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MARS: DSGE Model with Agent, Regional and Sectoral Heterogeneity

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A. Shulgin, A. Novak

Andrei Shulgin
Volga-Vyatka Main Branch of the Bank of Russia, Economic Department
E-mail: andrei.shulgin@gmail.com

Anna Novak
Volga-Vyatka Main Branch of the Bank of Russia, Economic Department
E-mail: ananova7@gmail.com

12 Neglinnaya Street, Moscow 107016
+7 499 300-30-00, +7 495 621-64-65 (fax)
Bank of Russia website: www.cbr.ru

Abstract

The paper presents a DSGE model (type: KANK) that allows using information on the heterogeneity of households, sectors and regions. We demonstrate how this model can be used for the purposes of regional macroeconomic analysis aimed at supporting the decision on monetary policy. The model is estimated using the Bayesian method on aggregated series of Russia, the Volga-Vyatka macroregion and foreign economy for the period 2014–2025. We demonstrate that in the presented version of the model, the role of shocks responsible for explaining heterogeneity is small. The contribution of such shocks to the dynamics of aggregated macroeconomic variables does not exceed 5%.

Keywords: DSGE, KANK, monetary policy, regional heterogeneity, sector heterogeneity, non-Ricardian households

JEL classification: E10, E17, E37, E58

Contents

Introduction

Dynamic stochastic general equilibrium (DSGE) models have firmly established their niche in the structural analysis of monetary policy. Monetary policy operates with an aggregate instrument to achieve aggregate goals, so the simplest DSGE model assumes homogeneity among households and firms and does not take geography into account. Is it necessary to complicate the model structure? Undoubtedly, if we want to better understand the country's business cycle. Heterogeneous elements allow for a more accurate description of shock transmission mechanisms in the model and more precise identification of the shocks themselves, thereby improving the quality of structural macroeconomic analysis.

We develop, evaluate, and analyse a DSGE model for monetary policy analysis, with a primary focus on the use of information about three types of heterogeneity. The first type is related to households' access to the financial market. Following the tradition of new Keynesian models with two types of agents (TANK models: see, for example, Gali et al., 2007), we assume that some of the agents are limited in their use of financial market instruments to optimise their consumption trajectory over time. Second, we assume that households and firms are geographically separated into non-identical regions. Regions vary in size, sector structure, demographic parameters, and firm pricing characteristics. Third, we assume that there is sectoral heterogeneity related both to the ability of firms in the sector to export goods and, consequently, to their response to external shocks and exchange rates, and to the characteristics of competition and pricing in the sectors.

The developed model has a practical aspect: it is designed for use in medium-term forecasting of inflation, GDP and other macro variables for one of the regional divisions of the Bank of Russia: the Volga-Vyatka Main Branch (VVMB) of the Bank of Russia[1] for participation in preliminary meetings of the Board of Directors (BoD) of the Bank of Russia. Further, we analyse the model's parameterisation specifically using its application for regional macroeconomic analysis purposes (in a two-region mode).

The practical orientation of the model under development means that it should accurately reproduce the main features of the business cycle of a small open export-oriented economy under conditions of restrictions on international capital flows. The model must also have the necessary properties for conducting structural analysis of business cycle sources and constructing conditional forecasts of macro variables. The emphasis on heterogeneity is placed, on the one hand, in order to be able to use additional sources of agent, regional and sectoral information to improve the quality of macroeconomic analysis. On the other hand, to use the model to analyse monetary policy, taking into account the state of the regional business cycle.

We take a conservative approach to interpreting the issue of heterogeneity, and first of all, proceed from the assumption that heterogeneity in itself is not a problem that monetary authorities must solve. Second, we do not experiment with various measures of heterogeneity that could be considered useful indicators for monetary policy purposes. In our work, heterogeneity is an opportunity for monetary authorities to take new information into account for more accurate identification of the business cycle and better conditional forecasting. At the same time, we do not rule out that taking into account heterogeneity in the style of Benigno (2004) and

[1] The regional structure of the Bank of Russia includes seven Main Branches (MB): North-Western MB, MB for the Central Federal District, Southern MB, Volga-Vyatka MB, Ural MB, Siberian MB, and Far Eastern MB.

Gelfer (2024)[2] could improve the quality of stabilisation monetary policy, but we leave further development of this topic for the future.

The inclusion of heterogeneous elements in DSGE models (households, firms, regions, markets, investors, banks, etc.) is a fairly standard practice, both for various research settings and for the purposes of applied macroeconomic analysis. We see the contribution of this work to the subject, first of all, in bringing together three types of heterogeneity into a single theoretical fragment at the same time. To achieve this, the model provides for interaction between heterogeneous elements, implemented using several channels through which spill-over effects are realised. The first channel is the goods market, where goods produced in different sectors and regions compete with each other for buyers. The second channel is the labour market, where firms from different sectors and regions compete for different types of labour resources from different regions. Both channels lead to positive connectivity of prices in regional and sectoral terms. Variables that are determined at the national level, such as the key rate and foreign exchange rates, also lead to the interconnection of various heterogeneous elements, creating a third channel for spills-over effects. Second, in our work, we propose a strategy for evaluating such a model based on available information about three types of heterogeneity. In particular, in the Bayesian estimation of model parameters, we justify constraints on shock variances that allow for the assessment of common shocks along with shocks responsible for the emergence of heterogeneity. Finally, in our work, we calculate the contribution of shocks responsible for explaining heterogeneity in the decomposition of historical Russian macroeconomic series (quarterly frequency) into structural shocks, and demonstrate their contribution to the variance of the forecast of key macroeconomic variables.

The model belongs to the class of regional macroeconomic general equilibrium models. Several semi-structural models with similar applications were developed by researchers for several macroregions (MR) of Russia: Nelyubina (2021) for the Central Federal District, Kryzhanovsky, Zykov (2022) for the Ural MR, Zhurakovsky et al. (2024) for the Far Eastern MR. Unlike the above-mentioned studies, we use the DSGE methodology to assess the business cycle of the Volga-Vyatka MR (VVMR), which allows us to introduce more diverse micro-based interregional spill-over effects into the model.

In terms of methodology, we use a KANK-type DSGE model (k-agent New Keynesian model: see, for example, Eskelinen, 2021) with the only modification being that heterogeneity applies not only to k groups of households, but also to regions and sectors. A more fundamental approach is considered to be the study of heterogeneity using models with heterogeneous agents (such as HANK: see, for example, Kaplan et al, 2018), in which it is possible to assign unique characteristics to each agent. On the one hand, we do not use this analysis technology because the basic model and its parameterised version with three types of heterogeneity are intended to solve applied tasks.[3] On the other hand, we proceed from the fact that the main results of HANK models can be reproduced using KANK technology (see, for example, Debortoli, Gali, 2017; Bilbiie, 2020).

We conducted a Bayesian assessment of model parameters for the Russian Federation over the historical period from 2014 to 2025. We limited the set of information on heterogeneous elements to series typical for regional macroeconomic analysis: five series characterising the

[2] These include information about heterogeneous elements in the monetary policy rule.

[3] This imposes certain operational constraints on the model, which at this stage do not allow for the use of HANK model analysis technology.

Volga-Vyatka MR;[4] a series on consumption by low-income households; three series characterising two sectors of the Russian Federation: services (prices and volumes) and extraction (volumes). An important feature of the assessment is the use of permanent shocks designed to reflect the impact of structural changes in the Russian economy on the business cycle. Following Aguiar and Gopinath (2007), we note the significant contribution of permanent shocks not only to the low-frequency but also to the high-frequency dynamics of variables (the 'trend generates cycle' principle). We have incorporated a structural view of the long-term dynamics of the Russian economy into our assessment: the assessment of long-term GDP growth rates, wages, consumption, labour force, and the assessment of the neutral rate are carried out in conjunction with a Bayesian assessment of the model's deep parameters.

We found that only a small share of the forecast variance of inflation (3–3.5%) and output (3.5–4%) in the Russian Federation is associated with shocks that we were able to associate with heterogeneity shocks. This estimate turned out to be lower than that of Deryugina et al. (2019). In addition to the fact that a different methodology was used for the assessment, the main explanation is that the selected historical interval of 2016–2025 was dominated by general and external shocks: during the Covid-19 pandemic, these were general plus external shocks to supply and demand; and the period 2022–2025 was dominated by general shocks to demand, supply, the public sector, the labour market and the trade balance. External financial market shocks of the supply of oil and materials (resources) also had a significant impact throughout the historical range. Against the backdrop of such strong disturbances, the share of heterogeneous shocks turns out to be small. The second explanation for the insignificant contribution of heterogeneity shocks to the forecast of aggregate series is that the assessment used limited information about the heterogeneity of the Russian economy.[5] To obtain a more realistic estimate of the contribution of heterogeneity shocks, it is necessary to use a complete set of regional and sectoral information.

The work consists of four parts. In the first part, we describe the DSGE model with three types of heterogeneity. In the second part, the parameterisation of the model is carried out. The third part is devoted to testing the adequacy of the estimated model. In the fourth part, we conduct several useful exercises with the model and also identify the contribution of heterogeneity to the variance of macro variables.

Model

Let us consider a small open economy (see Fig. 1), consisting of N_r regions (index r). In each region, there is sectoral differentiation of firms (N_m sectors with index m). In general, sectors differ in terms of the cost of transporting goods to another region or country (tradable and non-tradable), as well as in terms of pricing principles and types of competition. In the version of the model used for parameterisation, domestic firms produce tradable (T) and non-tradable (N) goods, utility services (S) and oil (O). All non-tradable goods and utility services produced in each region are consumed within that region. Tradable goods (H) produced in region r are partly consumed within that region (C_{rrH}), partly consumed in other regions of the country q (C_{rqH}), and partly exported abroad (X_{rH}). All produced oil is exported (X_{rO}).

[4] The Volga-Vyatka macroregion consists of 11 constituent territories of the Russian Federation representing approximately 13% of the Russian population, producing about 10% of Russia's GDP, and having an economic structure quite similar to that of Russia as a whole, with an emphasis on the machine-building industry.

[5] In this, our calculation differs significantly from the approach of Deryugina et al. (2019), which used a full set of regional and sectoral data on prices.

The imports of each region of the domestic economy consist of two parts. The first part includes imports of homogeneous goods, which domestic importing firms (F) transform into differentiated final goods and sell on the monopolistically competitive market (Y_{rF}). The second part consists of imports of homogeneous materials (resources), which domestic packaging firms (R) transform into differentiated intermediate goods and sell to domestic manufacturing firms in all sectors. The Cobb-Douglas type production function of all sectors includes: aggregate labour (L_r), capital (K_r), and materials (resources R_r). The diagram (Fig. 1) does not show the flows of materials from households to firms in each region and back in the form of profits: these are standard, and the diagram focuses on the geographical aspect of interaction between agents.

Non-tradable (N_r), tradable (H_r and F_r), goods, including exportable goods X_{rH}, as well as intermediate goods (R_r), are sold on monopolistically competitive markets according to the Calvo pricing model (1983) with indexation to previous CPI inflation. All markets use the principle of local currency pricing (LCP). Prices in the oil industry depend on international oil prices, taking into account the degree of government regulation (stabilisation at a steady level). Prices in the utility services sector are also regulated by the state, for which we determine the dynamics of relative prices in sector S of region r (in relation to the CPI of its region) as a first-order autoregression process.

