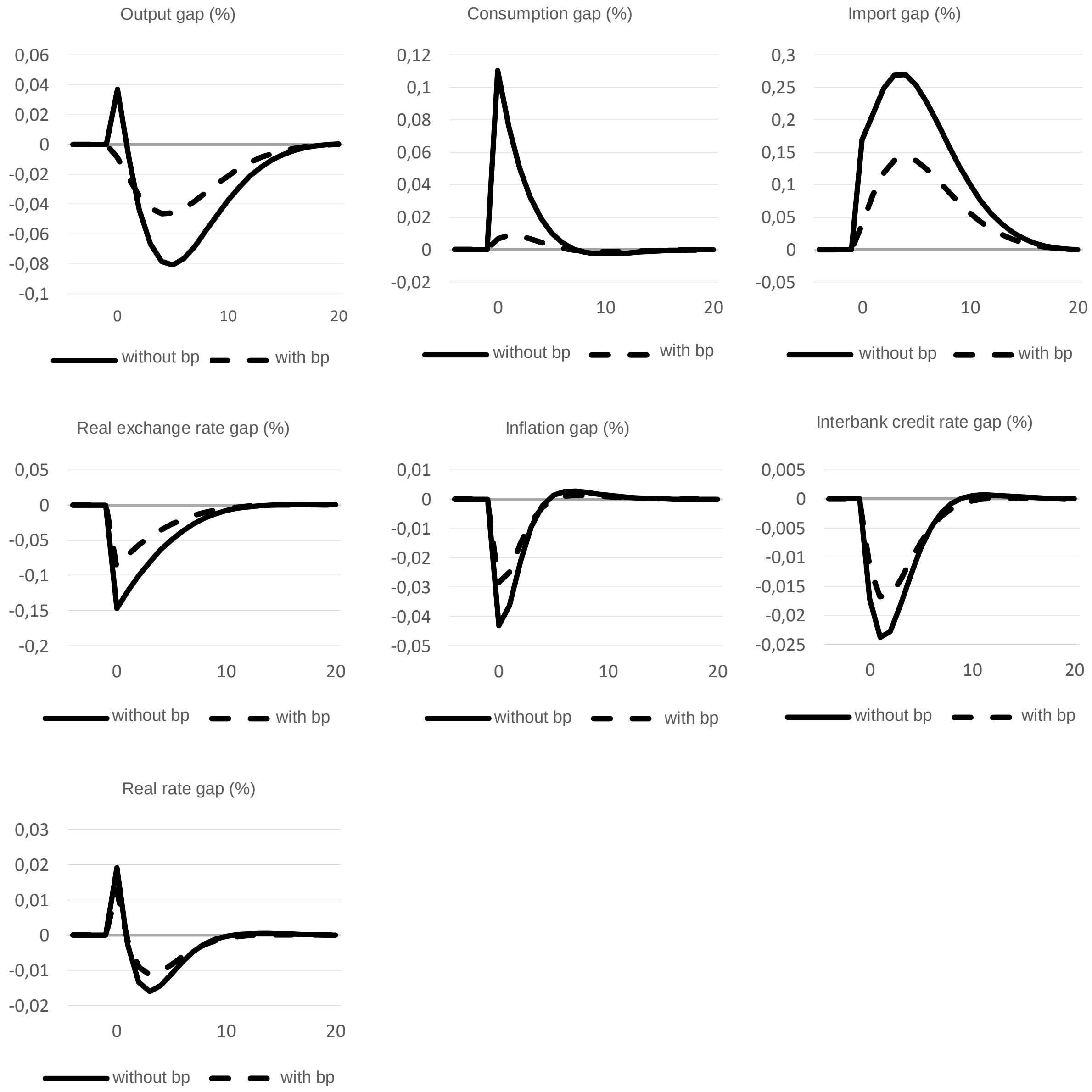


Chart 3.4 shows the responses of the main variables of the model to a 1% positive oil price shock (oil price is rising). An increase in the price of oil leads to a decrease in the risk premium and, accordingly, a strengthening of the exchange rate. With a stronger exchange rate, foreign goods become relatively cheaper. This leads to the opening of a negative inflation gap, and imports increase. The Central Bank cuts the key rate to bring inflation back to the target. Reducing the interest rate leads to a redistribution of consumer income in favour of consumption instead of saving, thus, consumption increases. Output is declining because consumption is growing less than imports. In the situation with the fiscal rule, the responses have the same direction as in the absence of it, but the impact of the shock on the model variables is lower, since the effect of the rule partially offsets the effects of the exchange rate.

Based on the analysis of impulse responses, we can say that the response of variables to model shocks reflects the mechanisms characteristic of neo-Keynesian DSGE models of a small open economy (for example, (Medina, 2007)).

8.3. Checking if the model fits the data

To assess the model adequacy and its compliance with the data, simulations are performed using a linear[11] model with a target of 4% per 100,000 periods. Simulations are made using the built-in function of the IRIS[12] package for Matlab. All shocks of the model are used for the simulation, except for the monetary policy shock. This is because, I assume, the central bank uses its key rate to respond to other shocks, but does not create shocks with its policies. The shocks for the simulations are taken from a normal distribution.

Table 3.2 compares the standard deviations and autocorrelation coefficients of observed data for the period 2003 Q2 to 2021 Q2 and simulated data.

Table 3.2. Comparison of characteristics of observed and simulated data

	Standard deviation		Ratio of autocorrelation AR(1)	
	Model	Data	Model	Data
Consumption gap (%)	5.99	5.87	0.69	0.80
Export gap (%)	3.05	3.33	0.32	0.77
Interbank credit rate gap (%)	2.28	2.59	0.87	0.88
Inflation gap (%)	4.41	3.62	0.43	0.48
Gap of the real ruble/dollar exchange rate (%)	11.52	13.87	0.45	0.92
Oil price gap (%)	20.69	18.56	0.61	0.75
External interest rate gap (%)	0.89	1.03	0.89	0.96
External inflation gap (%)	1.79	1.39	0.33	0.25

As can be seen from the table, the values of the considered characteristics of the simulated and actual data are close, and therefore, the resulting model reflects the actual dynamics of the variables quite well.

Table 3.3 provides a similar comparison for the fiscal rule model.

Table 3.3. comparison of characteristics of observed and simulated data for a model with a fiscal rule

	Standard deviation		Ratio of autocorrelation AR(1)	
	Model	Data	Model	Data

[11] Both the model with ZLB and the model without ZLB are loglinearised around the steady state. In this case, the model c ZLB does not become linear, since the ZLB condition itself is non-linear. Therefore, hereinafter, for brevity, the model without ZLB is called linear, and the model with ZLB is called nonlinear.
[12] IRIS is a package for macroeconomic modeling and forecasting in Matlab.

Consumption gap (%)	5.47	5.87	0.70	0.80
Export gap (%)	3.04	3.33	0.32	0.77
Interbank credit rate gap (%)	2.22	2.59	0.87	0.88
Inflation gap (%)	4.35	3.61	0.42	0.48
Gap of the real ruble/dollar exchange rate (%)	11.06	13.86	0.43	0.92
Oil price gap (%)	20.69	18.56	0.61	0.75
External interest rate gap (%)	0.89	1.03	0.89	0.96
External inflation gap (%)	1.78	1.39	0.33	0.25

As can be seen from the table, for this version of the model, the simulated characteristics are also close to those calculated from the data. It should also be noted that the standard deviations of the variables are less than or equal to those calculated for the model without the rule. This illustrates the operation of the fiscal rule, this mechanism reduces the volatility of variables.

It is worth noting that in the range I used from Q2 2003 to Q2 2021, several significant events occurred in the Russian economy: the adopted inflation targeting at the end of 2014, several revisions of the parameters of the fiscal rule, and periods when the rule was canceled, crises in 2008, 2014 and 2020. However, these periods are too short to give reliable estimates. For example, the period after the adopted inflation targeting includes only 32 points on the quarterly data. In addition, the purpose of this experiment is only to test the possibility of obtaining variables with characteristics close to those of real data using the model. The results show that these characteristics are quite close, and thus the conclusions drawn from the model can be considered relevant for the Russian economy.

9. Optimal inflation target level

This section is devoted to a discussion of the results of simulations on the constructed model. Simulations are made for 12,500[13] periods using the built-in function of the IRIS package for Matlab. The paper considered values from 0.5% per annum to 4% per annum with a step of 0.1.

Based on the data received, I calculate the probabilities of being at the ZLB and find the optimal target level based on the structural loss function. I also study the dependence of the optimal inflation target level on the real neutral interest rate.

9.1. Probability of being at the ZLB

The probabilities of being at the ZLB obtained from the simulated data for the model with the fiscal rule and without the fiscal rule are shown in Charts 4.1 and 4.2.

For the base model without a fiscal rule, the probability of being in the negative area of interest rates with an inflation target of 4% is about 1%.

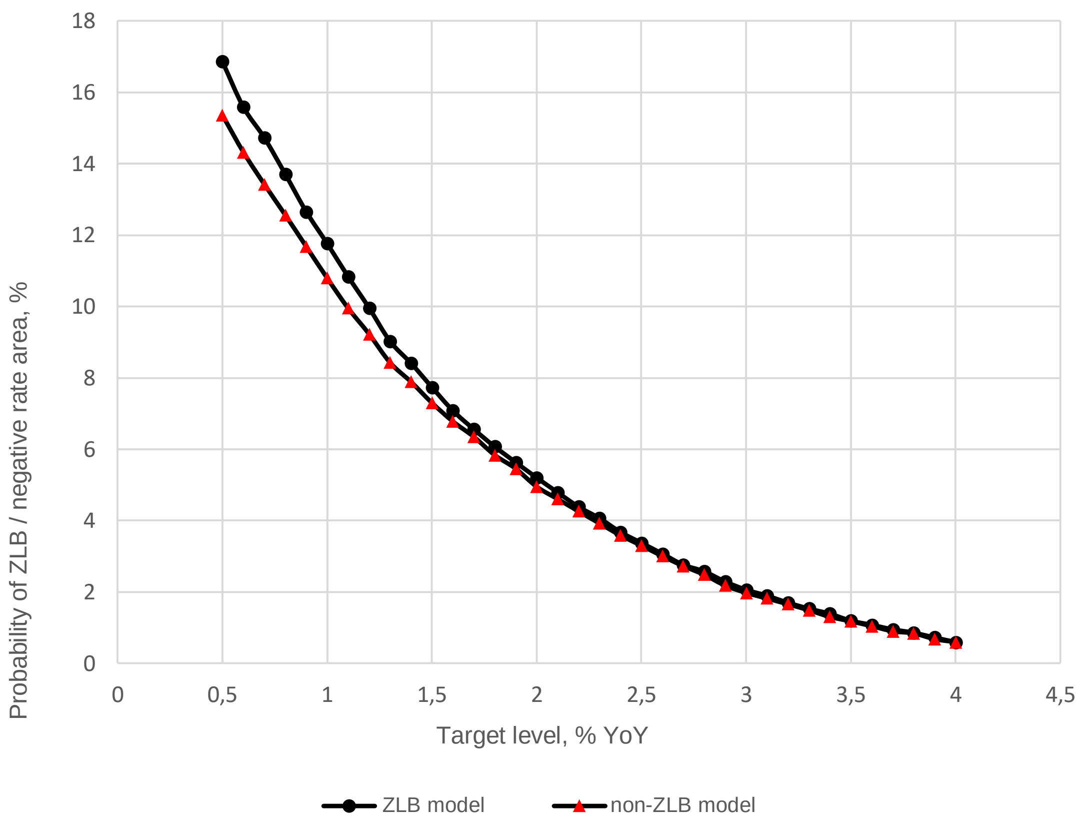
[13] This is due to a trade-off between accuracy and the amount of time required for calculations.

The probability was calculated with the following formula:

$$prob^{ZLB} = 100 * \frac{\sum_{i=0}^{N} i_t^{ZLB} \leq 0}{N},$$

where $prob^{ZLB}$ is the probability of being at the ZLB / in the negative area of interest rates, N is the number of simulation periods.

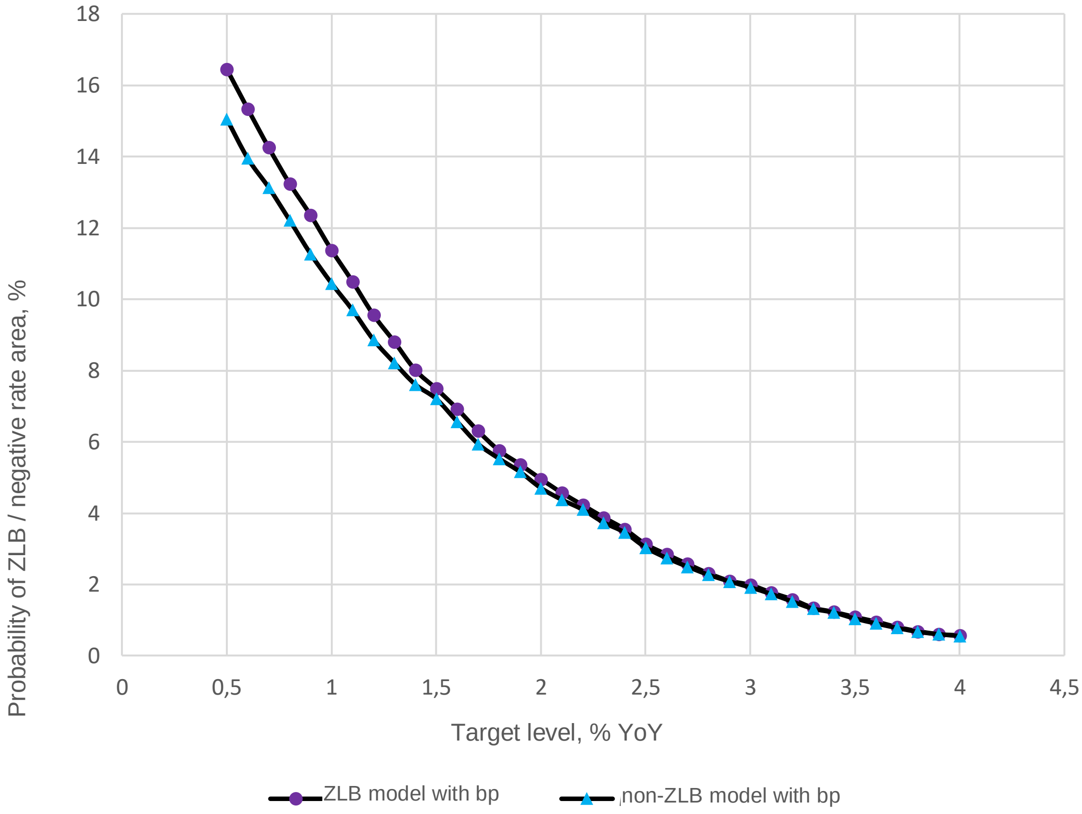
Chart 4.1. ZLB probability in the base model



The probability of ZLB increases as the level of the inflation target decreases. This is due to the fact that the nominal interest rate is the sum of the real interest rate and equilibrium inflation. With a constant real interest rate, as the inflation target decreases, the nominal interest rate also decreases and approaches the zero lower bound. With a sufficiently low nominal rate, even small shocks lead to a zero lower bound, and thus the probability of being at the ZLB increases as the inflation target is lowered.