Households of each region are divided into N_j groups (index j). We divide all households into two types: Ricardian (n) and non-Ricardian (h) households.[6] In the parameterised version of the model, each type of households corresponds to one group. Households offer their differentiated services in the labour markets of all regions. The labour of each group of households j from each region r is an imperfect substitute for the labour of households of the same group j from another region $q \neq r$. Wages for each group of households in each region are set according to the Calvo model with indexation to the previous inflation of their region. Firms in region r of sector m solve the cost minimisation problem by determining their demand for labour for each group of households j. As a result, monopolistic competition occurs in the labour market between different groups in the labour markets of each region, provided that there is an optimal distribution of labour demand among different groups, which is generated by firms in all industries in all regions.

The model introduces two channels of regional spill-over effects: through the goods market and the labour market. The flow of tradable goods between regions creates a spill-over effect through the goods market. This effect creates a positive correlation between regional prices. At the same time, the correlation of regional outputs will depend on which regional shock caused the price increase in one region. For example, a positive shock in demand for goods in one region results in increased demand in another region and creates a positive correlation between regional output. Conversely, a positive supply shock of goods in one region will lead to a decrease in demand in another region and will shift the correlation of GRP of regions into the negative area.

The flow of labour resources between regions creates a positive correlation between regional wages. If wage growth is linked to increased economic activity (aggregate demand and supply shocks), then the effect of this channel will be similar to the effect through the goods market. However, if wage growth is the result of labour market shocks, then labour market shocks in one region (formally aggregate supply shocks) lead to a transfer of labour resources

[6] From the English term 'hand-to-mouth' (h), denoting such a type of existence of an individual or household, when money is only enough for food.

between regions, and as a result, a negative correlation between the GRP of regions. The strength of each of the spill-over effects is specified in the model using a parameter that determines the ‘regional bias’ of demand in favour of goods (labour) from its own region. In the extreme case, when firms prefer goods (labour) exclusively from their own region, spill-over effects disappear.

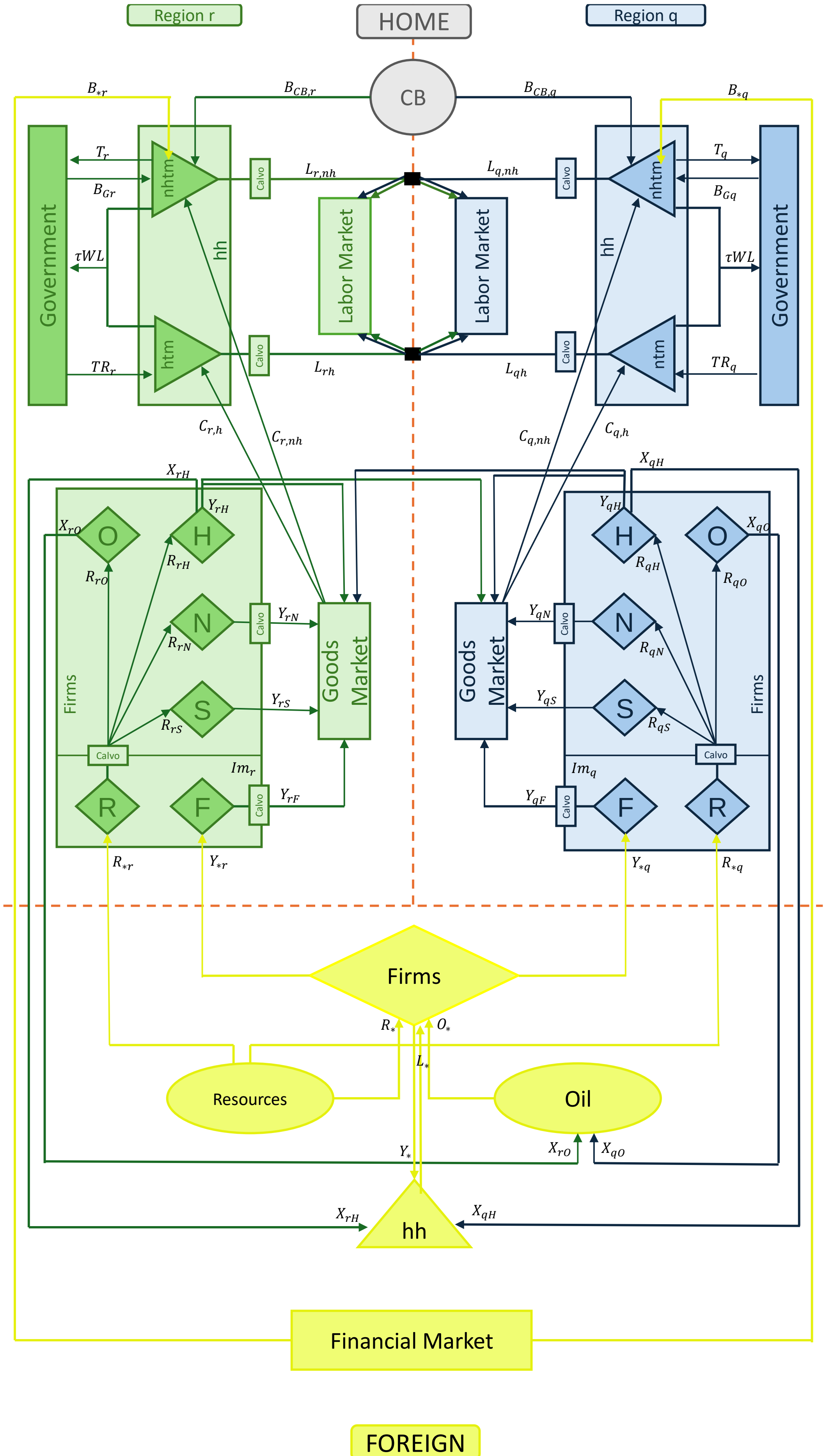

Figure 1. Model agent interaction diagram (geographical aspect)

Similar to regional spill-over effects, sectoral spill-over effects spread through the goods market and labour market channels due to competition effects: shocks occurring in one sector affect supply and demand in other sectors, creating inter-sectoral correlation between prices and volumes. Finally, competition in the labour market creates spill-over effects at the household level.

Another channel for spill-over effects arises from the fact that shocks occurring in one region or sector lead to changes in aggregate variables that determine the functioning of other regions or sectors. For example, increased demand in one region (sector) leads to higher prices across the country and prompts monetary authorities to raise interest rates, which reduces demand in other regions (sectors).

Traditionally, investment adjustment costs (a quadratic function that does not affect the stationary state) are introduced for production functions with capital.

The foreign economy is modelled using the New Keynesian closed economy model. The Cobb-Douglas type production function includes labour, as well as materials (R^*) and oil (O^*). Symmetrically included in the production function for the foreign economy, R^* and O^* have different effects on the domestic economy. An increase in international prices for domestic exports (O^*) leads to a strengthening of the national currency, growth in consumption and income, and a reduction in inflation and interest rates. An increase in international prices for domestic imports (R^*) leads to the exact opposite consequences. At the same time, external shocks unrelated to supply of R^* and O^*, which symmetrically affect their prices, will have a negligible impact on the domestic economy through the balance of payments channel (the effects arising from changes in oil and resource prices will largely offset each other). Foreign economy variables are invariant to the dynamics of the domestic economy.

We incorporate adjustment costs for the volume of foreign assets/debts into the model, as well as costs arising from deviations of foreign assets/debts from their long-term levels. As a result of the non-instantaneous adjustment of the financial account, deviations from the uncovered interest parity will arise, depending on fluctuations in the trade balance. Consequently, fluctuations in exports and imports, in addition to their direct effect on aggregate demand from domestic firms, create an opposite effect on demand through the exchange rate channel. For example, if demand shocks cause exports to grow while imports remain constant, the domestic currency will strengthen, which will have the effect on aggregate demand opposite to that of the initial shock. Similarly, a demand shock for imports will weaken the national currency and support demand for domestic goods. Thus, the exchange rate in a floating regime performs the traditional role of an automatic stabiliser.

All financial markets are incomplete. We introduce ad hoc risk premium functions, whose role is to explain the discrepancies between observed interest rates and to ensure the uniqueness and stability of the model solution.

The government manages public spending G_r by purchasing public goods on the open market. The government collects payroll taxes at flat rate τ and also issues lump-sum transfers to households of type h, taking funds in the form of lump-sum taxes from wealthier households of type n. The budget deficit is financed by issuing securities, which are evenly distributed among all households of type n. Fiscal stability is achieved through positive feedback of government spending with the accumulated volume of public debt.

The central bank manages the key rate, which follows the Taylor rule with inertia and a focus on expected annual inflation three quarters ahead. The key rate sets the level of all other rates in the economy, accurate to the risk premium functions. Spill-over effects through regional financial markets are ignored.

Households

Let us assume that in each region r there are N_j groups of households (group index j). Within each group, households are homogeneous. Household utility function:

$$U_{rjt} = \sum_{s=0}^{\infty} \beta^s \exp(u_{\beta rt+s} + u_{\beta jt+s} + u_{\beta t+s})\Lambda_{rjt+s} \quad (1)$$

where β is a subjective discount; $u_{\beta rt}$, $u_{\beta jt}$ and $u_{\beta t}$are AR(1) processes defining intertemporal preferences at the level of the region, household group, and the country as a whole, respectively:

$$u_{\beta rt} = \rho_{\beta r} u_{\beta rt-1} + \varepsilon_{\beta rt}, \qquad \rho_{\beta r} \in [0,1] \quad (2a)$$

$$u_{\beta jt} = \rho_{\beta j} u_{\beta jt-1} + \varepsilon_{\beta jt}, \qquad \rho_{\beta j} \in [0,1] \quad (2b)$$

$$u_{\beta t} = \rho_{\beta} u_{\beta t-1} + \varepsilon_{\beta t}, \qquad \rho_{\beta} \in [0,1] \quad (2c)$$

where $\varepsilon_{\beta rt}$, $\varepsilon_{\beta jt}$ and $\varepsilon_{\beta t}$ are intertemporal preference shocks;[7] $\rho_{\beta r}$, $\rho_{\beta j}$ and ρ_{β} are the corresponding autoregressive coefficients of intertemporal preference shocks; Λ_{rjt} is the instantaneous utility function, which for households h/h[8] takes the form:

$$\Lambda_{rjt}(h/h) = \frac{\left(\frac{C_{rjt(h/h)}}{N_t} - h\frac{C_{rjt-1}}{N_{t-1}}\right)^{1-\sigma}}{1-\sigma} - \frac{L_{rjt}^{1+\kappa}(h/h)}{1+\kappa}, \quad (3)$$

where $C_{rjt}(h/h)$ is consumption of households h/h of group j of region r; N_t is number of workers belonging to one household; L_{rjt} is hours worked by one worker of this household; σ is the parameter inverse to the elasticity of intertemporal substitution of consumption; κ is the value inverse to the elasticity of labour supply according to Frisch; h is the parameter of external consumption habits.

All household groups are divided into two types: n – Ricardian households; h – non-Ricardian households. The parameters of the utility function are assumed to be the same for both types of households and for all regions of the domestic economy.

Households of type n in each region make decisions regarding consumption and savings, demand/supply of borrowed funds in domestic and foreign currency, labour supply in each region, pricing of firms they own, investments in firms' capital.

The budget constraint of the household of type n in region r:

$$P_{crt}C_{rnt} + T_{0rnt} + S_t B^*_{rnt-1}(1 + i^*_{rnt-1}) + B_{rnt-1}(1 + i_{rnt-1}) + \frac{1}{(1-\lambda_{rh})} B_{rt} + S_t P^*_t Adj_{B^*t} = (1-\tau) W_{rnt} N_t L_{rnt} + \frac{1}{(1-\lambda_{rh})} \left\{ \sum_{m=1}^{N_m} \left(P_{rmt} Y_{rmt} - W_{rmt} N_t L_{rmt} - P_{crt} I_{rmt} - \frac{\varphi-1}{\varphi} S_t P^*_{Rt} P^*_t R_{rmt} \right) + (P_{rFt} - S_t MC^*_t P^*_t) Y_{rFt} + (S_t P^*_{rHt} - P_{rHt}) X_{rHst} + B_{rt-1}(1 + i_{rt-1}) \right\} + S_t B^*_{rnt} + B_{rnt} \quad (4)$$

where P_{crt} is CPI of the region; T_{0rnt} is lump-sum taxes on households; S_t is the foreign currency exchange rate; B^*_{rnt} is household debts in foreign currency; i^*_{rnt} is household borrowing rate abroad; B_{rnt} is household debts in domestic currency; i_{rnt} is household borrowing rate in

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[7] Hereinafter, a shock is defined as a random process with zero expected mean and non-zero variance, which is used to introduce exogenous factors that determine the dynamics of the system. All shocks in the model relate either to the region (index r), or households (index j), or sector (index m), or type of goods (index k), or the country as a whole (without an additional index), or the foreign economy (with an asterisk x^*).

[8] In all formulas, where appropriate, we omit the household and firm index if all elements of this type are homogeneous

domestic currency; B_{rt} is the volume of government-issued bonds; λ_{rh} is the share of households of type h in region r; Adj_{B^*t} is the real adjustment costs of foreign debts; W_{rnt} is nominal wages of households of type n; L_{rnt} is hours worked by households of type n; P_{rmt} is the price of goods in sector m; Y_{rmt} is the production volume of sector m; W_{rmt} is nominal wages in sector m; L_{rmt} is the volume of labour employed in sector m; I_{rmt} is investments in sector m; P^*_{Rt} is the real price of materials on the external market; P^*_t is foreign CPI; R_{rmt} is the volume of materials used in sector m; P_{rFt} is the price of imported goods; MC^*_t is the real marginal costs of producing imported goods; Y_{rFt} is the volume of consumption of imported goods; P^*_{rHt} is the price of domestic tradable goods for the foreign market (in units of foreign currency); X_{rHst} is the volume of exports of tradable goods; i_{rt} is the yield of government bonds.

Households of type h in each region consume all their [labour] income after paying taxes and receiving transfers, and also choose the labour supply in each region. Budget constraint for households of type h:

$$P_{crt}C_{rht} + T_{0rht} = (1-\tau)W_{rht}N_tL_{rht} \tag{5}$$

Financial market

The paper uses an incomplete financial market approach: all functions determining the difference in financial asset returns are set ad hoc. Households of type n have three types of financial assets on their balance sheet. Two financial assets in domestic currency: government bonds and direct borrowing/investment with the Central Bank are absolute substitutes, so we assume that to optimise consumption over time, households can use two financial instruments: in domestic currency with a return of i_{rnt} and in foreign currency with a return of i^*_{rnt}.

We model the liquid domestic financial instrument in a simplified manner, assuming that the Central Bank sets the key rate i_{kt}, and the borrowing/investment rate for households in different regions may differ from i_{kt} by the amount of the risk premium rp_{rnt}:

$$i_{rnt} = i_{kt} + rp_{rnt} \tag{6}$$

The risk premium for households of group $j \in n$ in region r is set by an ad hoc function based on their accumulated assets A_{rnt} compared to the similar indicator of this group for the country as a whole A_{nt}:

$$rp_{rnt} = -\psi_i\left\{\frac{A_{rnt}-\bar{A}_{rn}}{\bar{C}_{rn}} - \frac{A_{nt}-\bar{A}_n}{\bar{C}_{rn}}\right\} + u_{irt} + u_{ijt} + u_{it}, \tag{7}$$

where $\psi_i \geq 0$ is semi-elasticity of the risk premium by asset volume; A_{rnt} is the volume of real assets (+)/debts (-) of households of region r; A_{nt} is the volume of real assets (+) / debts (-) of the country's households as a whole; $\bar{A}_{rn}$ is variables with an overline denote stationary values of the variables; u_{irt}, u_{ijt} and u_{it} are AR(1) processes:

$$u_{irt} = \rho_{ir}u_{irt-1} + \varepsilon_{irt}, \qquad \rho_{ir} \in [0,1] \tag{8a}$$

$$u_{ijt} = \rho_{ij}u_{ijt-1} + \varepsilon_{ijt}, \qquad \rho_{ij} \in [0,1] \tag{8b}$$

$$u_{it} = \rho_{i}u_{it-1} + \varepsilon_{it}, \qquad \rho_{i} \in [0,1] \tag{8c}$$

where ε_{irt}, ε_{ijt} and ε_{it} are credit rate shocks of households; ρ_{ir}, ρ_{ij} and ρ_i are the corresponding autoregression coefficients of credit rate shocks.

Real assets of households:

$$A_{rnt} = \frac{1}{(1-\lambda_{rh})}\frac{B_{rt}}{P_{crt}} - \frac{B_{rnt}}{P_{crt}} - Q_t\frac{P_{ct}}{P_{crt}}\frac{B^*_{rnt}}{P^*_t}, \tag{9}$$

where $Q_t \equiv \frac{S_tP^*_t}{P_{ct}}$ is the real foreign currency exchange rate; P_{ct} is the country's CPI level.

The risk premium response to household assets in region r ensures the stability of the solution for the multi-regional model, preventing excessive accumulation/borrowing of assets in region r (for other solutions to a similar problem, see Schmitt-Grohe and Uribe, 2003).[9]

In operations with foreign debts/assets B^*_{rnt} households incur adjustment costs Adj_{B^*t}. We assume that the costs are related both to the deviation from the stationary level $B^*_{rnt} - \bar{B}^*_{rn}$ (justification in the style of Gabaix, Maggiory, 2015), and to the magnitude of the increase $B^*_{rnt} - B^*_{rnt-1}$ (justification – transaction costs associated with the volume of purchase/sale of foreign securities):

$$Adj_{B^*t} = \left\{ \frac{\psi_{B^*}}{2} \left(\frac{\frac{B^*_{rnt}}{P^*_t} - \bar{b}^*_{rn}}{\bar{b}^*_{rn}} \right)^2 + \frac{\psi_{\Delta B^*}}{2} \left(\frac{\frac{B^*_{rnt}}{P^*_t} - \frac{B^*_{rnt-1}}{P^*_{t-1}}}{\bar{b}^*_{rn}} \right)^2 \right\} \bar{b}^*_{rn}, \quad (10)$$

where ψ_{B^*} and $\psi_{\Delta B^*}$ are parameters of the cost function related to the deviation of the current level of real debts $\frac{B^*_{rnt}}{P^*_t}$ from its stationary level $\bar{b}^*_{rn}$ and from the previous level $\frac{B^*_{rnt-1}}{P^*_{t-1}}$, respectively.

The delayed adjustment of foreign financial assets/debts leads to a differential in interest rates between the domestic and foreign economies, i.e. to a deviation from the condition of uncovered interest rate parity. The magnitude of the interest rate differential turns out to be a function of the current $\frac{B^*_{rnt} - \bar{B}^*_{rn}}{\bar{B}^*_{rn}}$, past $\frac{B^*_{rnt-1} - \bar{B}^*_{rn}}{\bar{B}^*_{rn}}$, and expected future $\frac{E_t B^*_{rnt+1} - \bar{B}^*_{rn}}{\bar{B}^*_{rn}}$ deviations of the level of foreign debts from its stationary level (see the condition for optimising consumption of households using foreign assets).

The interest rate at which households can borrow abroad depends on the foreign policy rate i^*_t, introduced above the risk premium rp_{rnt} and the external borrowing shock u_{rpt}:

$$i^*_{rnt} = i^*_t + rp_{rnt} + u_{rpt} \quad (11)$$

where u_{rpt} is the AR(1) process:

$$u_{rpt} = \rho_{rp} u_{rpt-1} + \varepsilon_{rpt}, \qquad \rho_{rp} \in [0, 1] \quad (12)$$

where ε_{rpt} is the shock of foreign borrowing/investment rates for households; ρ_{rp} is the autoregression coefficient of shocks ε_{rpt}.

We assume that government securities B_{rt} are absolute substitutes for direct borrowing/investment at the rate i_{rnt}, therefore the yield on government securities:

$$i_{rt} = i_{rnt} \quad (13)$$

The adjustment of the volume of assets of households to their consumer behaviour within the business cycle occurs due to the volume of foreign debt B^*_{rnt}. The ability to purchase domestic assets is limited by the supply of securities from the government and the supply of liquidity from the domestic financial market, which we set through the condition on the aggregated volume of domestic loans/borrowings of households from the Central Bank:

$$B_{nt} = \rho_B B_{nt-1} + \varepsilon_{Bt} \quad (14)$$

where ε_{Bt} is the shock of the volume of domestic borrowings/investments of households; ρ_B is the autoregression coefficient of shocks ε_{Bt}.

[9] Assets are normalised by the amount of stationary consumption $\bar{C}_{rn}$, rather than the assets themselves $\bar{A}_{rn}$, solely for the convenience of perceiving the coefficient $\psi_i \geq 0$, since $\bar{A}_{rn} < 0$.

Optimisation of household consumption

The first-order condition for the dynamics of consumption of type $j \in n$ households, using domestic financial assets:

$$\beta_{adj} E_t \left\{ \frac{(1+i_{rjt})}{(1+\pi_{crt+1})} \frac{\exp(u_{\beta rt+1}+u_{\beta jt+1}+u_{\beta t+1})}{\exp(u_{\beta rt}+u_{\beta t})} \frac{N_t}{N_{t+1}} \frac{\Lambda_{Crjt+1}}{\Lambda_{Crjt}} \right\} = 1, \qquad j \in n \qquad (15)$$

where $\Lambda_{Crjt} \equiv \frac{\partial \Lambda_{rjt}}{\partial C_{rjt}} = (\frac{C_{rjt}}{N_t} - h\frac{C_{rjt-1}}{N_{t-1}})^{-\sigma}$ is the marginal utility from additional consumption of type n households in region r; $\pi_{crt} \equiv \frac{P_{crt}-P_{crt-1}}{P_{crt-1}}$ is the consumer price inflation rate; β_{adj} is the adjusted discount factor:

$$\beta_{adj} = \beta \exp(-n - \sigma(g_Y - n)), \qquad (16)$$

where g_Y and n are the long-term growth rates of output and labour resources, respectively.

The adjusted discount factor determines the level of the equilibrium (neutral) real rate:

$$r = \frac{1-\beta_{adj}}{\beta_{adj}}, \qquad (17)$$

The discount factor (17) reflects three reasons why people value the present more than the future: a) subjective discount in utility $\beta < 1$; b) decrease in the marginal utility of consumption Λ_{Ct} with a positive growth rate of output per capita (consumption per capita) $g_Y - g_N > 0$; c) decrease in the volume of assets/debts per person with a long-term increase in labour resources $g_N > 0$.