Moreover, in a situation where the central bank cannot lower the rate below zero, it needs more time to stabilise the economy than in a situation where there is no such restriction.

Chart 4.2. Probability of ZLB by model with fiscal rule



If there is a fiscal rule in the model, the ratio of the model with ZLB / model without ZLB and the negative relationship between the probability of ZLB and the target level, as described above, remain unchanged. However, for each level of the target, this probability becomes lower, since some of the shocks are leveled by the rule.

9.2. Optimal level choice criteria

As discussed above, to investigate the optimal inflation target level, we need to use a structural loss function. I use the function with micro-foundations as in Woodford (2001). This function is a second-order approximation of the consumer utility function. This kind of function has several important advantages. First, it is a structural function that naturally takes into account the mechanisms and parameters of the model. Secondly, it reflects the welfare of consumers, that is, the inflation target is chosen based not on the abstract task of the central bank, but based on the utility of households. Third, this function is considered in deviations from the natural level of variables (that is, variables in an economy without rigidities) and, thus, allows taking into account the costs of inflation considering the rigidities existing in a particular economy. It follows from the last property that the value of this function is negative, since in an economy without rigidities, nominal variables do not affect real variables, and thus consumers do not bear the costs of high inflation. Thus, the function reflects the loss to society in terms of the deviation from an economy with flexible prices.

For my model specification, this function looks like:

$$\log(C_t - \eta * C_{t-1} * e^{-\zeta_{z,t}}) - \frac{1}{1+\sigma_L} * \int_0^1 (N_t(h))^{1+\sigma_L} dh =$$

$$\left.\begin{aligned} &\frac{1}{1-\eta} * \left[\frac{C_t - C^n}{C_t} - \eta * \frac{C_{t-1} - C^n}{C_t} - \frac{1}{2} * \frac{1}{1-\eta} * \left(\frac{C_t - C^n}{C_t}\right)^2 \right. \\ &+ \frac{1}{1-\eta} * \left(\frac{C_t - C^n}{C_t}\right) * \left(\frac{C_{t-1} - C^n}{C_t}\right) - \frac{1}{2} * \frac{\eta^2}{1-\eta} \\ &* \left(\frac{C_{t-1} - C^n}{C_t}\right)^2 + \zeta_{c,t} * \frac{C_t - C^n}{C_t} - \eta * \zeta_{c,t} \\ &* \frac{C_{t-1} - C^n}{C_t} - \frac{\eta}{1-\eta} * \zeta_{z,t} * \frac{C_t - C^n}{C_t} + \frac{\eta}{1-\eta} \\ &\left. * \zeta_{z,t} * \left(\frac{C_{t-1} - C^n}{C_t}\right)\right] \end{aligned}\right\} \boxed{\text{Consumption}} \quad (9.2.1)$$

$$\left.\begin{aligned} &- \int_0^1 \left((N^n)^{1+\sigma_L} * \frac{N_t(h) - N^n}{N^n} + \frac{1}{2} * \sigma_L * (N^n)^{1+\sigma_L} \right. \\ &\left. * \left(\frac{N_t(h) - N^n}{N^n}\right)^2\right) dh \end{aligned}\right\} \boxed{\text{Labour}} \quad (9.2.2)$$

,

где $C - consumption, N - labour, C^n - natural\ level\ of\ consumption,$

$N^n - natural\ level\ of\ labor, h - labor\ type\ index, \eta - habits\ in\ concumptin,$

$\zeta_c - \ consumption\ shock, \zeta_z - productivity\ shock,$

$\sigma_L - Frish\ elasticity.$

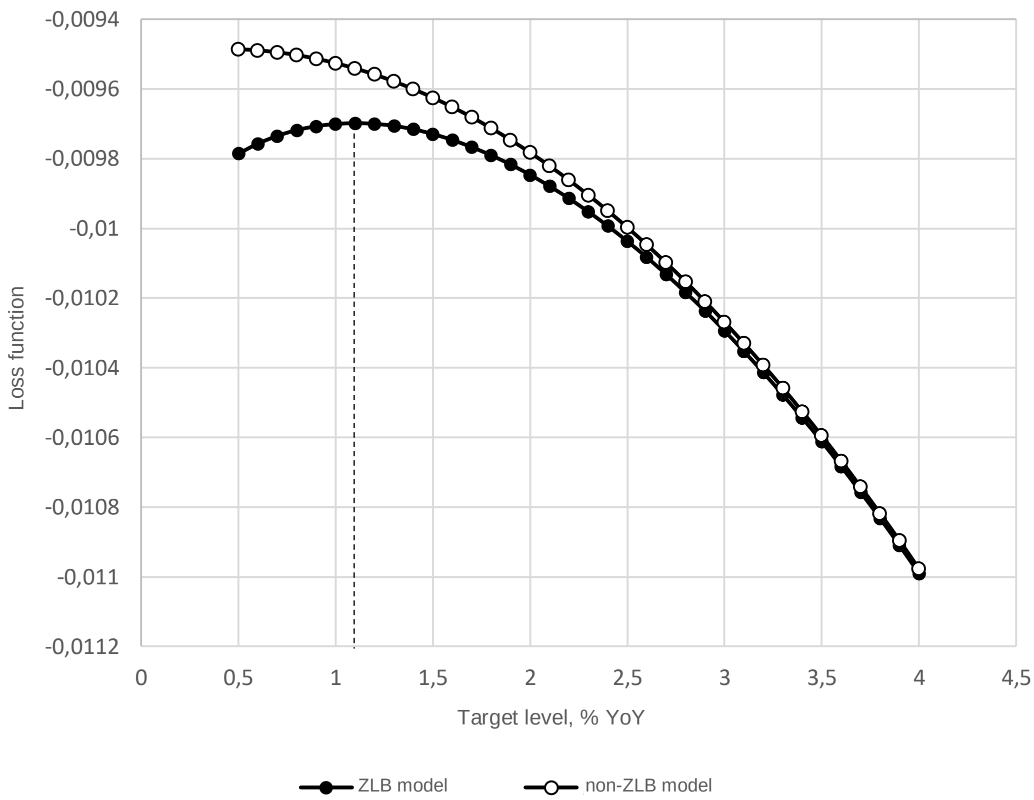
9.3. CHOICE OF OPTIMAL TARGET LEVEL AND ZLB PROBABILITY

The choice of the optimal inflation target level is based on the loss function given in Section 4.2. For each target level, based on the simulated variables, the value of the loss function is calculated, then its average value for all periods is calculated, that is, the unconditional mathematical expectation is calculated. However, I assume that the loss function has the same value for each period. This premise is due to the fact that the number of simulation periods is not a direct analogy of the time scale, but rather a repetition of the experiment in order to bring the sample mean closer to the actual one.

By calculating the loss function in this way, we get that, as a result, each target level corresponds to a certain value of the loss function. The optimal target means a target that corresponds to the smallest (modulo) value of the loss function.

I run simulations for targets from 0.5% to 4% in increments of 0.1. As described above, for each such target, the corresponding value of the loss function is obtained. Such pairs are calculated both for the model with ZLB (non-linear) and for the model without ZLB (linear). The results are shown in Chart 4.3.

Chart 4.3. Loss function for the base model



I would like to draw attention to a few points. First, the values of the loss function for each target level are negative for both the linear and non-linear models. As discussed above, this function is given in terms of deviations from variables in an economy without rigidities and reflects the loss to the economy from price dispersion. Second, for the linear model, the loss function increases (in absolute value) as the target level decreases, if there is no zero lower bound, the lower inflation is, the better consumers are in terms of their welfare. Third, if a zero lower bound on interest rates is added (the ZLB model), there is a trade-off between losses from too high inflation and a zero bound on interest rates. Moreover, at sufficiently high levels of the target, the first effect prevails, and as the level of the target decreases, the second begins to predominate. The optimum in terms of the smallest (modulo) value of the consumer loss function is achieved at a target level of 1.1%.

Chart 4.4. Loss function for a model with a fiscal rule

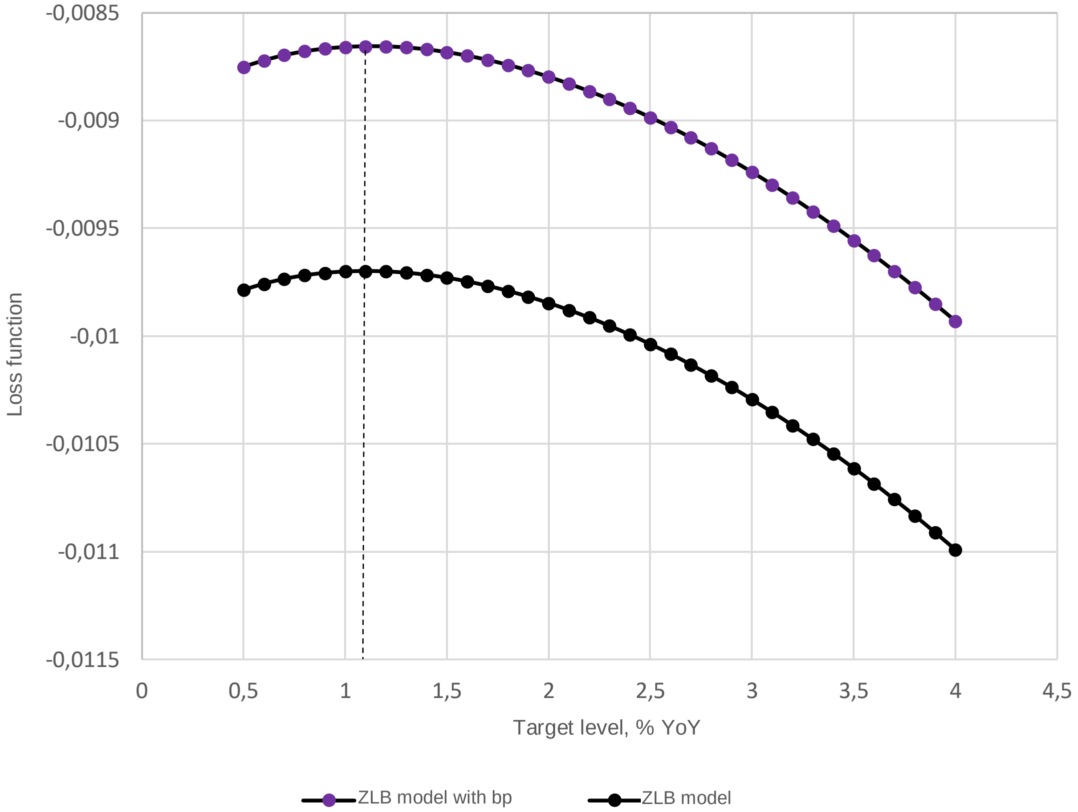


Chart 4.4 shows the loss function for the model with the fiscal rule (bp) and for the base model. If there is a fiscal rule in the model, the optimal target level remains the same of 1.1%. It should be noted that, in general, for a model with a fiscal rule, the modulo loss function is less than for an economy without such a rule at each corresponding target level, since the fiscal rule eliminates some of the shocks, and other things being equal, the ZLB probability is to decrease. Probably, when considering a smaller step of the chosen optimal rates for a model with a fiscal rule, a lower target would be chosen. But given that the choice of the target even with an accuracy of 0.1 is of academic rather than practical interest, and also taking into account the duration of the calculations, it was decided not to conduct such experiments.

9.4. OPTIMAL INFLATION AND REAL INTEREST RATE

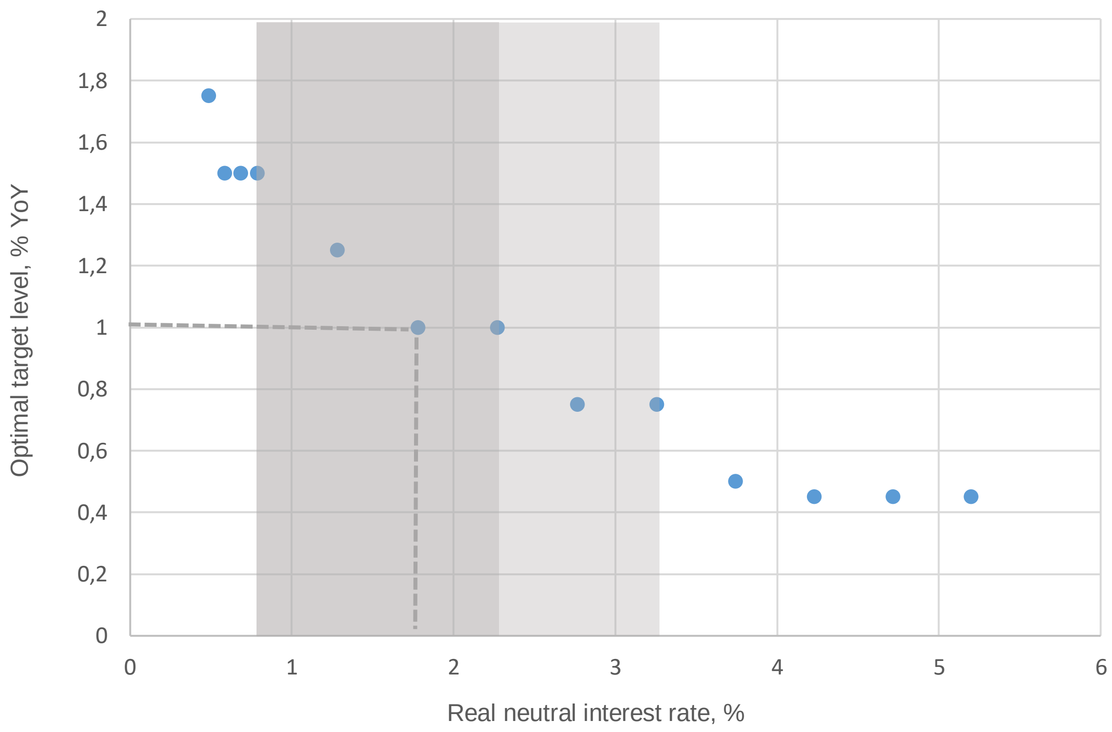
When studying the issue of choosing the optimal target level, it is necessary to take into account the value of the equilibrium real interest rate. As discussed in section 8, I assume this value is 1.78% in base calibration.