Condition (15) determines the optimal consumption trajectory of households. A decrease in the real interest rate, an increase in expected future consumption, and a positive preference shock stimulate an increase in household consumption in the current period.

The first-order condition for the dynamics of consumption of type $j \in n$ households, using foreign financial assets, determines the interest rate differential between two countries. We derive this condition, taking into account adjustment costs (10) and condition (15):

$$E_t \left\{ \frac{(1+i_{rjt})}{(1+i^*_{rjt})} \frac{S_t}{S_{t+1}} \right\} = E_t \left\{ \frac{1 + \frac{\psi_{\Delta B^*}}{(1+i^*_{rjt})} \frac{\frac{B^*_{rjt+1}}{P^*_{t+1}} - \frac{B^*_{rjt}}{P^*_t}}{\bar{b}^*_{rj}}}{1 + \psi_{B^*} - (\psi_{B^*} + \psi_{\Delta B^*}) \frac{\frac{B^*_{rjt}}{P^*_t}}{\bar{b}^*_{rj}} + \psi_{\Delta B^*} \frac{\frac{B^*_{rjt-1}}{P^*_{t-1}}}{\bar{b}^*_{rj}}} \right\} \qquad (18)$$

Condition (18) can be rewritten in the style of Gabaix, Maggiori (2015): expressing the dynamics of real foreign debts as a function of the interest rate differential:

$$\frac{\frac{B^*_{rjt}}{P^*_t} - \bar{b}^*_{rj}}{\bar{b}^*_{rj}} - \frac{\psi_{\Delta B^*}}{\psi_{B^*}} \frac{1}{(1+\bar{\iota}^*_{rj})} \frac{\frac{B^*_{rjt+1}}{P^*_{t+1}} - \frac{B^*_{rjt}}{P^*_t}}{\bar{b}^*_{rj}} - \frac{\psi_{\Delta B^*}}{\psi_{B^*}} \frac{\frac{B^*_{rjt-1}}{P^*_{t-1}} - \frac{B^*_{rjt}}{P^*_t}}{\bar{b}^*_{rj}} = \frac{1}{\psi_{B^*}} \left\{ i_{rjt} - i^*_{rjt} - \frac{E_t S_{t+1} - S_t}{S_t} \right\} \qquad (19)$$

where $\frac{1}{\psi_{B^*}}$ is a parameter similar to 'risk-taking ability' à la Gabaix, Maggiori.

Condition (19) demonstrates the role of two types of adjustment costs in (10). The parameter ψ_{B^*} is responsible for how strongly the debts of households B^*_t react to the emerging interest rate differential. The parameter $\psi_{\Delta B^*}$ determines how quickly foreign debts will adjust to $\frac{1}{\psi_{B^*}} \left\{ i_{rjt} - i^*_{rjt} - \frac{E_t S_{t+1} - S_t}{S_t} \right\}$.

Since the dynamics of foreign debts are related to the country's trade balance, conditions (18–19) can be interpreted as the exchange rate's response to the trade balance. The higher the ψ_{B^*}, the more strongly the exchange rate will react to the trade balance surplus, and consequently, the more strongly export and import shocks will affect the exchange rate and inflation through the exchange rate channel.

Households of type h do not optimise their consumption trajectory, as they do not have the tools to do so. They consume their current labour income, taking into account taxes and transfers.

Household choice between types of goods

Households of group j in region r consume domestic tradable C_{rHjt}, non-tradable C_{rNjt}, utility services C_{rSjt}, and foreign tradable goods C_{rFjt}, aggregating them using CES technology:

$$(C_{rjt})^{\frac{1-\delta_C}{\delta_C}} = \sum_{k\in\{H,N,S,F\}}[\lambda_{rk}(1+u_{kt}+z_{kt})]^{\frac{1}{\delta_C}}(C_{rkjt})^{\frac{1-\delta_C}{\delta_C}}, \tag{20}$$

where λ_{rk} is the stationary share of each type of goods in the basket; $\sum_{k\in\{H,N,S,F\}}\lambda_{rk}=1$; δ_C is the elasticity of substitution between different types of goods; u_{kt} is the temporary shock of the share of demand (preferences) for good k in the basket; z_{kt} is the permanent shock of the demand share for good k in the basket. The constraint on preference shocks is the condition of equality of the sum of the shares of all goods, taking into account shocks, that is:

$$\sum_{k\in\{H,N,S,F\}}\lambda_{rk}u_{kt}=0 \tag{21a}$$

$$\sum_{k\in\{H,N,S,F\}}\lambda_{rk}z_{kt}=0 \tag{21b}$$

Shocks of two types, u_k and z_{kt}, are intended to explain part of the fluctuations in relative output volumes of sectors (sectoral heterogeneity) due to relative demand shocks: those that do not directly affect aggregate demand.

To explain the observed dynamics of imports and production of non-tradable goods, we introduce: temporary and permanent shocks to import preferences u_{IMt} and z_{IMt}, as well as a temporary shock to consumption preferences for non-tradable good u_{dNt}. To satisfy the requirement (21a, b) these preference shocks must meet the condition:

$$\begin{bmatrix} u_{Ft} \\ u_{Nt} \\ u_{St} \\ u_{Ht} \end{bmatrix} = \begin{pmatrix} 1 & -\frac{\lambda_{rN}}{1-\lambda_{rN}} \\ -\frac{\lambda_{rF}}{1-\lambda_{rF}} & 1 \\ -\frac{\lambda_{rF}}{1-\lambda_{rF}} & -\frac{\lambda_{rN}}{1-\lambda_{rN}} \\ -\frac{\lambda_{rF}}{1-\lambda_{rF}} & -\frac{\lambda_{rN}}{1-\lambda_{rN}} \end{pmatrix} \begin{bmatrix} u_{IMt} \\ u_{dNt} \end{bmatrix} \tag{22a}$$

$$\begin{bmatrix} z_{Ft} \\ z_{Nt} \\ z_{St} \\ z_{Ht} \end{bmatrix} = \begin{bmatrix} 1 \\ -\frac{\lambda_{rF}}{1-\lambda_{rF}} \\ -\frac{\lambda_{rF}}{1-\lambda_{rF}} \\ -\frac{\lambda_{rF}}{1-\lambda_{rF}} \end{bmatrix} z_{IMt} \tag{22b}$$

where u_{IMt} and u_{dNt} are AR(1) processes; z_{IMt} is a component of permanent shocks[10] to the demand for imported goods that is close to I(1) (hereafter quasi-I(1)):

$$u_{IMt} = \rho_{IM}u_{IMt-1}+\varepsilon_{IMt} \qquad \rho_{IM}\in[0,1] \tag{22c}$$

[10] For all shocks that we refer to as permanent in this work, we assume that their dynamics are close to a random walk I(1), but formally are a converging AR1 process with an autoregression coefficient of 0.9999. This practically does not change the form of the response to such shocks compared to a random walk process, but simplifies the standard procedure of eliminating unit roots, which in a model with such a number of variables and permanent shocks is practically inapplicable for the necessary manipulations with the model. The price for such simplification is a reduction in the accuracy of the model with significant deviations from the initial stationary state under the influence of permanent shocks (linearisation error around the initial stationary point).

$$u_{dNt} = \rho_{dN} u_{dNt-1} + \varepsilon_{dNt} \qquad \rho_{IM} \in [0,1] \qquad (22d)$$
$$z_{IMt} = 0.9999 z_{IMt-1} + \upsilon_{IMt}, \qquad (22e)$$
where ε_{IMt} and ε_{dNt} are the corresponding temporary preference shocks; υ_{Wt} is the permanent import shock.

The first-order conditions for the problem of optimal distribution of household consumption among types of goods require:

$$C_{rkt} = \lambda_{rk}(1 + u_{kt} + z_{kt})\left(\frac{P_{rkt}}{P_{Crt}}\right)^{-\delta_C} C_{rt}, \qquad k \in \{H,N,S,F\} \qquad (23)$$

where the consumer price index of the region is:

$$(P_{crt})^{1-\delta_C} = \sum_{k\in\{H,N,S,F\}} \lambda_{rk}(1 + u_{kt} + z_{kt})(P_{rkt})^{1-\delta_C} \qquad (24)$$

Households of group j in region r consume domestic tradable goods H produced in all regions of the domestic economy:

$$\left(C_{rHjt}\right)^{\frac{1-\delta_R}{\delta_R}} = \sum_q \lambda_{rq}{}^{\frac{1}{\delta_R}}\left(C_{rqHjt}\right)^{\frac{1-\delta_R}{\delta_R}}, \qquad (25)$$

where λ_{rs} is the share of tradable domestic goods produced in region r in the total consumption of these goods in region q; $\sum_r \lambda_{rq} = 1$ C_{rqHjt} is the consumption by households of group j in region q of tradable domestic goods H produced in region r; δ_R is the elasticity of substitution between tradable domestic goods from different regions.

The first-order condition for the problem of optimal distribution by the household of group j of consumption of tradable domestic goods between regions:

$$C_{rqHjt} = \lambda_{rq}\left(\frac{P_{rHst}}{P_{qHt}}\right)^{-\delta_R} C_{qHjt}, \qquad (26)$$

where P_{rHst} is the producer price of tradable goods in region r; P_{qHt} is the aggregated price of tradable domestic goods in region q:

$$\left(P_{qHt}\right)^{1-\delta_R} = \sum_r \lambda_{rq}(P_{rHst})^{1-\delta_R} \qquad (27)$$

To prevent the number of parameters from growing quadratically with the number of regions, we will link the shares λ_{rq} with the shares of regional GRP in the total GRP λ_r in the spirit of the gravity model of trade:

$$\lambda_{rq} = \lambda_{Yr}\Omega \qquad r \neq q \qquad (28a)$$
$$\lambda_{qq} = 1 - (1 - \lambda_{Yq})\Omega, \qquad (28b)$$

where $\Omega \in [0, 1]$ is the parameter of regional bias in consumption. If $\Omega \to 1$, the regional bias disappears, and the structure of consumption of tradable domestic goods in all regions becomes the same, corresponding to the shares λ_{Yr} of the regions in the total GRP. If $\Omega \to 0$, then interregional trade disappears, and households consume goods of type H produced in their own region. In all intermediate cases Ω goods H from other regions are present in the consumption basket, but the weight in the basket of goods from their own region will be greater than the share of the GRP of that region in the total sum.

Labour market

In the labour market, we assume the mobility of labour resources both between sectors of one region and between different regions. Each household allocates its labour resources between sectors m and regions r. Firms hire households from different groups j and different regions r. Thus, the labour market creates spill-over effects associated with competition for all types of heterogeneity.

Households of one group j and one region r interact in the monopolistically competitive market, setting wages according to the Calvo model with indexing to the previous inflation of their region. The task of distributing labour demand between different regions and different types is solved by firms based on their contribution to aggregates.

Labour supply

We assume that each household possesses unique skills that make it a monopolistic competitor in the labour market. Firms aggregate the supply of households (index $i \in [0,1]$) of group j for each sector m of each region r using CES technology:

$$L_{rmjt} = \left[\int_0^1 [L_{rmjt}(i)]^{\frac{\varphi_L - 1}{\varphi_L}} di\right]^{\frac{\varphi_L}{\varphi_L - 1}}, \tag{29}$$

where φ_L is the elasticity of substitution between the labour of households of one group in one region.

By solving the cost minimisation problem to hire L_{rmjt} units of labour, we can obtain the condition for labour demand by household i of group j in sector m:

$$L_{rmjt}(i) = \left(\frac{W_{rmjt}(i)}{W_{rmjt}}\right)^{-\varphi_L} L_{rmjt}, \tag{30}$$

where the aggregated wage W_{rmjt} is:

$$W_{rmjt} = \left[\int_0^1 [W_{rmjt}(i)]^{1-\varphi_L} di\right]^{\frac{1}{1-\varphi_L}} \tag{31}$$

The wages of households of group j from region r are set according to the Calvo (1983) model with indexing to the previous inflation of their region, the same for all sectors and regions. The household i has the opportunity to adjust the wage to the optimal W^o_{rjt} in the current period with a probability of $1 - \theta_{Wrj}$, and with a probability of θ_{Wrj} indexes its previous wage to the inflation in region r in the previous period: $W_{rjt-1}(1 + \pi_{crt-1})^{\chi_W}$, where χ_W is the degree of indexing.

Then the dynamics of the aggregate real wage index set by households of group j is:

$$\frac{W_{rjt}}{P_{crt}} = \left(\theta_{Wrj}\left(\frac{(1+\pi_{crt-1})^{\chi_W}}{(1+\pi_{crt})}\frac{W_{rjt-1}}{P_{crt-1}}\right)^{1-\varphi_W} + (1 - \theta_{Wrj})\left(\frac{W^o_{rjt}}{P_{crt}}\right)^{1-\varphi_W}\right)^{\frac{1}{1-\varphi_W}} \tag{32}$$

The task of households is to maximise the infinite sum of the form:

$$\max_{W^o_{rjt}} E_t \sum_{s=0}^{\infty} (\beta\theta_{Wrj})^s \exp(u_{\beta rt+s} + u_{\beta jt+s} + u_{\beta t+s}) \Lambda_{rjt+s}\big|_{W^o_{rjt}} \tag{33}$$

under constraint (30).

The solution to this problem can be represented as a system of forward-looking difference equations:

$$\frac{W^o_{rjt}}{P_{crt}} = \frac{\varphi_L}{\varphi_L - 1}\frac{J_{Wrjt}}{N_{Wrjt}} \exp(u_{Wrt} + u_{Wjt} + u_{Wt} + z_{Wt}), \tag{34}$$

where u_{Wrt}, u_{Wjt} and u_{Wt} are AR(1) processes; z_{Wt} is the quasi-I(1) component of the permanent shock of monopolistic markup in wages:

$$u_{Wrt} = \rho_{Wr} u_{Wrt-1} + \varepsilon_{Wrt} \qquad \rho_{\mu r} \in [0,1] \tag{35a}$$

$$u_{Wjt} = \rho_{Wj} u_{Wjt-1} + \varepsilon_{Wjt} \qquad \rho_{Wj} \in [0,1] \tag{35b}$$

$$u_{Wt} = \rho_W u_{Wt-1} + \varepsilon_{Wt}, \qquad \rho_W \in [0,1] \tag{35c}$$

$$z_{Wt} = 0.9999 z_{Wt-1} + \upsilon_{Wt}, \tag{36}$$

where ε_{Wrt}, ε_{Wjt} and ε_{Wt} stand for the shock of monopolistic markup in wages of households of region r, group j and overall; υ_{Wt} is the permanent shock of monopolistic markup in wages; auxiliary forward-looking variables J_{Wrjt} and N_{Wrjt}:

$$J_{Wrjt} = \exp(u_{\beta rt} + u_{\beta jt} + u_{\beta t}) L_{rjt} \left(\frac{W_{rjt}}{P_{crt}}\right)^{\varphi_L} \frac{-\Lambda_{Lrjt}}{1-\tau} + \beta_{adj}\theta_{Wrj} E_t \left[\left(\frac{1+\pi_{crt+1}}{(1+\pi_{crt})^{\chi W}}\right)^{\varphi_L} J_{Wrjt+1}\right] \qquad (37)$$

$$N_{Wrjt} = \exp(u_{\beta rt} + u_{\beta jt} + u_{\beta t}) L_{rjt} \left(\frac{W_{rjt}}{P_{crt}}\right)^{\varphi_L} \Lambda_{Crjt} + \beta_{adj}\theta_{Wrj} E_t \left[\left(\frac{1+\pi_{crt+1}}{(1+\pi_{crt})^{\chi W}}\right)^{\varphi_L - 1} N_{Wrjt+1}\right] \qquad (38)$$

where $\Lambda_{Lrjt} = -L_{rjt}^{\kappa}$ is the negative marginal utility from an additional unit of labour effort.

Equations (34), (37)–(38) describe the process of adjusting the labour market to the optimal frictionless labour supply, which corresponds to the optimisation condition of the utility function (1) under the constraint (4)-(5):

$$\frac{W_{rj}}{P_{cr}} = -\frac{\Lambda_{Lrj}}{\Lambda_{Crj}} = L_{rj}^{\kappa}((1-h)\frac{C_{rj}}{N})^{\sigma} \qquad (39)$$

Selection of households of group j from region r is unified for all sectors and regions where these households offer their labour services.

The labour supply of households of group j from region r is distributed across regions q and sectors m:

$$L_{rjt} = \sum_q \sum_m L_{rqmjt}, \qquad (40)$$

where L_{rqmjt} is the labour of households of group j from region r in sector m of region q.

Demand for labour

The total demand for labour for a firm in sector m of region r L_{rmt} is formed based on the marginal product of labour in competition with other production factors: capital K_{rmt} and materials R_{rmt} (see 'Firms' section). In order to form the aggregate L_{rmt} of the firm, the first step is to solve the problem of optimal demand for labour from different regions L_{qrmjt} and the aggregation of labour L_{rmjt} from interregional flows L_{qrmjt} for $\forall s$. In the second step, the optimal demand for labour of households of group j is determined, and the task of aggregating labour L_{rmt} from L_{rmjt} is solved. Both steps are associated with aggregation based on the CES function.

In the first step, we will obtain aggregated (from interregional flows) labour for each household group j in sector m in region r:

$$L_{rmjt} = \left\{\sum_q (\gamma_{Wqrmj})^{\frac{1}{\delta_{LR}}} (\lambda_q \lambda_{qj} L_{qrmjt})^{\frac{\delta_{LR}-1}{\delta_{LR}}}\right\}^{\frac{\delta_{LR}}{\delta_{LR}-1}}, \qquad (41)$$

where γ_{Wqrmj} is a parameter characterising the share of households of group j from region q in sector m of region r; $\sum_q \gamma_{Wqrmj} = 1$; δ_{LR} is the elasticity of substitution between labour of one group of households from different regions; λ_r is the share of the population of region r in the population of the whole country; λ_{rj} is the share of households of group j in the total number of households in region r.

As with the interregional goods market, we will reduce the number of interregional labour market parameters by introducing a regional shift parameter in the labour market $\Omega_L \in [0,1]$, which, like the similar indicator Ω, will characterise the share of households in their region in the labour supply. At $\Omega_L \to 1$, the share of households of group j in region r in the labour markets of all regions will be the same. $\Omega_L \to 0$ in the market of each region, only households from that region will work. In all intermediate cases, the share of households in the labour market of their region will be higher than their share in the country as a whole.[11]

The optimal demand for labour of household group j in region q, firms in sector m from region r:

$$\lambda_r \lambda_{rj} L_{rqmjt} = \gamma_{Wrqmj} \left(\frac{W_{rjt}}{W_{qmjt}}\right)^{-\delta_{LR}} L_{qmjt}, \tag{42}$$

where the aggregated wage W_{qmjt} is:

$$W_{rmjt} = \left\{\sum_q \gamma_{Wqrmj} W_{qjt}{}^{1-\delta_{LR}}\right\}^{\frac{1}{1-\delta_{LR}}} \tag{43}$$

In the second step, we will collect the aggregate for sector m of region r:

$$L_{rmt} = \left\{\sum_j (\gamma_{Wrmj})^{\frac{1}{\delta_L}} (L_{rmjt})^{\frac{\delta_L - 1}{\delta_L}}\right\}^{\frac{\delta_L}{\delta_L - 1}}, \tag{44}$$

where γ_{Wrmj} is a parameter characterising the share of households of group j in sector m; $\sum_j \gamma_{Wrmj} = 1$; δ_L is the elasticity of substitution between labour of different household groups in the region.

The optimal demand for labour of household group j in region r, firms of industry m is:

$$L_{rmjt} = \gamma_{Wrmj} \left(\frac{W_{rmjt}}{W_{rmt}}\right)^{-\delta_L} L_{rmt}, \tag{45}$$

where the aggregated wage W_{rmt} is:

$$W_{rmt} = \left\{\sum_j \gamma_{Wrmj} W_{rmjt}{}^{1-\delta_L}\right\}^{\frac{1}{1-\delta_L}} \tag{46}$$

Since households choose their wage level independently of the sector, the share of households in group j will be the same for all sectors: $\gamma_{Wrj} = \gamma_{Wrmj}$, as will the wage in region: $W_{rt} = W_{rmt}$.

Such a labour market structure forms a spill-over effect channel, creating a positive correlation (due to competition effects) of wages across regions, sectors, and household groups. This also positively links prices in regions and industries (through the cost effect). This channel weakens the negative feedback of outputs of heterogeneous elements, which arises during idiosyncratic supply shocks, as it promotes faster dissemination of such shocks across the entire set of elements. In several models, the output of other heterogeneous elements is introduced into the production function of one heterogeneous element (region, sector) (see, for example, Votinov et al., 2023). The additional spill-over channel arising in such models will be similar in its effect to the spill-over channel through the labour market.

Government

We model the behaviour of the government at the regional level, assuming that the budget of each region must be balanced in the long term, independently of other regions. The actions of the federal authorities are modelled by the existence of shocks common to all regional governments.

Taxes on labour income:

$$T_{Lrt} = \tau \sum_m W_{rmt} N_t L_{rmt}, \tag{47}$$

[11] The peculiarity of parameter calibration is that Ω_L is the average regional shift for the country. For each sector m of each region r the magnitude of the regional shift in the labour market will differ slightly from Ω_L.

where τ is the income tax rate uniform for all regions and households and does not depend on their income.

The government collects lump-sum taxes from wealthy households (type n) in its region: $T_{0rn} > 0$ and pays lump-sum transfers to poor households (type h) in its region: $T_{0rh} < 0$. Total lump-sum taxes:

$$T_{0rt} = \sum_j \lambda_{rj} T_{0rjt} \quad (48)$$

For long-term fiscal balance, we introduce a dependence of lump-sum taxes on household group j on the accumulated government debts B_{rt}:

$$T_{0rjt} = P_{ct} T_{orj} \exp\left(\frac{T_{orj}}{|T_{orj}|}\left[u_{Trt} + u_{Tjt} + u_{Tt} + z_{Tt} + \psi_G \frac{\frac{B_{rt}}{P_{crt}} - \bar{b}_r}{\bar{b}_r}\right]\right) \quad (49)$$

where $\bar{b}_r \equiv \frac{\bar{B}_r}{\bar{P}_{cr}}$ is the long-term level of real government debts; $\psi_G > 0$ is the elasticity of lump-sum taxes to the level of real government debt;[12] u_{Trt}, u_{Tjt} and u_{Tt} are AR(1) processes; z_{Tt} is the quasi-I(1) component of the permanent shock of lump-sum taxes:

$$u_{Trt} = \rho_{Tr} u_{Trt-1} + \varepsilon_{Trt}, \qquad \rho_{Tr} \in [0,1] \quad (50a)$$

$$u_{Tjt} = \rho_{Tj} u_{Tjt-1} + \varepsilon_{Tjt}, \qquad \rho_{Tj} \in [0,1] \quad (50b)$$

$$u_{Tt} = \rho_T u_{Tt-1} + \varepsilon_{Tt}, \qquad \rho_T \in [0,1] \quad (50c)$$

$$z_{Tt} = 0.9999 z_{Tt-1} + \upsilon_{Tt}, \quad (50d)$$

where ε_{Trt}, ε_{Tjt} and ε_{Tt} are regional, agent and general lump-sum tax shocks; υ_{Tt} is a permanent lump-sum tax shock; ρ_{Tr}, ρ_{Tj} and ρ_T are the corresponding autoregression coefficients.