To study the influence of different values of the real interest rate on the choice of the optimal target in my model, I run simulations (the parameters are described at the beginning of this section), varying the real rate using the economic growth parameter (I assume growth from 0.2% to 5%, which corresponds to the real rate from 0.48% to 5.2%), and for such real rates I choose the optimal target level from 0.5% to 4% in increments of 0.25. The optimal target level is chosen in the same way as before, based on the consumer loss function.

Chart 4.5 points mean pairs (r^*, π^*), where r^* is the real neutral interest rate, π^* is the optimal target level. I have concluded that with an increase in the real rate in the economy, the optimal target level decreases.

The light grey area in Chart 4.5 shows the area of 1-3% - the current consensus estimate of the real neutral rate of analysts and researchers for Russia. Dark gray indicates the range of 1-2%, which is given in the reports on monetary policy by the Bank of Russia. As seen in Chart 4.5, for a wider range, the optimum lies in the range [0.75; 1.5], and for a narrower range - in the range [1; 1.5].

Chart 4.5. Optimal target level and real neutral interest rate



9.5. Robustness to changing model parameters

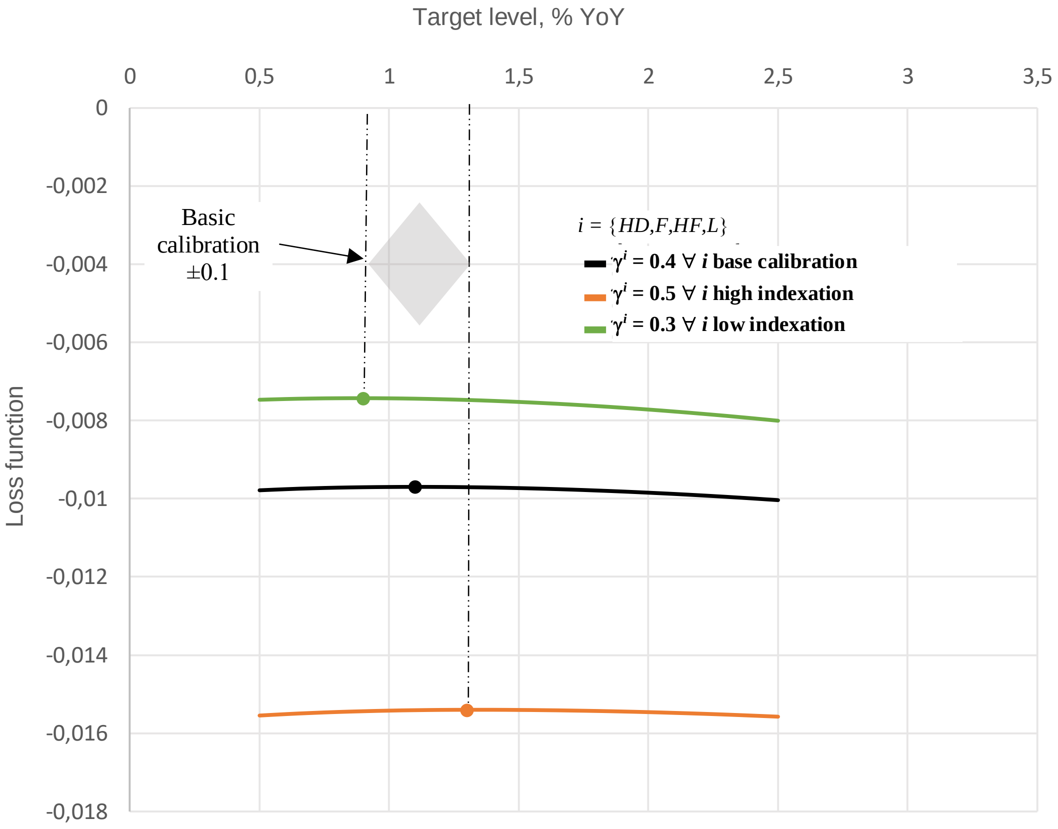
To check the robustness of the results, I consider the loss functions when changing some model parameters, such as the degree of indexing and the Calvo coefficient. These parameters are chosen because estimating their true values from macro data is the most difficult and they tend to be best estimated from micro data. However, for Russia, there are very few studies evaluating these coefficients. In addition, rigidities in the economy directly create a mechanism for the impact of inflation on real variables, and in this regard, it is important to understand how the conclusions of the model about the optimal target change when the coefficients change.

I build loss functions for coefficient deviations on the basis of a base calibration of ±0.1. For coefficients lying in the range $(0,1)$, such a change is at least 10% of the original value.

Alternative calibration - degree of indexation

The first set of coefficients I consider is the degree of indexing. As seen in Chart 4.6, under the assumption that the degree of indexation lies in the range $[0,3;\ 0,5]$, the optimal inflation rate lies in the range $[0,9;\ 1,3]$. It is important that for all the studied deviations of the coefficients, the logic predicted by the theory is preserved, and the closer the degree of indexation to perfect is (since it is equal to one), the higher the target level is chosen as optimal. The logic of this relationship is as follows: at a very low degree of indexation, firms practically do not change prices taking into account inflation, the price dispersion increases, having a negative impact on consumers. Thus, in terms of consumer welfare, lower inflation is optimal, despite losses from the ZLB.

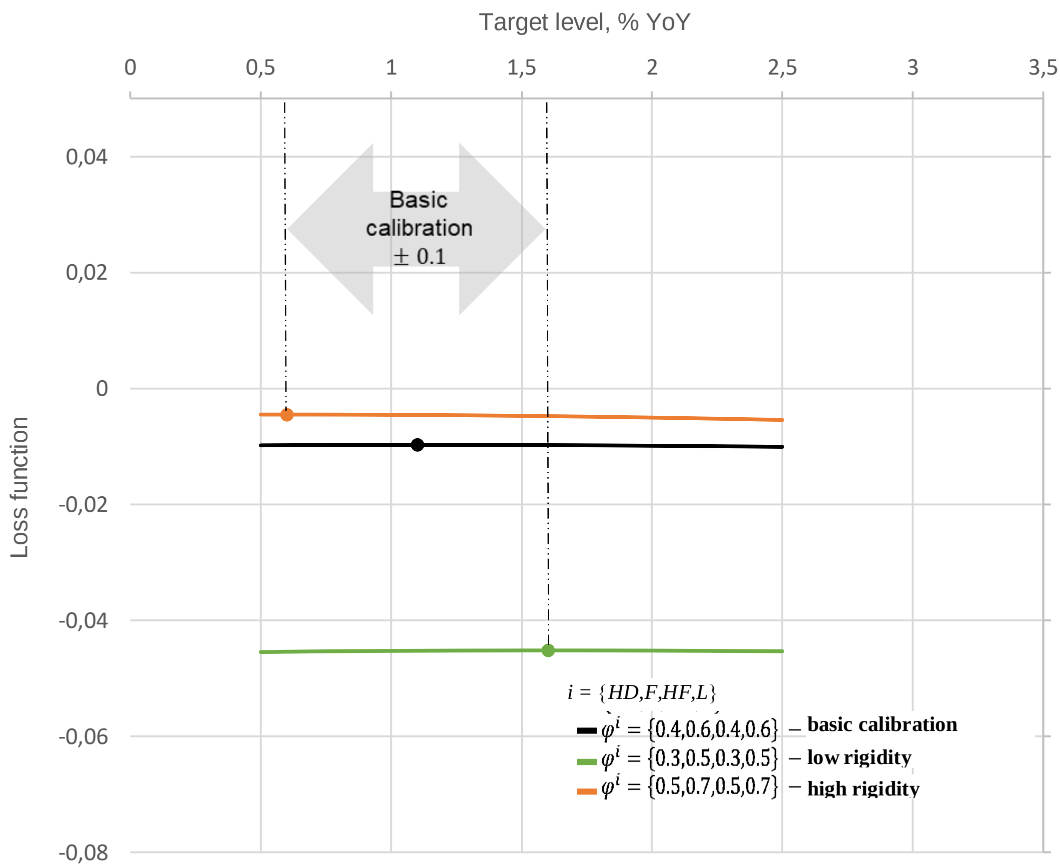
Chart 4.6. Loss function. Alternative calibration - degree of indexation



Alternative calibration - Calvo coefficient

The second set of coefficients I am considering are the Calvo coefficients. As seen in Chart 4.7, assuming that the coefficient deviates from the base calibration by ±0.1, the optimal inflation rate lies in the range $[0{,}6;\ 1{,}6]$. As in the previous paragraph, it is important that for all the studied coefficient deviations, the logic predicted by the theory is preserved - the higher the rigidity, the lower the inflation rate is chosen. The logic of this relationship is similar to the logic of the previous paragraph. If prices are very tight, i.e. the Calvo ratio is high, then price dispersion increases and a lower target becomes preferable.

Chart 4.7. Loss function. Alternative calibration - Calvo coefficient

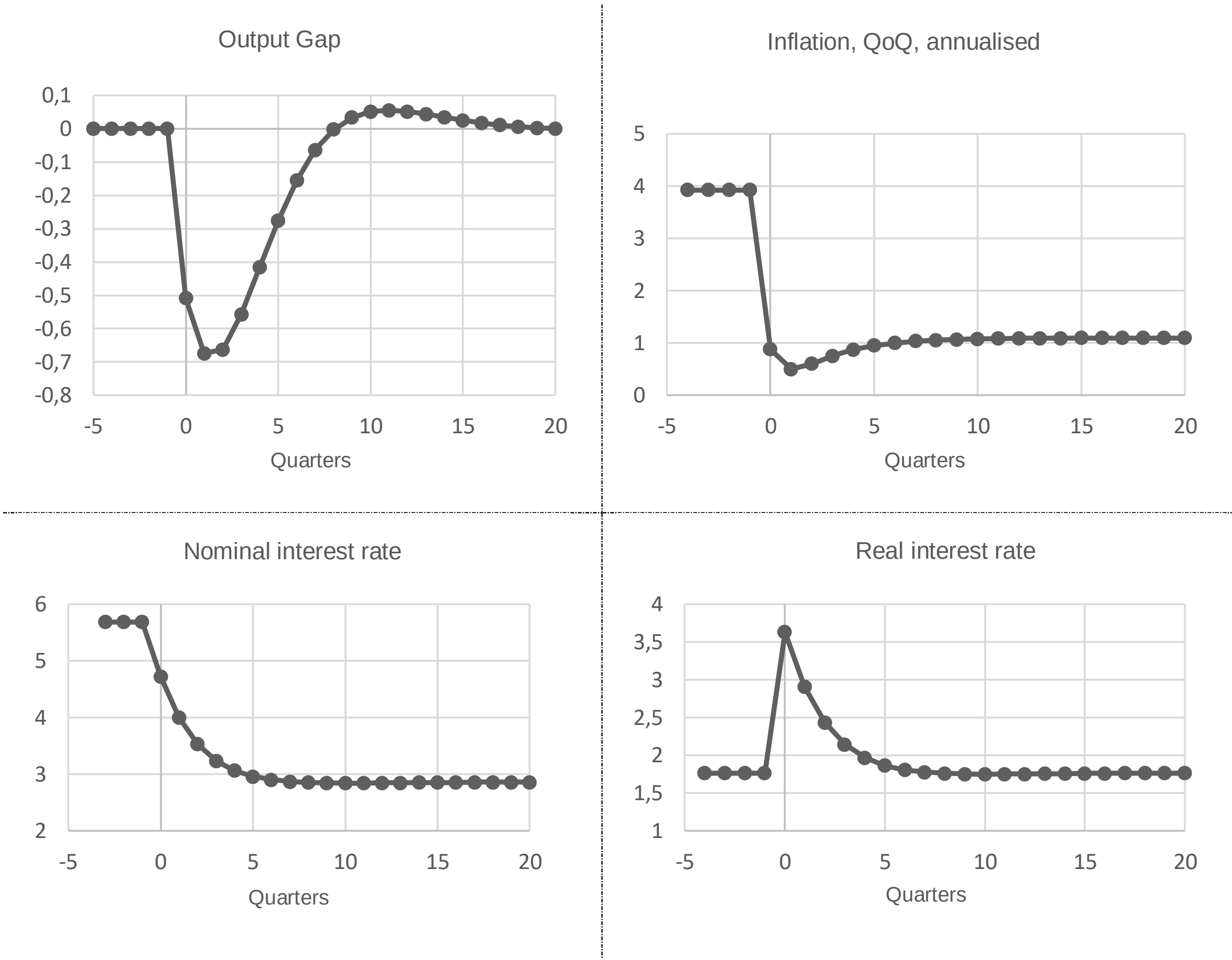


9.6. THE COSTS OF DISINFLATION

The key issue in this section is the cost of moving to a new (lower) target level. First, I examine the dynamics of the transition from the current 4% inflation rate targeted by the Bank of Russia to the 1.1% target that I have found optimal for the Russian economy.

I investigate the adoption of a new target based on the impulse responses of the variables of the model constructed in this paper to the shock of the initial conditions. The resulting paths are shown in Chart 4.8.

Chart 4.8. Impulse responses of variables when moving to a new target



As seen in Chart 4.8, our economy needs about 20 quarters to adjust to a new equilibrium and return output to potential levels. Similar results are obtained by Ascari and Ropele (2012) for the US economy. Meanwhile, the cumulative losses in quarterly output for the Russian economy are about 3%.

As discussed in Ascari and Ropele (2012), the sacrifice ratio SR is commonly used as a loss indicator due to lower target:

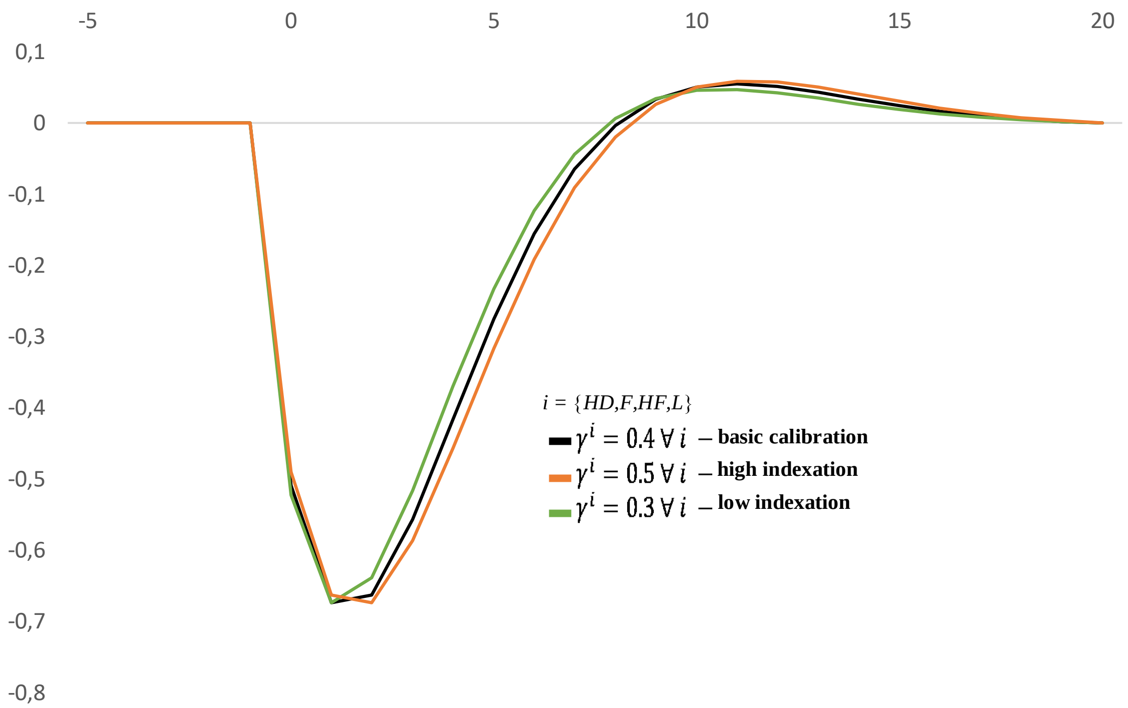
$$SR = -\frac{\sum_{t=0}^{T}(\hat{y}_t)}{\pi^*_{high} - \pi^*_{low}},$$

where $\hat{y}_t$ is the deviation of output from the equilibrium value, *T* is the number of periods for which the output gap closes.

I calculate this ratio based on the above impulse responses. For my model, this coefficient is 1.03.

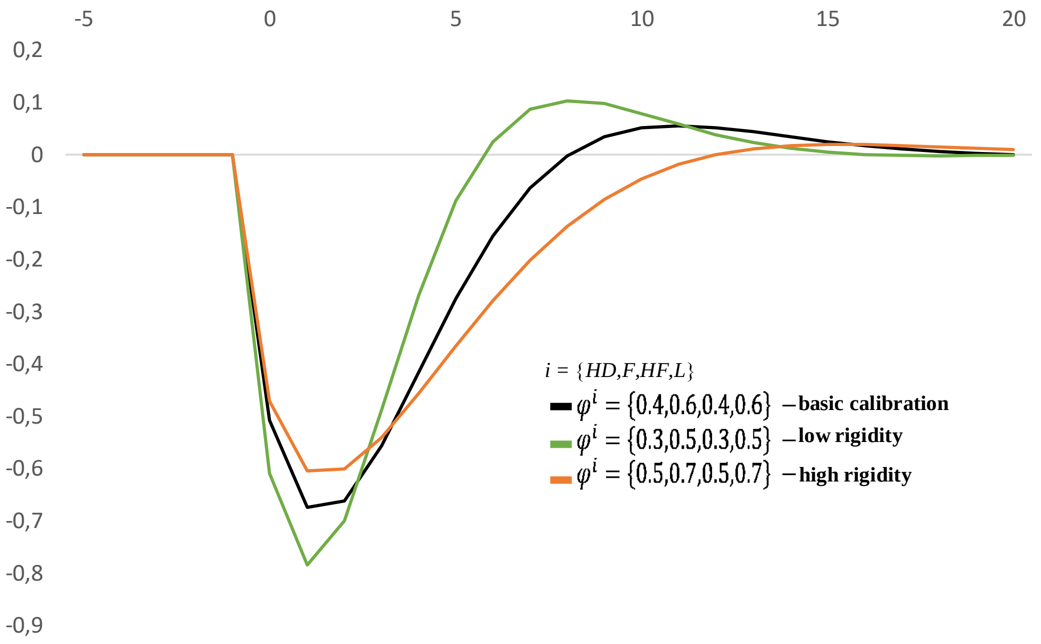
To test the robustness of the results, I calculate the sacrifice ratio for alternative calibrations. I use the same set of parameters as in the previous paragraph. The impulse responses of the output gap for various calibration options are shown in Charts 4.9 and 4.10. I conclude that for different calibrations the sacrifice ratio lies in the range $[0{,}84; 1{,}27]$.

Chart 4.9. Output gap impulse responses for different degrees of indexation



As seen in Chart 4.9, the paths of the return of the output gap to equilibrium are quite close at different degrees of indexation. The sacrifice ratio for varying the degree of indexation lies in the range [0.98; 1.08].

Chart 4.10. Output gap impulse responses for different Calvo coefficients



For deviations in the Calvo ratio, the trajectories of closing the output gap differ somewhat more (Chart 4.10). At lower rigidity, the output falls more strongly, though its recovery is faster; at higher stiffness, the reverse situation is observed. As a result, the sacrifice ratio is in the $[0{,}84; 1{,}27]$ range for this deviation.

As discussed in Ascari and Ropele (2012), the SR ratio typically ranges from 0.5 to 3. Thus, the coefficient 1.03 obtained by me for the basic calibration lies in this range. The range for SR $[0{,}84; 1{,}27]$ obtained for alternative calibrations is also in this range and lies closer to its lower bound.

The SR coefficient obtained for Russia means that the output costs of reducing the target level for the Russian economy are small compared to the economies of the US, euro area or UK, which may be an additional argument in favour of lower target. However, this result should be approached with caution, since some coefficients from this range were calculated not on the basis of structural models, but on the basis of econometric models, which could play a role in the discrepancy between estimates. In addition, as discussed above, the issue of adopted new equilibrium involves many practical issues and requires further study.

10. Conclusions

In this paper, I explore the issue of choosing the optimal inflation target level, taking into account the trade-off between the costs of high inflation and the increasing probability of facing the zero lower bound (ZLB) issued with a lower inflation target. I have concluded that 1.1% for the base model is the optimal target level for the Russian economy. This corresponds to a probability of being at the ZLB of about 11%.

In addition, examining the dependence of the optimal target level on the real interest rate, I have concluded that this dependence is negative, that is, ceteris paribus, a higher real rate allows you to set a lower inflation target level, and each percentage point of the rate increase allows you to lower the target by about 0.5 percentage points.

I also describe the adoption of a new inflation target. To do this, the paper calculates the sacrifice ratio, which is the cumulative decline in output divided by the difference between the old and new inflation targets. For my model, this coefficient is 1.03, which is closer to the lower bound of similar indicators for the US, euro area are and UK, meaning that for Russia, lower target is associated with relatively small GDP costs. It is also worth noting that the sacrifice ratio does not take into account the positive effects that a decrease in the target level entails, for example, a decrease in price volatility. Thus, positive effects can offset some of the losses. On the other hand, the standard neo-Keynesian DSGE model is based on the logic of rational expectations, and this premise is also fulfilled in my model. If this assumption is weakened, the losses from its adoption may be greater, as expectations will not immediately adapt to new conditions, and the central bank may need to reduce (or even raise) the nominal interest rate more slowly. The final benefits/costs of moving to a lower target depend on many factors (including the benefits/costs of which economic agents we are considering: consumers or firms; the period we analyse, whether it is short, medium or long term; types of agent expectations that may be rational, adaptive, learning, and so on) and require further study.

Thus, based on the study, I have come to the conclusion that, first, with the current target of 4%, the probability of facing the ZLB issue for the Russian economy is quite low and amounts to about 1%. This is consistent with historical data, since the Russian economy has never actually faced such problem. Second, the optimal inflation target for the Russian economy in terms of consumer welfare is 1.1%. Moreover, the target level negatively depends on the real interest rate. The range of the real rate of 1% to 3% corresponds to the optimal level of 0.75% to 1.5%. In addition, the optimal target level depends on model parameters such as Calvo rigidity and indexation degree. The optimal target value amid the above uncertainty lies in the range from 0.6% to 1.6%, which is lower than the current target of the Bank of Russia.

Loss of output during the adoption of a new level of the target, calculated on the basis of the sacrifice ratio, is closer to the lower bound of the range calculated for the economies of the US, Europe and the UK, which is an additional argument in favour of lowering the inflation target.

When interpreting the results, it should be noted that if there is some consensus in the academic literature regarding the very fact of society's losses from high inflation and the mechanism according to which this occurs, then for the costs of low inflation and the mechanism(s) for spreading these costs, there are quite a lot of opinions.

First, instead of the zero lower bound issue, the Effective Lower Bound (ELB) issue can be considered. The presence of ELB in the economy can lead to the fact that when a certain level (greater than zero) of the key rate is reached, its further reduction will not lead to a stimulating effect, and in some cases may even have a deterrent effect[14]. Thus, some positive ELB in the economy may require to set an inflation target higher than in the case when the ZLB issue is considered. Although, in line with the considerations described above, ELB could theoretically be relevant for the Russian economy (as well as for other emerging market countries). However, there are difficulties that prevent it from being used instead of ZLB as a lower inflation target. First of all, ELB (unlike ZLB) is probably not a constant. This indicator may depend on the development of financial institutions, the risks that have developed in the economy, the expectations of economic agents, their confidence in the policy being pursued, the historical level of rates in the economy (for example, if consumers are used to low rates, then ELB may decrease), and so on. In addition, the chosen target level itself can influence the ELB value. Today, for the Russian economy, not only are there no estimates of factors affecting ELB, but even point estimates of the ELB value. Thus, its use as a lower bound on the target value requires additional studies of this mechanism.

Second, my model assumes that non-oil output is homogeneous, that is, it does not take into account the influence of relative prices on the choice of the optimal target level. Including several sectors in the model is likely to cause additional costs from low inflation. However, such an extension of the model significantly complicates its structure and may complicate the interpretation of the results.

There are several other mechanisms to model the costs of low inflation. For example, Abbritti et al. (2021) include labour market friction, endogenous productivity, and downward wage rigidity (DWR) in the neo-Keynesian DSGE model. It leads to asymmetry, which creates the prerequisites for setting a higher target than that which is recognised as optimal in models without such prerequisites.

Diercks (2017) shows that more detailed modeling of the financial sector (and related non-linearities) than is accepted in standard neo-Keynesian DSGE models leads to a higher optimal target level. A detailed list of papers focused on the optimal target level is given in Dierks (2019).

In general, the optimal level of the inflation target is likely to remain a key topic of economic research in the foreseeable future. This is due to its practical significance for inflation targeting central banks as well as the poor development of the topic, which may suggest further research.

It is also worth noting that there are several practical aspects that may be relevant when moving to a new target level outside the scope of this study. For example, I do not focus on the following questions: the moment of adoption of a new target, what should be the economic conditions and the moment of the economic cycle, whether its adoption should be carried out at once or in several stages, how the central bank should conduct an information policy when adopting a new target.

Finally, I deliberately do not include capital restrictions and other changes that have been taking place in the Russian economy since the end of February 2022 in the model. This is due to the fact that this paper focuses on the choice of the optimal target, which is a fundamental matter, and such choice must be made based on the long-term equilibrium structure of the economy. As with any model of the DSGE type, the conclusions of my model depend on its structure. Now, in a period of significant restructuring of economic ties, the equilibrium that the economy of Russia and other world economies will come to remains uncertain. And when the dust settles and it becomes clear which of the current changes will remain temporary shocks and which ones will turn into a new reality, we will inevitably return to this subject.

[14] This can happen if, in the wake of a rate cut, the following financial stability risks occur: following the dollarisation of deposits and the outflow of capital, the national currency weakens, the probability of defaults on foreign exchange obligations of individuals and legal entities goes up, the burden on the capital of financial institutions increases, and, as a result, the number of loan offers decreases.

REFERENCES

Abbritti, Mirko, Consolo, Agostino and Weber, Sebastian. 2021. *Endogenous Growth, Downward Wage Rigidity and Optimal Inflation*, Working Paper Series 2635, European Central Bank.

Andrade P., Jordi Galí, Hervé Le Bihan, Julien Matheron. *The Optimal Inflation Target and the Natural Rate of Interest.* 2019.

Ascari, G. and Ropele, T. *Sacrifice Ratio in a Medium-Scale New Keynesian Model.* s.l. : Journal of Money, Credit and Banking, Vol. 44, No. 2-3 (March-April 2012), 2012.

Ball, L. *What Determines the Sacrifice Ratio?* s.l. : Monetary Policy, pp. 155–82. Chicago : University of Chicago Press, 1994b.

Cecchetti, Stephen G., and Robert W. Rich. *Structural Estimates of the U.S. Sacrifice Ratio.* s.l. : Journal of Business and Economic Statistics, 19, 416–27, 2001.

Céspedes L.F., García-Cicco J. , Saravia D. *Monetary Policy at the Zero lower bound: the chilean experience.* 2014.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. *Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.* s.l. : Journal of Political Economy, 113, 1–45, 2005.

Chung H., Laforte D., Reifschneider D., Willams J. C. *Have We Underestimated the Likelihood and Severity of Zero Lower Bound Events?* s.l. : Journal of Money, Credit and Banking, 44, 47–82, 2012.

Coibion, Gorodnichenko, & Wieland (2012) и Andrade, Gali, Bihan, & Matheron. *The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?* 2019.

Durand, Jean-Jacques, Maryline Huchet-Bourdon, and Julien Licheron. *Sacrifice Ratio Dispersion within the Euro Zone: What Can Be Learned about Implementing a Single Monetary Policy?* s.l. : International Review of Applied Economics, 22, 601–21, 2008.

Gordon, Robert J., and Stephen King. *The Output Cost of Disinflation in Traditional and Vector Autoregressive Models.* s.l. : Brookings Papers on Economic Activity, 1, 205–42, 1982.

IMF. *Country report.* 2021.

International Monetary Fund, Russian Federation. *Staff Report for the 2019 Article IV Consultation.* 2019.

Isakov A., Latypov R. *The Ibsen Manoeuvre: Yet Another R* Estimate.* s.l. : VTB Capital Research Alert, (Very) Technical Brief series, 2019.

Kiley M., Roberts J. M. *Monetary Policy in a Low Interest RateWorld.* s.l. : Brookings Papers on Economic Activity, 48, 317–396, 2017.

Lawrence J. Christiano, Martin S. Eichenbaum, and Mathias Trabandt. *On DSGE Models.* 2018.

Leif B., Husebo T. A., Maih J., Olsen K., Ostnor M. *Finding NEMO: Documentation of the Norwegian economy model.* 2005.

Lindé, J. *DSGE models: still usefull in policy analysis?* 2018.

Medina, Soto. *The chilean business cycles through the lens of a stochastic general equilibrium model.* 2007.

Mikael Apel, Hanna Armelius and Carl Andreas. *The level of the inflation target. 2017.*

S., Bernanke B., T., Kiley M. and J.M., Roberts. *Monetary Policy Strategies for a Low-Rate.* s.l. : AEA Papers and Proceedings, 109, 421–426, 2019.