General taxes:

$$T_{rt} = T_{Lrt} + T_{0rt} \quad (51)$$

The government purchases goods on the open market to meet society's demand for public goods. We assume that the demand for public goods from the government is invariant to household decisions regarding the consumption of private goods:[13]

$$G_{rt} = \bar{G}_r \exp(u_{Grt} + u_{Gt} + z_{Gt}) \quad (52)$$

where u_{Grt} and u_{Gt} are AR(1) processes:

$$u_{Grt} = \rho_{Gr} u_{Grt-1} + \varepsilon_{Grt}, \qquad \rho_{Gr} \in [0,1] \quad (53a)$$

$$u_{Gt} = \rho_G u_{Gt-1} + \varepsilon_{Gt}, \qquad \rho_G \in [0,1] \quad (53b)$$

where ε_{Grt} and ε_{Gt} are regional and general government spendign shocks; ρ_{Gr} and ρ_G are the corresponding autoregression coefficients of government spendign shocks.

z_{Gt} is the quasi-I(1) component of permanent government spending shocks:

$$z_{Gt} = 0.9999 z_{Gt-1} + \upsilon_{Gt}, \quad (54)$$

where υ_{Gt} is a permanent government spending shock.

The resulting budget deficit of the regional government is financed by issuing securities B_{rt}, which are purchased by households of type n in their region:

$$B_{rt} = B_{rt-1}(1 + i_{rt-1}) + P_{crt} G_{rt} - T_{rt} \quad (55)$$

Primary budget deficit:

$$BD_{0rt} = P_{crt} G_{rt} - T_{rt} \quad (56)$$

Overall budget deficit:

$$BD_{rt} = P_{crt} G_{rt} + B_{rt-1} i_{rt-1} - T_{rt} \quad (57)$$

[12] The multiplier $\frac{T_{orj}}{|T_{orj}|}$ is used to ensure that tax shock and excessive public debt have a positive effect on T_{0rjt}, regardless of the sign T_{orj}.

[13] For example, assuming that the utility function for households is additively separable with respect to the volume of private goods and public goods.

The distribution of consumption of public goods across $k \in \{H, N, S, F\}$ types is similar to how households distribute private consumption among goods of these types:

$$(G_{rt})^{\frac{1-\delta_C}{\delta_C}} = \sum_k [\lambda_{rk}(1 + u_{kt} + z_{kt})]^{\frac{1}{\delta_C}} (G_{rkt})^{\frac{1-\delta_C}{\delta_C}}, \quad (58)$$

$$G_{rkt} = \lambda_{rk}(1 + u_{kt} + z_{kt}) \left(\frac{P_{rkt}}{P_{crt}}\right)^{-\delta_C} G_{rt}, \quad (59)$$

$$(G_{rHt})^{\frac{1-\delta_R}{\delta_R}} = \sum_q \lambda_{rq}^{\frac{1}{\delta_R}} (G_{rqHt})^{\frac{1-\delta_R}{\delta_R}}, \quad (60)$$

where G_{rqHt} is government consumption in region q of domestic tradable goods H produced in region r.

$$G_{rqHt} = \lambda_{rq} \left(\frac{P_{rHst}}{P_{qHt}}\right)^{-\delta_R} G_{qHt}. \quad (61)$$

Firms

All firms belong to households of type n, refer to one of sectors $m \in \{H, N, S, O\}$, and have a Cobb-Douglas production function with an identical set of production factors: labour L_{rmt}, capital K_{rmt}, and materials R_{rmt}. Sectors may differ in terms of competition and pricing.

Production function:

$$Y_{rmt} = A_{rmt}(N_t L_{rmt})^{[\alpha_L + u_{Lt}]} (K_{rmt})^{[\alpha_K - \frac{\alpha_K}{1-\alpha_L} u_{Lt}]} (R_{rmt})^{[\alpha_R - \frac{\alpha_R}{1-\alpha_L} u_{Lt}]}, \quad (62)$$

where α_L, α_K and α_R are the elasticity parameters of the production function for labour, capital, and materials, respectively; $\alpha_L + \alpha_K + \alpha_R = 1$; u_{Lt} is an AR(1) process that defines the change in the elasticity of the production function for labour:

$$u_{Lt} = \rho_L u_{Lt-1} + \varepsilon_{Lt}, \qquad \rho_L \in [0, 1] \quad (63)$$

where ε_{Lt} is the shock to the labour share in total factor income.

A_{rmt} is the total factor productivity (TFP) of sector m:

$$A_{rmt} = A_{rm} \exp(gt + u_{Art} + u_{Amt} + u_{At} + z_{At}), \quad (64)$$

where g is the total factor productivity (TFP) growth rate; u_{Art}, u_{Amt} and u_{At} are AR(1) processes of TFP dynamics (regional, sectoral and overall, respectively); z_{At} is a quasi-I(1) process of TFP:

$$z_{At} = 0.9999 z_{At-1} + \upsilon_{At} \quad (65)$$

$$u_{Art} = \rho_{Ar} u_{Art-1} + \varepsilon_{Art} \qquad \rho_{Ar} \in [0, 1] \quad (66a)$$

$$u_{Amt} = \rho_{Am} u_{Amt-1} + \varepsilon_{Amt} \qquad \rho_{Am} \in [0, 1] \quad (66b)$$

$$u_{At} = \rho_A u_{At-1} + \varepsilon_{At}, \qquad \rho_A \in [0, 1] \quad (66c)$$

where υ_{At} is the permanent TFP shock; ε_{Art}, ε_{Amt} and ε_{At} are regional, sectoral and general productivity shocks, respectively.

Labour force dynamics:

$$N_t = N \exp(nt + z_{Nt} + u_{Nt}), \quad (67)$$

where N is the normalisation constant; n is the deterministic growth rate of the labour force; z_{Nt} is the quasi-I(1) process in the labour force; u_{Nrt} is the AR(1) process:

$$z_{Nt} = 0.9999 z_{Nt-1} + \upsilon_{Nt} \quad (68)$$

$$u_{Nt} = \rho_N u_{Nt-1} + \varepsilon_{Nt}, \qquad \rho_N \in [0, 1] \quad (69)$$

where υ_{Nt} is the permanent labour shock; ε_{Nt} is the temporary labour shock.

Optimal demand of firms for resources:

$$p_{Krmt} K_{rmt} = \alpha_K Y_{rmt} MC_{rmt} \quad (70)$$

$$w_{rmt} N_t L_{rmt} = \alpha_L Y_{rmt} MC_{rmt} \quad (71)$$

$$p_{Rrmt} R_{rmt} = \alpha_R Y_{rmt} MC_{rmt}, \quad (72)$$

where p_{Krmt} is the real price of renting a unit of capital; $w_{rmt} \equiv \frac{W_{rmt}}{P_{rmt}}$ is the real wage; $p_{Rrmt} = \frac{P_{Rrt}}{P_{rmt}}$ is the real price of a unit of materials; MC_{rmt} is the real marginal cost. All of the above real values are expressed in units of goods of sector m.

Optimal investments

When forming investments, the firm takes into account that the lag in capital input into production is one quarter, as well as the costs of adjusting investments Adj_{Irmt}:

$$Adj_{Irmt} = \frac{\phi_I}{2}\left(\frac{I_{rmt}-I_{rmt-1}}{I_{rmt-1}}\right)^2 I_{rmt}, \tag{73}$$

where ϕ_I is the parameter of the cost function.

Then the dynamics of capital is:

$$K_{rmt} = (1-\delta_K)K_{rmt-1} + I_{rmt-1} - Adj_{Irmt-1} \tag{74}$$

Solving the problem of maximising the expected utility of type n household managing the capital accumulation of its own firm, we obtain the condition for optimal investment:

$$\Lambda_{Crnt}\exp\left(u_{\beta rt} + u_{\beta nt} + u_{\beta t}\right) - \beta_{adj}\left(1 - \phi_I\frac{I_{rmt}-\bar{I}_{rm}}{\bar{I}_{rm}}\right) Z_{Irmt+1}\exp\left(u_{Irt} + u_{It}\right) = 0 \tag{75}$$

where u_{Irt} and u_{It} are AR(1) processes of investment efficiency shocks:

$$u_{Irt} = \rho_{Ir}u_{Irt-1} + \varepsilon_{Irt} \qquad \rho_{Ir} \in [0,1] \tag{76a}$$

$$u_{It} = \rho_I u_{It-1} + \varepsilon_{It} \qquad \rho_I \in [0,1] \tag{76b}$$

where ε_{Irt} and ε_{It} are regional and general investment efficiency shocks, respectively.

Z_{Irmt+1} is an auxiliary forward-looking process characterising the expected effect of introducing a unit of capital:

$$Z_{Irmt} = \Lambda_{Crnt}\exp\left(u_{\beta rt} + u_{\beta nt} + u_{\beta t}\right)\alpha_K\frac{Y_{rmt}}{K_{rmt}}\frac{P_{rmt}}{P_{crt}}MC_{rmt} + \beta_{adj}(1-\delta_K)E_t Z_{Irmt+1} \tag{77}$$

The total volume of the firm's investments is distributed among $k \in \{H, N, S, F\}$ types of goods in the same proportion as household consumption and government spending:

$$\left(I_{rmt}\right)^{\frac{1-\delta_C}{\delta_C}} = \sum_k\left[\lambda_{rk}(1 + u_{kt} + z_{kt})\right]^{\frac{1}{\delta_C}}\left(I_{rmkt}\right)^{\frac{1-\delta_C}{\delta_C}} \tag{78}$$

where I_{rmkt} is the investment demand in sector m of region r for good k:

$$I_{rmkt} = \lambda_{rk}(1 + u_{kt} + z_{kt})\left(\frac{P_{rkt}}{P_{crt}}\right)^{-\delta_C} I_{rmt}, \tag{79}$$

$$\left(I_{rmHt}\right)^{\frac{1-\delta_R}{\delta_R}} = \sum_q \lambda_{rq}^{\frac{1}{\delta_R}}\left(I_{rqmHt}\right)^{\frac{1-\delta_R}{\delta_R}}, \tag{80}$$

where I_{rqmHt} is the investment demand in sector m of region q for domestic tradable goods H, produced in region r:

$$I_{rqmHt} = \lambda_{rq}\left(\frac{P_{rHst}}{P_{qHt}}\right)^{-\delta_R} I_{qmHt}. \tag{81}$$

Inflation

Let us consider the channels through which structural shocks are transmitted to inflation, for which purpose we will define the principles of pricing in all markets.