Smets, Frank, and Raf Wouters. *An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area.* s.l. : Journal of the European Economic Association, 1, 1123–75, 2003.

—.*Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach.* s.l. : American Economic Review, 97, 586–606, 2007.

Woodford. *Interest and Prices.* s.l. : Princeton, NJ: Princeton University Press, 2003.

WorldBank. *Russian economic report.* 2021.

Андреев М.Ю., Полбин А.В. *Optimal monetary policy rules for a resource-rich economy and Zero Lower Bound. 2021*

Дробышевский С., Трунин П., Синельникова-Мурылева Е., Макеева Н., Гребенкина А. *Optimal inflation in Russia: theory and practice.* s.l. : Экономическая политика Т.15 №4 с.8-29, 2020.

Крепцев Д., Поршаков А., Селезнев С., Синяков А. Equilibrium interest rate: estimates for Russia. 2016.

ЦБРФ. *Monetary Policy Report No. 2(38). May 2022.*

—. *Monetary Policy Report. July 2022.*

—. *Monetary Policy Guidelines for 2022–2024. 2021.*

ЦБРФ, ДИП. *Drivers of price inertia: survey evidence. 2017.*

APPENDICES

I OPTIMISATION PROBLEMS

Producers of intermediate domestic goods

Intermediate goods are produced by firms under monopolistic competition in accordance with the production function:

$$Y_t^{HD}(f) = Z_t L_t(f),$$

where Z_t is the stochastic performance trend and

$$Z_t = Z_{t-1} e^{\zeta_t^a}.$$

Intermediate goods are produced with nominal à la Calvo price rigidities. This means that firms are ϕ^i probability to face an inability to change prices, $i \in \{HD, HF\}$, HD are domestic goods sold domestically, HF is domestic goods sold abroad.

If the firm cannot optimise its price in period *t*, then it sets it according to the following rule:

$$P_t^i(f) = \left(\Pi_{t-1}^i\right)^{\gamma_i} P_{t-1}^i(f),$$

$$where\ i \in \{HD, HF\}, \Pi_t^i \equiv \frac{P_t^i}{P_{t-1}^i},$$

$$\Pi^i - steady - state\ value\ of\ \ inflation\ , \gamma_i - degree\ of\ indexation\ и\ 0 \le \gamma_i < 1.$$

If the firm can revise its price for domestic goods sold domestically in period *t*, then it chooses it based on the profit maximisation condition:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi^{HD}) \Lambda_{t+s} \left(\frac{V_{t,t+s}^{HD} P_t^{HD*}(f)}{P_{t+s}} Y_{t,t+s}^{HD} - \frac{W_{t+s}}{P_{t+s}} \frac{Y_{t,t+s}^{HD}}{Z_{t,t+s}} \right),$$

where Λ_t – is the marginal utility of consumers and $Y_{t,t+s}^i(f)$ is the demand for the products of a monopolist who fixed the price in period *t*, in period *t* + *s*, which has the form:

$$Y_{t,t+s}^i(f) = \left(\frac{V_{t,t+s}^i P_t^{i*}}{P_{t+s}} \right)^{-\epsilon_i} Y_{t+s}^i,$$

where V_t^i is the cumulative effect of price indexation on inflation in previous periods:

$$V_{t,t+s}^i = \prod_{j=t}^{t+s-1} \left(\Pi_j\right)^{\gamma_i}.$$

The first order condition for this problem is:

$$\Sigma_{s=0}^{\infty} (\beta \phi^{HD}) \Lambda_{t+s} \left(\frac{\left(V_{t,T}^{HD} P_t^{HD*}(f)\right)^{1-\theta^{HD}}}{P_{t+s}} \left(\frac{1}{P_{t+s}^{HD}}\right)^{-\theta^{HD}} Y_{t+s}^{HD} - \frac{\theta^{HD}}{\theta^{HD}-1} \frac{W_{t+s}}{P_{t+s}} \left(\frac{V_{t,t+s}^{HD} P_t^{HD*}(f)}{P_{t+s}^{HD}} \right)^{-\theta^{HD}} \frac{Y_{t+s}^{HD}}{Z_{t+s}} \right) = 0.$$

Transforming, we get:

$$\frac{P_t^{HD*}(f)}{P_t} \frac{P_t}{P_t^{HD}} = \frac{\theta^{HD}}{\theta^{HD} - 1} \frac{K_t^{HD}}{F_t^{HD}},$$

where

$$K_t^{HD} = \Lambda_{z,t} \frac{W_{z,t}}{P_t} Y_{z,t}^{HD} + \beta\phi^{HD} \mathbb{E}_t \left(\frac{(\Pi_t^{HD})^{\gamma^{HD}}}{\Pi_{t+1}^{HD}} \right)^{-\theta^{HD}} K_{t+1}^{HD},$$

$$F_t^{HD} = \Lambda_{z,t} Y_{z,t}^{HD} + \beta\phi^{HD} \mathbb{E}_t \left(\frac{1}{\Pi_{t+1}^{HD}} \right)^{-\theta^{HD}} \frac{1}{\Pi_{t+1}} \left((\Pi_t^{HD})^{\gamma^{HD}} \right)^{1-\theta^{HD}} \frac{P_t^{HD}}{P_t} F_{t+1}^{HD},$$

$$\Pi_t^{HD} \equiv \frac{P_t^{HD}}{P_{t-1}^{HD}}, \Pi_t \equiv \frac{P_t}{P_{t-1}}.$$

In addition, by transforming

$$P_t^{HD\,1-\theta^{HD}} = \int_0^1 P_t^{HD}(f)^{1-\theta^{HD}} df = (1-\phi^{HD}) * P_t^{HD*\,1-\theta^{HD}} + \int_0^1 \left(\Pi_{t-1}^{HD\;\gamma^{HD}} P_{t-1}^{HD}(f) \right)^{1-\theta^{HD}} df,$$

we see that

$$\left(\frac{P_t^{HD*}}{P_t} \right)^{1-\theta^{HD}} \left(\frac{P_t}{P_t^{HD}} \right)^{1-\theta^{HD}} = \frac{1-\phi^{HD} \left(\frac{\Pi_{t-1}^{HD\;\gamma^{HD}}}{\Pi_t^{HD}} \right)^{1-\theta^{HD}}}{1-\phi^{HD}}.$$

Similarly, the price is chosen for domestic goods sold abroad:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\phi^{HF}) \Lambda_{t+s} \left(\frac{V_{t,t+s}^{HF} P_t^{HF*}(f)}{P_{t+s}} Y_{t+s}^{HF} - \frac{1}{\mathcal{E}_{t+s}} \frac{W_{t+s}}{P_{t+s}} \frac{Y_{t+s}^{HF}}{Z_{t+s}} \right).$$

The first order condition for this problem is:

$$\sum_{s=0}^{\infty} (\beta\phi^{HF}) \Lambda_{t+s} \left(\frac{\left(V_{t,T}^{HD} P_t^{HF*}(f) \right)^{1-\theta^{HF}}}{P_{t+s}} \left(\frac{1}{P_{t+s}^{HF}} \right)^{-\theta^{HD}} Y_{t+s}^{HF} - \frac{\theta^{HF}}{\theta^{HF}-1} \frac{1}{\mathcal{E}_{t+s}} \frac{W_{t+s}}{P_{t+s}} \left(\frac{V_{t,t+s}^{HF} P_t^{HF*}(f)}{P_{t+s}} \right)^{-\theta^{HF}} \frac{Y_{t+s}^{HF}}{Z_{t+s}} \right) = 0.$$

Transforming, we get

$$\frac{P_t^{HF*}(f)}{P_t} \frac{P_t}{P_t^{HF}} = \frac{\theta^{HF}}{\theta^{HF}-1} \frac{K_t^{HF}}{F_t^{HF}},$$

where

$$K_t^{HF} = \Lambda_{z,t} \frac{1}{\mathcal{E}_{t+s}} \frac{W_{z,t}}{P_t} Y_{z,t}^{HF} + \beta\phi^{F} \mathbb{E}_t \left(\frac{(\Pi_t^{HF})^{\gamma^{HF}}}{\Pi_{t+1}^{HF}} \right)^{-\theta^{HF}} K_{t+1}^{HF},$$

$$F_t^{HF} = \Lambda_{z,t} Y_{z,t}^{HF} + \beta\phi^{HF} \mathbb{E}_t \left(\frac{1}{\Pi_{t+1}^{HF}} \right)^{-\theta^{HF}} \frac{P_t^{HF}}{P_t} \left((\Pi_t^{HF})^{\gamma^{HF}} \right)^{1-\theta^{HF}} F_{t+1}^{HF},$$

$$where\ \Pi_t^{HF} \equiv \frac{P_t^{HF}}{P_{t-1}^{HF}}.$$

Transforming,

$$P_t^{HF\,1-\theta^{HF}} = \int_0^1 P_t^{HF}(f)^{1-\theta^{HF}} df = (1-\phi^{HF}) * P_t^{HF*\,1-\theta^{HF}} + \int_0^1 \left(\Pi_{t-1}^{HF\;\gamma^{HF}} P_{t-1}^{HF}(f) \right)^{1-\theta^{HF}} df,$$

we see that

$$\left(\frac{P_t^{HF*}}{P_t}\right)^{1-\theta^{HF}} \left(\frac{P_t}{P_t^{HF}}\right)^{1-\theta^{HF}} = \frac{1-\phi^{HF}\left(\frac{\Pi_{t-1}^{HF}{}^{\gamma^{HF}}}{\Pi_t^{HF}}\right)^{1-\theta^{HF}}}{1-\phi^{HF}}.$$

Producers of intermediate imported goods

Intermediate goods are produced by firms under monopolistic competition from foreign goods.

Just like domestic goods, foreign goods are produced with nominal à la Calvo price rigidities. This means that firms are ϕ^F probability to be unable to change prices.

If the firm can revise its price for domestic goods sold domestically in period *t*, then it chooses it based on the profit maximisation condition:

$$\mathbb{E}\sum_{s=0}^{\infty}(\beta\phi^F)\,\Lambda_{t+s}\left(\frac{V_{t,t+s}^F P_t^{F*}}{P_{t+s}}Y_{t,t+s}^F - \mathcal{E}_{t+s}\frac{P_t^{For}}{P_{t+s}}\frac{Y_{t,t+s}^F}{Z_{t,t+s}}\right),$$

where P_t^{For} is the price of intermediate goods abroad, P_t^{F*} is the effective price of importers.

If the firm cannot optimise its price in period *t*, then it sets it according to the following rule:

$$P_t^F(f) = (\Pi_{t-1}^F)^{\gamma_F} P_{t-1}^F(f),$$

$$\Pi_t^F \equiv \frac{P_t^F}{P_{t-1}^F}, \Pi^F - inflation\ stady\ state, 0 \le \gamma_F < 1.$$

As in the case of producers of domestic goods, price indexation for importers is imperfect.

The first order condition for this problem is:

$$\sum_{s=0}^{\infty}(\beta\phi^F)\,\Lambda_{t+s}\left(\frac{\left(V_{t,T}^F P_t^{F*}(f)\right)^{1-\epsilon^F}}{P_{t+s}}\left(\frac{1}{P_{t+s}^F}\right)^{-\epsilon^F} Y_{t+s}^F - \frac{\epsilon^F}{\epsilon^F-1}\mathcal{E}_{t+s}\frac{P_t^{For}}{P_{t+s}}\left(\frac{V_{t,t+s}^F P_t^{F*}(f)}{P_{t+s}}\right)^{-\epsilon^F}\frac{Y_{t+s}^F}{Z_{t+s}}\right) = 0.$$

Transforming, we get

$$\frac{P_t^{F*}(f)}{P_t}\frac{P_t}{P_t^F} = \frac{\epsilon^F}{\epsilon^F-1}\frac{K_t^F}{F_t^F},$$

where

$$F_t^F = \Lambda_{z,t}Y_{z,t}^F + \beta\phi^F\mathbb{E}_t\left(\frac{(\Pi_t^F)^{\gamma^F}}{\Pi_{t+1}^F}\right)^{-\epsilon^F} K_{t+1}^F,$$

$$K_t^F = \Lambda_{z,t}\mathcal{E}_t\frac{P_t^{For}}{P_t}Y_{z,t}^F + \beta\phi^F\mathbb{E}_t\left(\frac{1}{\Pi_{t+1}^F}\right)^{-\epsilon^F}\frac{P_t^F}{P_t}\left((\Pi_t^F)^{\gamma^F}\right)^{1-\epsilon^F} F_{t+1}^F,$$

$$\Pi_t^F \equiv \frac{P_t^F}{P_{t-1}^F}.$$

Transforming,

$$P_t^{F^{1-\theta^F}} = \int_0^1 P_t^F(f)^{1-\epsilon^F}df = (1-\phi^F) * P_t^{F*^{1-\epsilon^F}} + \int_0^1\left(\Pi_{t-1}^F{}^{\gamma^F} P_{t-1}^F(f)\right)^{1-\epsilon^F} df,$$

we will ascertain that

$$\left(\frac{P_t^{F*}}{P_t}\right)^{1-\epsilon^F}\left(\frac{P_t}{P_t^F}\right)^{1-\epsilon^F}=\frac{1-\phi^F\left(\frac{\Pi_{t-1}^{F}{}^{\gamma^F}}{\Pi_t^F}\right)^{1-\epsilon^F}}{1-\phi^F}.$$

II LOSS FUNCTION

First of all, let us derive some definitions.

Let us define the deviation from the equilibrium state in the model with rigid prices as:

$$\frac{X_t-X}{X}=\widehat{x_t}+\frac{1}{2}\widehat{x_t^2}+\mathcal{O}(\|\zeta\|^3).$$

Let us define the deviation from the equilibrium state in the model with flexible prices as:

$$\frac{X_t-X^n}{X^n}=\widetilde{x_t}+\frac{1}{2}\widetilde{x_t^2}+\mathcal{O}(\|\zeta\|^3).$$

Two ratios for further use according to the Taylor expansion:

$$g(x)=g(x^*)+\frac{g'(x^*)}{1!}(x-x^*)+\frac{g''(x^*)}{2!}(x-x^*)^2+\cdots. \tag{II.1}$$

Let's take the mathematical expectation of the left and right parts:

$$E\big(g(x)\big)=E\big(g(x^*)\big)+E\left(\frac{g'(x^*)}{1!}(x-x^*)\right)+E\left(\frac{g''(x^*)}{2!}(x-x^*)^2\right)+\mathcal{O}(\|x\|^3).$$

Given that $x^*=E(x)$ and $E\left(\big(x-E(x)\big)^2\right)=V(x)$ by definition (where $V(x)$ is the variance of a random variable), we obtain that

$$E\big(g(x)\big)=g(E(x))+\frac{1}{2}g''\big(E(x)\big)V(x)+\mathcal{O}(\|x\|^3). \tag{II.2}$$

Now take the variance from both sides of the equation (II.1):

$$V\big(g(x)\big)=V\big(g(x^*)\big)+V\left(\frac{g'(x^*)}{1!}(x-x^*)\right)+V\left(\frac{g''(x^*)}{2!}(x-x^*)^2\right)+\mathcal{O}(\|x\|^3).$$

Transforming, we get

$$V\big(g(x)\big)=\big(g'(E(x))\big)^2V(x)+\mathcal{O}(\|x\|^3). \tag{II.3}$$

Now let's go directly to the loss function.

The value of the welfare function is used as a criterion for choosing the optimal inflation target level. This function is an approximation (Taylor expansion) of the second order of the consumer's utility function and has the form:

$$\log(C_t - \eta * C_{t-1} * e^{-\zeta_{z,t}}) - \frac{1}{1+\sigma_L} * \int_0^1 (N_t(h))^{1+\sigma_L} dh =$$

$$\frac{1}{1-\eta} * \left[\frac{C_t - C^n}{C^n} - \eta * \frac{C_{t-1} - C^n}{C^n} - \frac{1}{2} * \frac{1}{1-\eta} * \left(\frac{C_t - C^n}{C^n}\right)^2 + \frac{\eta}{1-\eta} * \left(\frac{C_t - C^n}{C^n}\right) * \left(\frac{C_{t-1} - C^n}{C^n}\right) - \frac{1}{2} * \frac{\eta^2}{1-\eta} * \left(\frac{C_{t-1} - C^n}{C^n}\right)^2 + \zeta_{c,t} * \frac{C_t - C^n}{C^n} - \eta * \zeta_{c,t} * \frac{C_{t-1} - C^n}{C^n} - \frac{\eta}{1-\eta} * \zeta_{z,t} * \frac{C_t - C^n}{C^n} + \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\frac{C_{t-1} - C^n}{C^n}\right)\right] \quad \text{Consumption} \quad \text{(II.4)}$$

$$-\int_0^1 \left((N^n)^{1+\sigma_L} * \frac{N_t(h) - N^n}{N^n} + \frac{1}{2} * \sigma_L * (N^n)^{1+\sigma_L} * \left(\frac{N_t(h) - N^n}{N^n}\right)^2\right) dh \quad \text{Labour} \quad \text{(II.5)}$$

where C consumption, N labour, C^n natural level of consumption,

N^n natural level of labour, h labour type, η consumption habits,

ζ_c consumption shock, ζ_z producivity shock, σ_L Frisch elasticity of labour supply.

Let's first transform the consumption part. Given that

$$\frac{C_t - C^n}{C^n} = \frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1,$$

let us rewrite (9.2.1) as

$$\frac{1}{1-\eta} * \left[\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1 - \eta * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \frac{1}{2} * \frac{1}{1-\eta} * \left(\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right)^2 + \frac{\eta}{1-\eta} * \left(\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \frac{1}{2} * \frac{\eta^2}{1-\eta} * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right)^2 + \zeta_{c,t} * \left(\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \eta * \zeta_{c,t} * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\frac{C_t - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) + \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\frac{C_{t-1} - C}{C}\frac{C}{C^n} + \frac{C}{C^n} - 1\right)\right].$$

Using

$$\frac{C_t - C}{C} = \hat{c}_t + \frac{1}{2}\hat{c}_t^2 + \mathcal{O}(\|\zeta\|^3),$$

let us rewrite as

$$\frac{1}{1-\eta}*\Bigg[\left(\hat{c}_t+\frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n}-\eta*\left(\hat{c}_t+\frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n}-\frac{1}{2}*\frac{1}{1-\eta} \tag{II.6}$$

$$*\left(\hat{c}_t^2\left(\frac{C}{C^n}\right)^2+2\left(\hat{c}_t+\frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n}*\left(\frac{C}{C^n}-1\right)\right)+\frac{\eta}{1-\eta}$$

$$*\left(\frac{C}{C^n}^2\hat{c}_t\hat{c}_{t-1}+\left(\frac{C}{C^n}-1\right)\left(\hat{c}_t+\frac{1}{2}\hat{c}_t^2\right)+\left(\frac{C}{C^n}-1\right)\left(\hat{c}_{t-1}+\frac{1}{2}\hat{c}_{t-1}^2\right)\right)-\frac{1}{2}*\frac{\eta^2}{1-\eta}$$

$$*\left(\hat{c}_{t-1}^2\left(\frac{C}{C^n}\right)^2+2\left(\hat{c}_{t-1}+\frac{1}{2}\hat{c}_{t-1}^2\right)\frac{C}{C^n}\left(\frac{C}{C^n}-1\right)\right)+\zeta_{c,t}*\left(\hat{c}_t\frac{C}{C^n}+\frac{C}{C^n}-1\right)-\eta$$

$$*\zeta_{c,t}*\left(\hat{c}_{t-1}\frac{C}{C^n}+\frac{C}{C^n}-1\right)-\frac{\eta}{1-\eta}*\zeta_{z,t}*\left(\hat{c}_t\frac{C}{C^n}+\frac{C}{C^n}-1\right)+\frac{\eta}{1-\eta}*\zeta_{z,t}$$

$$*\left(\hat{c}_{t-1}\frac{C}{C^n}+\frac{C}{C^n}-1\right)\Bigg]+t.i.p.+\mathcal{O}(\|\zeta\|^3),$$

where t.i.p. means terms independent of policy.

Now let's transform the labour part.

Given that

$$\frac{N_t-N^n}{N^n}=\frac{N_t-N}{N}\frac{N}{N^n}+\frac{N}{N^n}-1,$$

let us transform (9.2.2):

$$\int_0^1\left((N^n)^{1+\sigma_L}*\left(\frac{N_t(h)-N}{N}\frac{N}{N^n}+\frac{N}{N^n}-1\right)+\frac{1}{2}*\sigma_L*(N^n)^{1+\sigma_L}*\left(\frac{N_t(h)-N}{N}\frac{N}{N^n}+\frac{N}{N^n}-1\right)^2\right)dh.$$

Using

$$\frac{N_t-N}{N}=\hat{n}_t+\frac{1}{2}\hat{n}_t^2+\mathcal{O}(\|\zeta\|^3)$$

the integrand has the form:

$$\frac{1}{1+\sigma_L}(N_t(h))^{1+\sigma_L}=(N^n)^{1+\sigma_L}*\left(\hat{n}_t\frac{N}{N^n}\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)+\hat{n}_t^2\frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)\right)+\text{t.i.p.}+\mathcal{O}(\|\zeta\|^3).$$

Integrating by type of labour:

$$\frac{1}{1+\sigma_L}\int_0^1(N_t(h))^{1+\sigma_L}dh$$

$$=(N^n)^{1+\sigma_L}\left[\mathbb{E}_h(\hat{n}_t(h))\frac{N}{N^n}\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)+\mathbb{E}_h(\hat{n}_t^2(h))\frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)\right]+\text{t.i.p.}$$

$$+\mathcal{O}(\|\zeta\|^3).$$

Since the variance can be written as

$$\mathbb{V}_h(\hat{n}_t(h))=\mathbb{E}_h(\hat{n}_t(h)^2)-\mathbb{E}_h(\hat{n}_t(h))^2,$$

then

$$\frac{1}{1+\sigma_L}\int_0^1 \left(N_t(h)\right)^{1+\sigma_L} dh$$
$$= (N^n)^{1+\sigma_L}\left[\mathbb{E}_h(\hat{n}_t(h))\frac{N}{N^n}\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)+\frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)\right. \tag{II.7}$$
$$\left. *\,(\mathbb{V}_h(\hat{n}_t(h))+\mathbb{E}_h\hat{n}_t(h))^2)\right]+\mathrm{t.i.p.}+\mathcal{O}(\|\zeta\|^3).$$

Aggregation of labour and output

Take the logarithm (7.5.1) and get:

$$\frac{\epsilon_L-1}{\epsilon_L}\hat{n}_t = \log\left(\int_0^1 \left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}} dh\right).$$

Using (II.2), we transform as

$$\hat{n}_t = \mathbb{E}_h\big(\hat{n}_t(h)\big)+\frac{1}{2}\frac{\epsilon_L-1}{\epsilon_L}\mathbb{E}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right)^{-2}\mathbb{V}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right)+\mathcal{O}(\|\zeta\|^3). \tag{II.8}$$

Using the logarithm property:

$$\mathbb{V}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right)=\mathbb{V}_h\left(\exp\left((1-\epsilon_L^{-1})\log\left(\frac{N_t(h)}{N^n}\right)\right)\right).$$

Then, using (II.3), we transform it as

$$\mathbb{V}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right)=(1-\epsilon_L^{-1})^2\exp\left((1-\epsilon_L^{-1})\mathbb{E}_h\big(\hat{n}_t(h)\big)\right)^2\mathbb{V}_h\big(\hat{n}_t(h)\big)+\mathcal{O}(\|\zeta\|^3).$$

Using the logarithm property again

$$\mathbb{E}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right)=\mathbb{E}_h\left(\exp\big((1-\epsilon_L^{-1})\hat{n}_t(h)\big)\right)$$

and using (II.3), we transform it as

$$\mathbb{E}_h\left(\left(\frac{N_t(h)}{N^n}\right)^{\frac{\epsilon_L-1}{\epsilon_L}}\right)=\exp\left((1-\epsilon_L^{-1})\mathbb{E}_h\big(\hat{n}_t(h)\big)\right)\left(1+\frac{1}{2}(1-\epsilon_L^{-1})^2\mathbb{V}_h\big(\hat{n}_t(h)\big)\right)+\mathcal{O}(\|\zeta\|^3).$$

Then (II.8) takes the form:

$$\hat{n}_t = \mathbb{E}_h(\hat{n}_t(h)) + \frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\left(\exp\left((1-\epsilon_L^{-1})\mathbb{E}_h(\hat{n}_t(h))\right)\left(1+\frac{1}{2}(1-\epsilon_L^{-1})^2\mathbb{V}_h(\hat{n}_t(h))\right)\right)^{-2}(1$$
$$- \epsilon_L^{-1})^2 \exp\left((1-\epsilon_L^{-1})\mathbb{E}_h(\hat{n}_t(h))\right)^2 \mathbb{V}_h(\hat{n}_t(h)) + \mathcal{O}(\|\zeta\|^3)$$
$$= \mathbb{E}_h(\hat{n}_t(h)) + \frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\left(\left(1+\frac{1}{2}(1-\epsilon_L^{-1})^2\mathbb{V}_h(\hat{n}_t(h))\right)\right)^{-2}(1-\epsilon_L^{-1})^2\mathbb{V}_h(\hat{n}_t(h)) + \mathcal{O}(\|\zeta\|^3)$$
$$= \mathbb{E}_h(\hat{n}_t(h)) + \frac{\frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}(1-\epsilon_L^{-1})^2\mathbb{V}_h(\hat{n}_t(h))}{\left(\left(1+\frac{1}{2}(1-\epsilon_L^{-1})^2\mathbb{V}_h(\hat{n}_t(h))\right)\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let us define $\mathbb{V}_h(\tilde{n}_t(h)) \equiv \Delta_{h,t}$, then (II.8):

$$\hat{n}_t = \mathbb{E}_h(\hat{n}_t(h)) + \frac{\frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\Delta_{h,t}}{\left(\left(1+\frac{1}{2}(1-\epsilon_L^{-1})^2\Delta_{h,t}\right)\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let's expand in a Taylor series:

$$\frac{\frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\Delta_{h,t}}{\left(\left(1+\frac{1}{2}(1-\epsilon_L^{-1})^2\Delta_{h,t}\right)\right)^2} = \frac{\frac{1-\epsilon_L{}^{-1}}{2}\Delta_n}{\left(1+\frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n\right)^2} +$$
$$+\frac{1-\epsilon_L{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n}{\left(1+\frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n\right)^3}(\Delta_{h,t}-\Delta_n).$$

Then we finally get

$$\hat{n}_t = \mathbb{E}_h(\hat{n}_t(h)) + \frac{\frac{1-\epsilon_L{}^{-1}}{2}\Delta_n}{\left(1+\frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n\right)^2} + \\ +\frac{1-\epsilon_L{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n}{\left(1+\frac{1}{2}(1-\epsilon_L{}^{-1})^2\Delta_n\right)^3}(\Delta_{h,t}-\Delta_n) \\ +\mathcal{O}(\|\zeta\|^3). \tag{II.9}$$

Similarly for output:

$$\hat{y}_t^{HD} = \mathbb{E}_f(\hat{y}_t^{HD}(f)) + \frac{\frac{1}{2}\frac{\epsilon_{HD}}{\epsilon_{HD}-1}\Delta_{y^{HD},t}}{\left(\left(1+\frac{1}{2}(1-\epsilon_{HD}^{-1})^2\Delta_{y^{HD},t}\right)\right)^2} + \mathcal{O}(\|\zeta\|^3)$$

$$\hat{y}_t^{HF} = \mathbb{E}_f(\hat{y}_t^{HF}(f)) + \frac{\frac{1}{2}\frac{\epsilon_{HF}}{\epsilon_{HF}-1}\Delta_{y^{HF},t}}{\left(\left(1+\frac{1}{2}(1-\epsilon_{HF}^{-1})^2\Delta_{y^{HF},t}\right)\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let's expand in a Taylor series:

$$\frac{\frac{1}{2}\frac{\epsilon_{HD}-1}{\epsilon_{HD}}\Delta_{y,t}}{\left(\left(1+\frac{1}{2}(1-\epsilon_{HD}^{-1})^2\Delta_{y,t}\right)\right)^2}=\frac{\frac{1-\epsilon_{HD}{}^{-1}}{2}\Delta_{y^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}\right)^2}+$$

$$+\frac{1-\epsilon_{HD}{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}\right)^3}(\Delta_{y^{HD},t}-\Delta_{y^{HD}})$$

$$\frac{\frac{1}{2}\frac{\epsilon_{HF}-1}{\epsilon_{HF}}\Delta_{y^{HF},t}}{\left(\left(1+\frac{1}{2}(1-\epsilon_{HF}^{-1})^2\Delta_{y^{HF},t}\right)\right)^2}=\frac{\frac{1-\epsilon_{HF}{}^{-1}}{2}\Delta_{y^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}\right)^2}+$$

$$+\frac{1-\epsilon_{HF}{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}\right)^3}(\Delta_{y^{HF},t}-\Delta_{y^{HF}}).$$