Pricing according to the Calvo model in domestic goods markets

Domestic and foreign tradable and non-tradable goods are sold in a monopolistically competitive market with pricing based on the Calvo model (1983) and indexation to previous inflation. Let us denote the segments of the domestic market where pricing occurs according to the Calvo model (1983) as $m_K \in \{H, F, N\}$. Pricing and indexation according to the Calvo model are carried out in the currency of the country where the good is sold (LCP – Local Currency Pricing).

With probability θ_{rm_K}, the firm indexes the price to previous inflation (indexation degree χ). With probability $1-\theta_{rm_K}$, the producer gets the opportunity to adjust the price to the optimal $P^{opt}_{rm_K t}$.

Then the aggregated price level is:

$$P^{1-\varphi}_{rm_K t} = \theta_{rm_K}\left((1+\pi_{crt-1})^{\chi} P_{rm_K t-1}\right)^{1-\varphi} + \left(1-\theta_{rm_K}\right)\left(P^{opt}_{rm_K t}\right)^{1-\varphi} \tag{82}$$

The firm i solves the problem of maximising the flow of expected utility from the consumption of profit for its owner (household type n), which can be expressed as:

$$\max_{P^{opt}_{rm_K t}} E_t \sum_{s=0}^{\infty}\left(\theta_{rm_K}\beta\right)^s \Lambda_{Crnt+s} \exp\left(u_{\beta rt+s} + u_{\beta nt+s} + u_{\beta t+s}\right)\left[\frac{1}{N_{t+s}}\left(\frac{P_{crt+s}}{P_{crt}}\right)^{\chi} \frac{P^{opt}_{rm_K t}}{P_{crt+s}}(1 - MC_{rm_K t+s})\right] Y_{rm_K t+s}\Big|_{P^{opt}_{rm_K t}}(i) \tag{83}$$

where $Y_{rm_K t+s}\Big|_{P^{opt}_{rm_K t}}(i)$ is the demand for the products of firm i in period $t+s$, assuming that the price t set by the firm in period $P^{opt}_{rm_K t}$ was not reset but was indexed to previous inflation (Yun, 1996) s times. This demand is found similarly to the solution of the aggregate maximisation problem (84) with a given budget I (85):

$$\max_{Y_{rkt}(i)} Y_{rkt} = \left[\int_0^1 [Y_{rkt}(i)]^{\frac{\varphi-1}{\varphi}} di\right]^{\frac{\varphi}{\varphi-1}} \tag{84}$$

$$\text{s.t. } I = \int_0^1 Y_{rkt}(i) P_{rkt}(i) di \tag{85}$$

where φ is the elasticity of substitution between goods of type k for all agents consuming these goods (households, government, firms).

The solution to problem (84) with constraint (85) is the demand for a good of type k by firm i with elasticity $-\varphi$:

$$Y_{rkt}(i) = \left(\frac{P_{rkt}(i)}{P_{rkt}}\right)^{-\varphi} Y_{rkt}, \tag{86}$$

where the aggregated price level of goods of type k is:

$$P_{rkt} = \left[\int_0^1 [P_{rkt}(i)]^{1-\varphi} di\right]^{\frac{1}{1-\varphi}} \tag{87}$$

Then the conditional demand for period $t+s$ will be:

$$Y_{rm_K t+s}\Big|_{P^{opt}_{rm_K t}}(i) = \left[\left(\frac{P_{crt+s}}{P_{crt}}\right)^{\chi} \frac{P^{opt}_{rm_K t}}{P_{rm_K t+s}}\right]^{-\varphi} Y_{rm_K t+s} \tag{88}$$

The solution to problem (83) with constraint (88) can be represented as the ratio of two auxiliary forward-looking variables $J_{rm_K t}$ and $N_{rm_K t}$:

$$\frac{P^{opt}_{rm_K t}}{P_{rm_K t}} = \frac{\varphi}{\varphi-1}\frac{J_{rm_K t}}{N_{rm_K t}} \exp(u_{\mu rt} + u_{\mu m_K t} + u_{\mu t}), \tag{89}$$

where $u_{\mu rt}$, $u_{\mu m_K t}$ and $u_{\mu t}$ are AR(1) processes:

$$u_{\mu rt} = \rho_{\mu r} u_{\mu rt-1} + \varepsilon_{\mu rt} \qquad \rho_{\mu r} \in [0,1] \tag{90a}$$

$$u_{\mu m_K t} = \rho_{\mu m_K} u_{\mu m_K t-1} + \varepsilon_{\mu m_K t} \qquad \rho_{\mu m_K} \in [0,1] \tag{90b}$$

$$u_{\mu t} = \rho_{\mu} u_{\mu t-1} + \varepsilon_{\mu t}, \qquad \rho_{\mu} \in [0,1] \tag{90c}$$

where $\varepsilon_{\mu rt}$, $\varepsilon_{\mu m_K t}$ and $\varepsilon_{\mu t}$ are regional, sectoral and general monopolistic markup shocks, respectively.

Equations for determining forward-looking processes $J_{rm_K t}$ and $N_{rm_K t}$:

$$J_{rm_K t} = \Lambda_{Crnt}\exp\left(u_{\beta rt} + u_{\beta nt} + u_{\beta t}\right) Y_{rm_K t}\left[\frac{P_{rm_K t}}{P_{crt}}\right]^{\varphi} MC_{rm_K t} + \theta_{rm_K}\beta_{adj}E_t\left\{\left[\frac{1+\pi_{crt+1}}{(1+\pi_{crt})^{\chi}}\right]^{\varphi} J_{rm_K t+1}\right\} \tag{91}$$

$$N_{rm_K t} = \Lambda_{Crnt}\exp\left(u_{\beta rt} + u_{\beta nt} + u_{\beta t}\right) Y_{rm_K t}\left[\frac{P_{rm_K t}}{P_{crt}}\right]^{\varphi} + \theta_{rm_K}\beta_{adj}E_t\left\{\left[\frac{1+\pi_{crt+1}}{(1+\pi_{crt})^{\chi}}\right]^{\varphi-1} N_{rm_K t+1}\right\} \tag{92}$$

At the stationary point, the real producer price converges to the value of real marginal costs increased by the amount of the long-term monopolistic markup $\frac{1}{\varphi-1}$:

$$\frac{\bar{P}_{rm_K t}}{\bar{P}_{crt}} = \frac{\varphi}{\varphi-1}\overline{MC}_{rm_K t} \tag{93}$$

All structural shocks are transmitted to inflation through the channel of real marginal costs MC_{rmt}, which were determined in the section on optimal demand for resources by conditions (70)-(72). Part of the dynamics of observed inflation is determined by the action of monopolistic markup shocks $\varepsilon_{\mu rt}$, $\varepsilon_{\mu mt}$ and $\varepsilon_{\mu t}$ (regional, sectoral, and general, respectively). These shocks allow us to take into account fluctuations in the competitive conditions of firms, as well as the part of price dynamics that cannot be explained by other factors.

Real marginal costs for imported goods are:

$$MC_{rFt} = \frac{MC_t^* P_t^* S_t}{P_{rFt}}, \tag{94}$$

where MC_t^* are actual marginal costs of producing goods abroad.

Pricing of domestic tradable goods in the foreign segment

This segment also uses the Calvo model (1983) with indexation according to Yun (1996), with the only difference being that the price is set in foreign currency and indexed to foreign inflation. We assume that the elasticity of substitution is also φ, and the degree of indexation is also χ.

Then the solution to a similar problem for the foreign segment will be the following equations:

$$(P_{rH^*t}^*)^{1-\varphi} = \theta_{rH^*}((1+\pi_{t-1}^*)^{\chi} P_{rH^*t-1}^*)^{1-\varphi} + (1-\theta_{rH^*})((P_{rH^*t}^*)^{opt})^{1-\varphi} \tag{95}$$

$$\frac{(P_{rH^*t}^*)^{opt}}{P_{rH^*t}^*} = \frac{\varphi}{\varphi-1}\frac{J_{rH^*t}}{N_{rH^*t}}\exp(u_{\mu rt} + u_{\mu H^*t} + u_{\mu t}), \tag{96}$$

where $u_{\mu H^*t}$ is the AR(1) process:

$$u_{\mu H^*t} = \rho_{\mu H^*}u_{\mu H^*t-1} + \varepsilon_{\mu H^*t} \qquad \rho_{\mu H^*} \in [0, 1] \tag{97}$$

where $\varepsilon_{\mu H^*t}$ is the shock of monopolistic markup for the foreign segment of domestic tradable goods; $\rho_{\mu H^*}$ is the autoregression coefficient of shock $\varepsilon_{\mu H^*t}$.

$$J_{rH^*t} = \Lambda_{Crnt}\exp\left(u_{\beta rt} + u_{\beta nt} + u_{\beta t}\right) Y_{rH^*t}\left[\frac{P_{rH^*t}^*}{P_t^*}\right]^{\varphi} MC_{rH^*t} + \theta_{rH^*}\beta_{adj}E_t\left\{\left[\frac{1+\pi_{t+1}^*}{(1+\pi_t^*)^{\chi}}\right]^{\varphi} J_{rH^*t+1}\right\} \tag{98}$$

$$N_{rH^*t} = \Lambda_{Crnt}\exp\left(u_{\beta rt} + u_{\beta nt} + u_{\beta t}\right) Y_{rH^*t}\left[\frac{P_{rH^*t}^*}{P_t^*}\right]^{\varphi} + \theta_{rH^*}\beta_{adj}E_t\left\{\left[\frac{1+\pi_{t+1}^*}{(1+\pi_t^*)^{\chi}}\right]^{\varphi-1} N_{rH^*t+1}\right\} \tag{99}$$

Real marginal costs in the domestic tradable goods sector:

$$MC_{rH^*t} = MC_{rHt}\frac{P_{rHt}}{P^*_{rH^*t}S_t}, \tag{100}$$

where $\frac{MC_{rHt}P_{rHt}}{S_t}$ are nominal marginal costs in sector H expressed in units of foreign currency.

Pricing in the housing and utility services sector

Let us assume that prices in the housing and utility services sector S are set by the state in such a way that the stationary monopolistic markup in this sector is the same as in other sectors:

$$\bar{P}_{rSt} = \frac{\varphi}{\varphi-1}\overline{MC}_{rSt} \tag{101}$$

Let us also assume that as a result of government regulation, there are deviations of the real price level in the housing and utility services sector from the stationary level, but in the long term, the government aims to return the real price level in the housing and utility services sector to its stationary level:

$$p_{rSt} = (p_{rSt-1})^{\rho_{\mu S}}\exp(\varepsilon_{\mu St}), \tag{102}$$

where $p_{rSt} \equiv \frac{P_{rSt}}{P_{crt}}$ is the real price level in the housing and utility services sector; $\rho_{\mu S}$ is the autoregressive coefficient in the dynamics of real prices in the housing and utility services sector; $\varepsilon_{\mu St}$ is the real price shock in the housing and utility services sector.