Given that $\widehat{yh}_t=\hat{y}_t^{HD}+\hat{y}_t^{HF}$, and that $\mathbb{E}_f(\hat{y}_t^{HD}+\hat{y}_t^{HF})=\mathbb{E}_f(\hat{y}_t^{HD})+\mathbb{E}_f(\hat{y}_t^{HF})=\mathbb{E}_f\left(\widehat{yh}_t(f)\right)$,

we see that

$$\begin{aligned}\widehat{yh}_t=\mathbb{E}_f\left(\widehat{yh}_t(f)\right)&+\frac{\frac{1-\epsilon_{HD}{}^{-1}}{2}\Delta_{y^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}\right)^2}+\frac{1-\epsilon_{HD}{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD}{}^{-1})^2\Delta_{y^{HD}}\right)^3}(\Delta_{y^{HD},t}\\&-\Delta_{y^{HD}})+\frac{\frac{1-\epsilon_{HF}{}^{-1}}{2}\Delta_{y^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}\right)^2}\\&++\frac{1-\epsilon_{HF}{}^{-1}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF}{}^{-1})^2\Delta_{y^{HF}}\right)^3}(\Delta_{y^{HF},t}-\Delta_{y^{HF}})\\&+\mathcal{O}(\|\zeta\|^3).\end{aligned}\tag{II.10}$$

Now, note that (7.5.2) suggests that

$$\frac{N_t}{N}=\int_0^1\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}df+\int_0^1\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}df.$$

Given that, take the logarithm (7.5.2) and see

$$\hat{n}_t=log\left(\int_0^1\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}df+\int_0^1\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}df\right)=log\left(\mathbb{E}_f\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right)+\mathbb{E}_f\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right)\right).$$

Using (II.2) and (II.3) we see that

$$\hat{n}_t=\mathbb{E}_f\left(\widehat{yh}_t(f)\right)+\frac{\frac{1}{2}\mathbb{V}_f\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right)}{\left(\mathbb{E}_f\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right)\right)^2}+\frac{\frac{1}{2}\mathbb{V}_f\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right)}{\left(\mathbb{E}_f\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right)\right)^2}+\mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{V}_f\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right) = \exp\left(\mathbb{E}_f\left(\hat{y}_t^{HD}(f)\right)\right)^2 \mathbb{V}_f\left(\hat{y}_t^{HD}(f)\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{V}_f\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right) = \exp\left(\mathbb{E}_f\left(\hat{y}_t^{HF}(f)\right)\right)^2 \mathbb{V}_f\left(\hat{y}_t^{HF}(f)\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{E}_h\left(\frac{Y^{HD}{}_t(f)}{Y^{HD}{}_t}\right) = \exp\left(\mathbb{E}_h\left(\hat{y}_t^{HD}(f)\right)\right)\left(1 + \frac{1}{2}\mathbb{V}_f\left(\hat{y}_t^{HD}(f)\right)\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{E}_h\left(\frac{Y^{HF}{}_t(f)}{Y^{HF}{}_t}\right) = \exp\left(\mathbb{E}_h\left(\hat{y}_t^{HF}(f)\right)\right)\left(1 + \frac{1}{2}\mathbb{V}_f\left(\hat{y}_t^{HF}(f)\right)\right) + \mathcal{O}(\|\zeta\|^3).$$

Then

$$\hat{n}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{\frac{1}{2}\mathbb{V}_f\left(\hat{y}_t^{HD}(f)\right)}{\left(1 + \frac{1}{2}\mathbb{V}_f\left(\hat{y}_t^{HD}(f)\right)\right)^2} + \frac{\frac{1}{2}\mathbb{V}_f\left(\hat{y}_t^{HF}(f)\right)}{\left(1 + \frac{1}{2}\mathbb{V}_f\left(\hat{y}_t^{HF}(f)\right)\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

By defining $\mathbb{V}_f\left(\hat{y}_{t_t}^{HD}(f)\right) \equiv \Delta_{y^{HD,t}}$ and $\mathbb{V}_f\left(\hat{y}_{t_t}^{HF}(f)\right) \equiv \Delta_{y^{HF,t}}$,

then

$$\hat{n}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{\frac{1}{2}\Delta_{y^{HD,t}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD,t}}\right)^2} + \frac{\frac{1}{2}\Delta_{y^{HF,t}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF,t}}\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let's expand in a Taylor series:

$$\frac{\Delta_{y^{HD,t}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD,t}}\right)^2} = \frac{\Delta_{y^{HD}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD}}\right)^2} + \frac{1 - \frac{1}{2}\Delta_{y^{HD}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD}}\right)^3}\left(\Delta_{y^{HD,t}} - \Delta_{y^{HD}}\right),$$

$$\frac{\Delta_{y^{HF,t}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF,t}}\right)^2} = \frac{\Delta_{y^{HF}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF}}\right)^2} + \frac{1 - \frac{1}{2}\Delta_{y^{HF}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF}}\right)^3}\left(\Delta_{y^{HF,t}} - \Delta_{y^{HF}}\right).$$

Then we finally get

$$\begin{aligned}\hat{n}_t = {}& \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{\Delta_{y^{HD}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD}}\right)^2} + \frac{1 - \frac{1}{2}\Delta_{y^{HD}}}{\left(1 + \frac{1}{2}\Delta_{y^{HD}}\right)^3}\left(\Delta_{y^{HD,t}} - \Delta_{y^{HD}}\right), \\ & \frac{\Delta_{y^{HF}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF}}\right)^2} + \frac{1 - \frac{1}{2}\Delta_{y^{HF}}}{\left(1 + \frac{1}{2}\Delta_{y^{HF}}\right)^3}\left(\Delta_{y^{HF,t}} - \Delta_{y^{HF}}\right) + \mathcal{O}(\|\zeta\|^3).\end{aligned} \tag{II.11}$$

Aggregation of prices and wages

Let us recall the equations for the level of prices and wages from Section 2:

$$P_t^{HD} = \left(\int_0^1 P_t^{HD}(f)^{\epsilon_{HD}-1} df\right)^{\frac{1}{\epsilon_{HD}-1}}, \tag{7.2.2}$$

$$P_t^{HF} = \left(\int_0^1 P_t^{HF}(f)^{\epsilon_{HF}-1} df\right)^{\frac{1}{\epsilon_{HF}-1}}, \tag{7.2.2}$$

$$W_t = \left(\int_0^1 W_t(h)^{1-\epsilon_L} dh\right)^{1/(1-\epsilon_L)}. \tag{7.5.3}$$

Then, using (II.2), we rewrite it as

$$p_t^{HD} = \mathbb{E}_f\big(p_t^{HD}(f)\big) + \frac{\frac{1}{2}\frac{1}{1-\epsilon_{HD}}\mathbb{V}_f\big(P_t^{HD}(f)^{1-\epsilon_{HD}}\big)}{\Big(\mathbb{E}_f\big(P_t^{HD}(f)^{\epsilon_{HD}-1}\big)\Big)^2} + \mathcal{O}(\|\zeta\|^3),$$

$$p_t^{HF} = \mathbb{E}_f\big(p_t^{HF}(f)\big) + \frac{\frac{1}{2}\frac{1}{1-\epsilon_{HF}}\mathbb{V}_f\big(P_t^{HF}(f)^{1-\epsilon_{HF}}\big)}{\Big(\mathbb{E}_f\big(P_t^{HF}(f)^{\epsilon_{HF}-1}\big)\Big)^2} + \mathcal{O}(\|\zeta\|^3),$$

$$w_t = \mathbb{E}_h\big(w_t(h)\big) + \frac{\frac{1}{2}\frac{1}{1-\epsilon_L}\mathbb{V}_f\big(W_t(h)^{1-\epsilon_L}\big)}{\big(\mathbb{E}_h(W_t(h)^{1-\epsilon_L})\big)^2}.$$

Using (II.3), we will get that

$$\mathbb{V}_f\big(P_t^{HD}(f)^{1-\epsilon_{HD}}\big) = \mathbb{V}_f\Big(\exp\big((1-\epsilon_{HD})p_t^{HD}(f)\big)\Big) = (1-\epsilon_{HD})^2 \exp\Big((1-\epsilon_{HD})\mathbb{E}_f\big(p_t^{HD}(f)\big)\Big)^2 \mathbb{V}_f\big(p_t^{HD}(f)\big),$$

$$\mathbb{V}_f\big(P_t^{HF}(f)^{1-\epsilon_{HF}}\big) = \mathbb{V}_f\Big(\exp\big((1-\epsilon_{HF})p_t^{HF}(f)\big)\Big) = (1-\epsilon_{HF})^2 \exp\Big((1-\epsilon_{HF})\mathbb{E}_f\big(p_t^{HF}(f)\big)\Big)^2 \mathbb{V}_f\big(p_t^{HF}(f)\big),$$

$$\mathbb{V}_h\big(W_t(h)^{1-\epsilon_L}\big) = \mathbb{V}_h\Big(\exp\big((1-\epsilon_L)w_t(h)\big)\Big) = (1-\epsilon_L)^2 \exp\Big((1-\epsilon_L)\mathbb{E}_h\big(w_t(h)\big)\Big)^2 \mathbb{V}_h\big(w_t(h)\big).$$

Let us define:

$$\boldsymbol{\bar{p}_t^{HD}} = \mathbb{E}_f\big(p_t^{HD}(f)\big), \boldsymbol{\bar{p}_t}^{HF} = \mathbb{E}_f\big(p_t^{HF}(f)\big), \boldsymbol{\bar{w}_t} = \mathbb{E}_h\big(w_t(h)\big),$$

$$\Delta_{\boldsymbol{p^{HD}},\boldsymbol{t}} = \mathbb{V}_f\big(p_t^{HD}(f)\big), \Delta_{\boldsymbol{p}^{HF},\boldsymbol{t}} = \mathbb{V}_f\big(p_t^{HF}(f)\big), \Delta_{\boldsymbol{w},\boldsymbol{t}} = \mathbb{V}_h\big(w_t(h)\big).$$

Then

$$\mathbb{V}_f\big(P_t^{HD}(f)^{1-\epsilon_{HD}}\big) = (1-\epsilon_{HD})^2 \exp\big((1-\epsilon_{HD})\bar{p}_t^{HD}\big)^2 \Delta_{p^{HD},t},$$

$$\mathbb{V}_f\big(P_t^{HF}(f)^{1-\epsilon_{HF}}\big) = (1-\epsilon_{HF})^2 \exp\big((1-\epsilon_{HF})\bar{p}_t^{HF}\big)^2 \Delta_{p^{HF},t},$$

$$\mathbb{V}_h\big(W_t(h)^{1-\epsilon_L}\big) = (1-\epsilon_L)^2 \exp\big((1-\epsilon_L)\bar{w}_t\big)^2 \Delta_{w,t}.$$

Using (II.2), we will get

$$\mathbb{E}_f\big(P_t^{HD}(f)^{1-\epsilon_{HD}}\big) = \mathbb{E}_f\big(\exp\big((1-\epsilon_{HD})p_t^{HD}(f)\big) = \exp\big((1-\epsilon_{HD})\bar{p}_t^{HD}\big)\left(1 + \frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD},t}\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{E}_f\big(P_t^{HF}(f)^{1-\epsilon_{HF}}\big) = \mathbb{E}_f\big(\exp\big((1-\epsilon_{HF})p_t^{HF}(f)\big) = \exp\big((1-\epsilon_{HF})\bar{p}_t^{HF}\big)\left(1 + \frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF},t}\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\mathbb{E}_h\big(W_t(h)^{1-\epsilon_L}\big) = \mathbb{E}_h\big(\exp\big((1-\epsilon_L)w_t(h)\big) = \exp\big((1-\epsilon_L)\bar{w}_t\big)\left(1 + \frac{1}{2}(1-\epsilon_L)^2\Delta_{w,t}\right) + \mathcal{O}(\|\zeta\|^3).$$

Then we finally get

$$p_t^{HD} = \bar{p}_t^{HD} + \frac{\frac{1}{2}(1-\epsilon_{HD})\Delta_{p^{HD},t}}{\left(1 + \frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD},t}\right)^2},$$

$$p_t^{HF} = \bar{p}_t^{HF} + \frac{\frac{1}{2}(1-\epsilon_{HF})\Delta_{p^{HF},t}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF},t}\right)^2},$$

$$w_t = \bar{w}_t + \frac{\frac{1}{2}(1-\epsilon_L)\Delta_{w,t}}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_{w,t}\right)^2} + \mathcal{O}(\|\zeta\|^3).$$

Let's expand in a Taylor series:

$$\frac{\frac{1}{2}(1-\epsilon_{HD})\Delta_{p^{HD},t}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD},t}\right)^2} = \frac{\frac{1-\epsilon_{HD}}{2}\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^2} +$$

$$+\frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\Delta_{p^{HD},t}-\Delta_{p^{HD}}),$$

$$\frac{\frac{1}{2}(1-\epsilon_{HF})\Delta_{p^{HF},t}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF},t}\right)^2} = \frac{\frac{1-\epsilon_{HF}}{2}\Delta_{p^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}\right)^2} +$$

$$+\frac{1-\epsilon_{HF}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}\right)^3}(\Delta_{p^{HF},t}-\Delta_{p^{HF}}),$$

$$\frac{\frac{1}{2}(1-\epsilon_L)\Delta_{w,t}}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_{w,t}\right)^2} = \frac{\frac{1-\epsilon_L}{2}\Delta_w}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_w\right)^2} +$$

$$+\frac{1-\epsilon_L}{2}\frac{1-\frac{1}{2}(1-\epsilon_L)^2\Delta_w}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_w\right)^3}(\Delta_{w,t}-\Delta_w).$$

Then, as a result, for wages and prices we get

$$p_t^{HD} = \bar{p}_t^{HD} + \frac{\frac{1-\epsilon_{HD}}{2}\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^2} + \\ +\frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\Delta_{p^{HD},t}-\Delta_{p^{HD}}) + \mathcal{O}(\|\zeta\|^3), \tag{II.12}$$

$$p_t^{HF} = \bar{p}_t^{HF} + \frac{\frac{1-\epsilon_{HF}}{2}\Delta_{p^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}\right)^2} + \\ +\frac{1-\epsilon_{HF}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}}{\left(1+\frac{1}{2}(1-\epsilon_{HF})^2\Delta_{p^{HF}}\right)^3}(\Delta_{p^{HF},t}-\Delta_{p^{HF}}) + \mathcal{O}(\|\zeta\|^3), \tag{II.13}$$

$$w_t = \bar{w}_t + \frac{\frac{1-\epsilon_L}{2}\Delta_w}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_w\right)^2} + \tag{II.14}$$

$$+\frac{1-\epsilon_L}{2}\frac{1-\frac{1}{2}(1-\epsilon_L)^2\Delta_w}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_w\right)^3}(\Delta_{w,t}-\Delta_w)+\mathcal{O}(\|\zeta\|^3).$$

We loglinearise the demand functions:

$$\hat{y}^{HD}(f)=-\epsilon_{HD}(p_t^{HD}(f)-p_t^{HD})+\hat{y}^{HD}{}_t,$$

$$\hat{y}^{HF}(f)=-\epsilon_{HF}(p_t^{HF}(f)-p_t^{HF})+\hat{y}^{HF}{}_t,$$

$$\tilde{n}_t(h)=-\epsilon_L(w_t(h)-w_t)+\tilde{n}_t.$$

Take the variance from both parts and as a result we get

$$\Delta_{y^{HD},t}=\epsilon_{HD}^2\Delta_{p^{HD},t}, \tag{II.15}$$
$$\Delta_{y^{HF},t}=\epsilon_{HF}^2\Delta_{p^{HF},t}, \tag{II.16}$$
$$\Delta_{l,t}=\epsilon_L^2\Delta_{w,t}. \tag{II.17}$$

Price and wage dispersion

For convenience, we write the variance as

$$\Delta_{p^{HD},t}=\mathbb{V}_f(p_t^{HD}(f)-\bar{p}_{t-1}^{HD}),$$

$$\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD}=\phi_{HD}\gamma_{HD}\pi_{t-1}^{HD}+(1-\phi_{HD})(p^{*HD}-\bar{p}_{t-1}^{HD}).$$

The standard formula for dispersion is:

$$\Delta_{p^{HD},t}=\mathbb{E}_f((p_t^{HD}(f)-\bar{p}_{t-1}^{HD})^2)-\left(\mathbb{E}_f(p_t^{HD}(f)-\bar{p}_{t-1}^{HD})\right)^2.$$

Using this, we get that

$$\Delta_{p^{HD},t}=\mathbb{E}_f((p_{t-1}^{HD}(f)-\bar{p}_{t-1}^{HD}+\gamma_{HD}\pi_{t-1}^{HD})^2)+(1-\phi_{HD})(p^{*HD}-\bar{p}_{t-1}^{HD})^2-(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD})^2. \tag{II.18}$$

As

$$\begin{aligned}(1-\phi_{HD})&(p^{*HD}-\bar{p}_{t-1}^{HD})^2-(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD})^2\\ &=(1-\phi_{HD})\left(\frac{1}{1-\phi_{HD}}(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD})-\frac{\phi_{HD}}{1-\phi_{HD}}\gamma_{HD}\pi_t^{HD}\right)^2-(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD})^2\\ &=\frac{\phi_{HD}}{1-\phi_{HD}}(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD}-\gamma_{HD}\pi_t^{HD})^2-\phi_{HD}(\gamma_{HD}\pi_t^{HD})^2\end{aligned}$$

and

$$\phi_{HD}\mathbb{E}_f((p_{t-1}^{HD}(f)-\bar{p}_{t-1}^{HD}+\gamma_{HD}\pi_{t-1}^{HD})^2)=\phi_{HD}\mathbb{E}_f((p_{t-1}^{HD}(f)-\bar{p}_{t-1}^{HD}+\gamma_{HD}\pi_{t-1}^{HD})^2)-\phi_{HD}(\gamma_{HD}\pi_{t-1}^{HD})^2,$$

then (II.18) is rewritten as

$$\Delta_{p^{HD},t}=\phi_{HD}\mathbb{E}_f((p_{t-1}^{HD}(f)-\bar{p}_{t-1}^{HD})^2)+\frac{\phi_{HD}}{1-\phi_{HD}}(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD}-\gamma_{HD}\pi_t^{HD})^2=$$

$$\Delta_{p^{HD},t}=\phi_{HD}\Delta_{p^{HD},t-1}+\frac{\phi_{HD}}{1-\phi_{HD}}(\bar{p}_t^{HD}-\bar{p}_{t-1}^{HD}-\gamma_{HD}\pi_t^{HD})^2.$$

From

$$p_t^{HD} = \bar{p}_t^{HD} + \frac{\frac{1-\epsilon_{HD}}{2}\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^2} +$$

$$+\frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\Delta_{p^{HD},t} - \Delta_{p^{HD}}) + \mathcal{O}(\|\zeta\|^3)$$

follows that

$$\bar{p}_t^{HD} - \bar{p}_{t-1}^{HD} = \pi_t^{HD} - \frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}\left(\Delta_{p^{HD},t} - \Delta_{p^{HD},t-1}\right) + \mathcal{O}(\|\zeta\|^3),$$

$$\Delta_{p^{HD},t} = \phi_{HD}\Delta_{p,t-1} + \frac{\phi_{HD}}{1-\phi_{HD}}\left[\pi_t^{HD} - \frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\Delta_{p^{HD},t} - \Delta_{p^{HD},t-1}) - \gamma_{HD}\pi_{t-1}^{HD}\right]^2 + \mathcal{O}(\|\zeta\|^3).$$

Then the equilibrium value of the dispersion is equal to

$$\Delta_{p^{HD}} = \frac{(1-\gamma_{HD})^2\phi_{HD}}{(1-\phi_{HD})^2}(\pi^{HD})^2,$$

$$\Delta_{p^{HD},t} = \phi_{HD}\Delta_{p,t-1,}$$

$$+\frac{\phi_{HD}}{1-\phi_{HD}}\left[(1-\gamma_{HD})\pi^{HD} + \hat{\pi}_t^{HD} - \gamma_{HD}\hat{\pi}_{t-1}^{HD} - \frac{1-\epsilon_{HD}}{2}\frac{1-\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}}{\left(1+\frac{1}{2}(1-\epsilon_{HD})^2\Delta_{p^{HD}}\right)^3}(\hat{\pi}_t^{HD} - \hat{\pi}_{t-1}^{HD})\right]^2 + \mathcal{O}(\|\zeta\|^3).$$

Similarly

$$\Delta_{w,t} = \alpha_w\Delta_{w,t-1} + \frac{\alpha_w}{1-\alpha_w}\left[(1-\gamma_z)\mu_z + (1-\gamma_w)\pi + \hat{\pi}_t - \gamma_w\hat{\pi}_{t-1} - \frac{1-\epsilon_L}{2}\frac{1-\frac{1}{2}(1-\epsilon_L)^2\Delta_{p^w}}{\left(1+\frac{1}{2}(1-\epsilon_L)^2\Delta_{p^w}\right)^3}(\Delta_{p^w,t} - \Delta_{p^w,t-1})\right]^2 + \mathcal{O}(\|\zeta\|^3).$$

Considering that the stationary state $\Delta_{p^{HD}}$ is of the second order, the previously derived expressions can be simplified:

$$p_t^{HD} = \bar{p}_t^{HD} + + \frac{1-\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \mathcal{O}(\|\zeta\|^3), \tag{II.12}$$

$$p_t^{HF} = \bar{p}_t^{HF} + + \frac{1-\epsilon_{HF}}{2}\Delta_{p^{HF},t} + \mathcal{O}(\|\zeta\|^3), \tag{II.13}$$

$$w_t = \overline{w}_t + + \frac{1-\epsilon_L}{2}\Delta_{w,t} + \mathcal{O}(\|\zeta\|^3), \tag{II.14}$$

$$\hat{n}_t = \mathbb{E}_h\big(\hat{n}_t(h)\big) + \frac{1-\epsilon_L{}^{-1}}{2}\Delta_{n,t} + \mathcal{O}(\|\zeta\|^3), \tag{II.19}$$

$$\widehat{yh}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{1-\epsilon_{HD}{}^{-1}}{2}\Delta_{y^{HD},t} + \frac{1-\epsilon_{HF}{}^{-1}}{2}\Delta_{y^{HF},t} + \mathcal{O}(\|\zeta\|^3), \tag{II.20}$$

$$\hat{n}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{1}{2}\big(\Delta_{y^{HD},t} + \Delta_{y^{HF},t}\big) + \mathcal{O}(\|\zeta\|^3). \tag{II.21}$$

Loss function

Let us transform the loss function given at the beginning of the section. Let us do this separately for the consumption and labour parts.

Labour

Let us recall that

$$\frac{1}{1+\sigma_L}\int_0^1 \big(N_t(h)\big)^{1+\sigma_L} dh$$
$$= (N^n)^{1+\sigma_L}\left[\mathbb{E}_h(\hat{n}_t(h))\frac{N}{N^n}\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right) + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)\right. \tag{II.22}$$
$$\left. * \left(\mathbb{V}_h(\hat{n}_t(h)) + \mathbb{E}_h\hat{n}_t(h))^2\right)\right] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).$$

Using

$$\hat{n}_t = \mathbb{E}_h\big(\hat{n}_t(h)\big) + \frac{1-\epsilon_L{}^{-1}}{2}\Delta_{w,t} + \mathcal{O}(\|\zeta\|^3), \tag{II.19}$$

we will ascertain that

$$\frac{1}{1+\sigma_L}\int_0^1 \big(N_t(h)\big)^{1+\sigma_L} dh$$
$$= (N^n)^{1+\sigma_L}\left[\frac{N}{N^n}\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)\hat{n}_t + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)\hat{n}_t^2\right.$$
$$\left. + \left(1+\sigma_L\left(\frac{N}{N^n}-1\right) + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n}+\frac{N}{N^n}-\frac{1}{2}\right)\right) * \Delta_{w,t}\right] + \text{t.i.p.} \quad + \mathcal{O}(\|\zeta\|^3),$$

$$(N^n)^{1+\sigma_L}\left[-\frac{1}{2}\frac{\epsilon_L - 1}{\epsilon_L}\Delta_{h,t} + \frac{1}{2}\Delta_{h,t} + \frac{1}{2}\sigma_L\Delta_{h,t}\right] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).$$

Let us take $\widehat{yh}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right)$

from

$$\widehat{yh}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{1-\epsilon_{HD}{}^{-1}}{2}\Delta_{y^{HD},t} + \frac{1-\epsilon_{HF}{}^{-1}}{2}\Delta_{y^{HF},t} + \mathcal{O}(\|\zeta\|^3) \tag{II.23}$$

and put it to

$$\hat{n}_t = \mathbb{E}_f\left(\widehat{yh}_t(f)\right) + \frac{1}{2}\left(\Delta_{y^{HD},t} + \Delta_{y^{HF},t}\right) + \mathcal{O}(\|\zeta\|^3). \quad \text{(II.24)}$$

We will ascertain

$$\hat{n}_t = \widehat{yh}_t + \frac{{\epsilon_{HD}}^{-1}}{2}\Delta_{y^{HD},t} + \frac{{\epsilon_{HF}}^{-1}}{2}\Delta_{y^{HF},t} + \mathcal{O}(\|\zeta\|^3). \quad \text{(II.25)}$$

Given that (II.25) and $\Delta_{y^{HD},t} = \epsilon_{HD}^2\Delta_{p^{HD},t}$ (II.15), $\Delta_{y^{HF},t} = \epsilon_{HF}^2\Delta_{p^{HF},t}$ (II.16) and $\Delta_{l,t} = \epsilon_L^2\Delta_{w,t}$ (II.17), the loss function for labour has the form:

$$\begin{aligned}
&\frac{1}{1+\sigma_L}\int_0^1 \left(N_t(h)\right)^{1+\sigma_L} dh \\
&= (N^n)^{1+\sigma_L}\left[\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)\left(\widehat{yh}_t + \frac{\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \frac{\epsilon_{HF}}{2}\Delta_{p^{HF},t}\right)\right. \\
&+ \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\left(\widehat{yh}_t + \frac{\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \frac{\epsilon_{HF}}{2}\Delta_{p^{HF},t}\right)^2 \\
&\left. + \left(1+\sigma_L\left(\frac{N}{N^n}-1\right) + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\right) * \epsilon_L^2\Delta_{w,t}\right] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).
\end{aligned} \quad \text{(II.26)}$$

Combining (II.26) and (II.6), we get the final loss function:

$$\begin{aligned}
&\log(C_t - \eta * C_{t-1} * e^{-\zeta_{z,t}}) - \frac{1}{1+\sigma_L} * \int_0^1 (N_t(h))^{1+\sigma_L} dh \\
&= \frac{1}{1-\eta} \\
&* \left[\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n} - \eta * \left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n} - \frac{1}{2} * \frac{1}{1-\eta}\right. \\
&* \left(\hat{c}_t^2\left(\frac{C}{C^n}\right)^2 + 2\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)\frac{C}{C^n} * \left(\frac{C}{C^n} - 1\right)\right) + \frac{\eta}{1-\eta} \\
&* \left(\frac{C}{C^n}^2 \hat{c}_t\hat{c}_{t-1} + \left(\frac{C}{C^n} - 1\right)\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) + \left(\frac{C}{C^n} - 1\right)\left(\hat{c}_{t-1} + \frac{1}{2}\hat{c}_{t-1}^2\right)\right) - \frac{1}{2} * \frac{\eta^2}{1-\eta} \\
&* \left(\hat{c}_{t-1}^2\left(\frac{C}{C^n}\right)^2 + 2\left(\hat{c}_{t-1} + \frac{1}{2}\hat{c}_{t-1}^2\right)\frac{C}{C^n}\left(\frac{C}{C^n} - 1\right)\right) + \zeta_{c,t} * \left(\hat{c}_t\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \eta * \zeta_{c,t} \\
&* \left(\hat{c}_{t-1}\frac{C}{C^n} + \frac{C}{C^n} - 1\right) - \frac{\eta}{1-\eta} * \zeta_{z,t} * \left(\hat{c}_t\frac{C}{C^n} + \frac{C}{C^n} - 1\right) + \frac{\eta}{1-\eta} * \zeta_{z,t} \\
&* \left.\left(\hat{c}_{t-1}\frac{C}{C^n} + \frac{C}{C^n} - 1\right)\right] - (N^n)^{1+\sigma_L}\left[\left(1+\sigma_L\left(\frac{N}{N^n}-1\right)\right)\left(\widehat{yh}_t + \frac{\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \frac{\epsilon_{HF}}{2}\Delta_{p^{HF},t}\right)\right. \\
&+ \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\left(\widehat{yh}_t + \frac{\epsilon_{HD}}{2}\Delta_{p^{HD},t} + \frac{\epsilon_{HF}}{2}\Delta_{p^{HF},t}\right)^2 \\
&\left. + \left(1+\sigma_L\left(\frac{N}{N^n}-1\right) + \frac{N}{N^n}\left(\frac{1}{2}\sigma_L\frac{N}{N^n} + \frac{N}{N^n} - \frac{1}{2}\right)\right) * \epsilon_L^2\Delta_{w,t}\right] + \text{t.i.p.} + \mathcal{O}(\|\zeta\|^3).
\end{aligned}$$