Pricing in the oil sector

In the oil sector, which produces goods for export, we also assume state pricing. In this case, the long-term price level in the oil sector is linked to the steady real price of oil on the external market $\bar{P}^*_O$. The role of the government is to smooth out short-term fluctuations in the nominal price of oil on the international market $P^*_{Ot}P^*_t$ through various stabilisation mechanisms. In our work, we will define the operation of the stabilisation mechanism in a simplified manner by determining the dynamics of domestic oil prices:

$$P_{rOt} = (k_{SF}\bar{P}^*_O + (1-k_{SF})P^*_{Ot})P^*_tS_t, \tag{103}$$

where $k_{SF} \in [0,1]$ is a parameter characterising the degree of stabilisation of foreign oil price trends for the domestic market. When $k_{SF} \to 1$ the real price in foreign currency is fixed at a stationary level $\bar{P}^*_O$. Fluctuations in the domestic oil price P_{rOt} are only related to fluctuations in the foreign currency exchange rate S_t and foreign prices P^*_t.

We ignore the impact of funds accumulated as a result of the stabilisation scheme (88) on the country's business cycle.

Pricing of imported materials

Suppose that materials are imported into the country in the form of homogeneous goods with a total volume of R_{rt}, and the importing firm i, belonging to household type n, transforms the homogeneous goods into differentiated ones without costs. These differentiated goods are aggregated by means of the CES technology for use in the production function of all sectors:

$$R_{rt} = \sum_m R_{rmt} = \left[\int_0^1 [R_{rt}(i)]^{\frac{\varphi-1}{\varphi}}di\right]^{\frac{\varphi}{\varphi-1}} \tag{104}$$

Demand for materials from importer i:

$$R_{rt}(i) = \left(\frac{P_{Rrt}(i)}{P_{Rrt}}\right)^{-\varphi}R_{rt}, \tag{105}$$

where the aggregated price level of materials is:

$$P_{Rrt} = \left[\int_0^1 [P_{Rrt}(i)]^{1-\varphi} di\right]^{\frac{1}{1-\varphi}} \tag{106}$$

Prices of differentiated materials are set according to the Calvo model (1983) with indexation to previous inflation, similar to prices for goods in sector group $m_K \in \{H, F, N\}$.

We could write down the pricing conditions using auxiliary variables J and N, but for demonstration purposes, let us write down the Phillips curve equation for the rate of growth of material prices as a single reduced equation, structurally identical to the pricing scheme for goods described above:

$$\pi_{Rrt} - \chi_R \pi_{crt-1} = \beta_{adj}(E_t \pi_{Rrt+1} - \chi_R \pi_{crt}) + \frac{(1-\theta_{Rr})(1-\beta_{adj}\theta_{Rr})}{\theta_{Rr}} \frac{MC_{Rrt} - \overline{MC}_{Rr}}{\overline{MC}_{Rr}} \tag{107}$$

where θ_{Rr} is the probability of indexing the price of materials in one period; χ_R is the degree of indexing to previous inflation; MC_{Rrt} is the real marginal cost of producing one unit of differentiated materials:

$$MC_{Rrt} = \frac{\varphi-1}{\varphi} \frac{P^*_{Rt} P^*_t S_t}{P_{Rrt}}, \tag{108}$$

where P^*_{Rt} and $P^*_{Rt} P^*_t$ are the actual and nominal prices of materials abroad, respectively.

We assume that importing firms purchase materials at a price lower (by the amount of the monopolistic markup) than $P^*_{Rt} P^*_t$, which ensures that domestic and foreign material prices are equalised in a steady state:

$$\bar{P}^*_{Rt} \bar{P}^*_t \bar{S}_t = \overline{MC}_{Rrt} \bar{P}_{Rrt} \tag{109}$$

The nominal rigidity in the pricing of imports (goods F and materials R) makes the adjustment of the domestic economy to external shocks more protracted than the adjustment of foreign economies to the same shocks.

Central bank

Monetary authorities operating under an inflation targeting regime use the principle of inflation forecast targeting, in which their own inflation forecast is an intermediate target of monetary policy:

The Taylor rule for the key rate is based on expected inflation:

$$i_{kt} = \rho_{ik} i_{kt-1} + (1 - \rho_{ik}) \left({}^{k_\pi}/_{4} \frac{E_t P_{ct+3} - P_{ct-1}}{P_{ct-1}} \right) + \varepsilon_{ikt}, \tag{110}$$

where i_{kt} is the Central Bank's key rate; ρ_{ik} is the inertia indicator of the key rate dynamics; ε_{ikt} is the monetary policy shock characterising its discretionary component; $k_\pi > 1$ is the coefficient of response [adjusted to year][14] of the key rate to expected annual inflation.

The key rate responds to expected annual inflation (four quarters: current and three future). Due to the inertia of the inflationary process and the key rate, its response to most shocks takes the form of a hill. To make the interest rate response to economic shocks smoother, most studies include an autoregressive component in the Taylor rule.[15] Inertia prevents sharp fluctuations in the key rate in response to various shocks, i.e. the need to reverse current decisions in the future.

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[14] This indicator k_π is divided by 4 to align the annual CPI growth rate $\frac{E_t P_{ct+3} - P_{ct-1}}{P_{ct-1}}$ with the key rate i_{kt} adjusted to the quarter.

[15] Woodford (2001) believes that such a response may characterise optimal monetary policy, which is based not only on the needs arising from the optimisation of the current situation, but also on the historical dynamics of interest rates, which may characterise the fulfilment of the monetary authorities' commitments to society.

We assume that the monetary authorities do not make systematic interventions in the foreign exchange market, and the national currency is in a free-floating regime. At the same time, the non-instantaneous adjustment of the international capital market makes foreign exchange interventions potentially effective. In the model, the instrument of unsystematic foreign exchange interventions is implemented through the Central Bank's ability to issue securities B_{nt} (see equation (14)), which leads to the need for households of type n to adjust the volume of operations with foreign securities B^*_{rnt} (see budget constraint (4)). As a result, the issuance of B_{nt} affects the country's trade balance and, consequently, demand, exchange rates, output, inflation and other macroeconomic variables.

Foreign economy

We assume that the foreign economy is a [large] closed economy, the dynamics of which do not depend on processes in the [small open] domestic economy.

We use a new Keynesian closed economy model with constant returns to scale, in which labour L^*_t, materials R^*_t, and oil O^*_t are used to produce final goods Y^*_t. The Cobb-Douglas production function is:

$$Y^*_t = A^*_t(\exp(n^*)L^*_t)^{\alpha_{L^*}}(R^*_t)^{\alpha_{R^*}}(O^*_t)^{\alpha_{O^*}}, \qquad \alpha_{L^*} + \alpha_{R^*} + \alpha_{O^*} = 1 \tag{111}$$

where $\alpha_{L^*} \in (0,1)$ is the share of labour income in total income; $\alpha_{R^*} \in (0,1)$ is the share of material owners in total income; $\alpha_{O^*} \in (0,1)$ is the share of oil owners in total income; A^*_t is the total factor productivity of foreign goods, following an AR(1) process:

$$A^*_t = (A^*_{t-1})^{\rho_{A^*}}(A^*)^{1-\rho_{A^*}}\exp(g^* + \varepsilon_{A^*t}), \qquad \rho_{A^*} \in (0,1) \tag{112}$$

where g^* is the growth rate of productivity abroad; ε_{A^*t} is the shock to foreign total factor productivity.

The supply of materials and oil is assumed to be exogenous AR(1) processes:

$$R^*_t = (R^*_{t-1})^{\rho_{R^*}}(\bar{R}^*)^{1-\rho_{R^*}}\exp(g_{R^*} + \varepsilon_{R^*t}), \qquad \rho_{R^*} \in (0,1) \tag{113}$$

where g_{R^*} is the growth rate of material production abroad; ε_{R^*t} is material supply shocks.

$$O^*_t = (O^*_{t-1})^{\rho_{O^*}}(\bar{O}^*)^{1-\rho_{Y^*_{Oil}}}\exp(g_{O^*} + \varepsilon_{Y^*_{Oil}t}), \qquad \rho_{O^*} \in (0,1) \tag{114}$$

where g_{O^*} is the growth rate of oil production abroad; ε_{O^*t} is oil supply shocks.

Household utility function:

$$U^*_t = \sum_{s=0}^{\infty}(\beta^*)^s\exp(u_{\beta^*t+s})\,\Lambda^*_{t+s} \tag{115}$$

where β^* is the subjective discount; u_{β^*t+s} is the AR(1) process:

$$u_{\beta^*t} = \rho_{\beta^*}u_{\beta^*t-1} + \varepsilon_{\beta^*t} \qquad \rho_{\beta^*} \in [0,1] \tag{116}$$

Λ^*_{t+s} is an instantaneous utility function, which has the following form:

$$\Lambda^*_t = \frac{(C^*_t - h^*C^*_{t-1})^{1-\sigma^*}}{1-\sigma^*} - \frac{(L^*_t)^{1+\kappa^*}}{1+\kappa^*}, \tag{117}$$

where C^*_t is household consumption; L^*_t is hours worked by the household; σ^* is a parameter inverse to the elasticity of intertemporal substitution of consumption; κ^* is the value inverse to the elasticity of labour supply according to Frisch; h^* is a parameter of external consumption habits.

where ε_{β^*t} is the shock of intertemporal preferences;

First-order condition for the dynamics of foreign household consumption:

$$\beta^*E_t\left\{\frac{(1+i^*_t)}{(1+\pi^*_{t+1})}\frac{\exp(u_{\beta^*t+1})}{\exp(u_{\beta^*t})}\frac{\Lambda^*_{Ct+1}}{\Lambda^*_{Ct}}\right\} = 1, \tag{118}$$

where $\Lambda^*_{Ct} \equiv \frac{\partial \Lambda^*_t}{\partial C^*_t} = (C^*_t - h^* C^*_{t-1})^{-\sigma^*}$ is the marginal utility of additional consumption; $\pi^*_t \equiv \frac{P^*_t - P^*_{t-1}}{P^*_{t-1}}$ is the rate of inflation of foreign consumer prices.

Household labour supply:

$$\frac{W^*_t}{P^*_t} = ((1-h^*)C^*_t)^{\sigma^*}(L^*_t)^{\kappa^*} \tag{119}$$

Foreign monopolistic competitor firms set the optimal price according to the Calvo model with indexation to foreign inflation of the previous quarter. The Phillips curve for foreign inflation is as follows:

$$\pi^*_t = \frac{\chi^*}{1+\beta^*\chi^*}\pi^*_{t-1} + \frac{\beta^*}{1+\beta^*\chi^*}E_t\pi^*_{t+1} + \frac{(1-\theta^*)(1-\beta^*\theta^*)}{\theta^*}\frac{MC^*_t - \overline{MC}^*}{\overline{MC}^*}, \tag{120}$$

where χ^* is the indexation rate; θ^* is the Calvo model parameter (probability of price non-optimisation in each period); MC^*_t is the real marginal costs of firms:

Optimal demand for resources under monopolistic competition:

$$L^*_t = \alpha_{L^*} Y^*_t \left(\frac{W^*_t}{P^*_t}\right)^{-1} MC^*_t \tag{121}$$

$$R^*_t = \alpha_{R^*} Y^*_t \left(\frac{P^*_{Rt}}{P^*_t}\right)^{-1} MC^*_t \tag{122}$$

$$O^*_t = \alpha_{O^*} Y^*_t \left(\frac{P^*_{Ot}}{P^*_t}\right)^{-1} MC^*_t \tag{123}$$

All produced goods are consumed by foreign households:

$$Y^*_t = C^*_t \tag{124}$$

For clarity, from the solutions of the pairs of equations (113) and (122); (114) and (123), we derive the equations of dynamics for the prices of materials and oil:

$$P^*_{Rt} = \alpha_{R^*} \frac{MC^*_t}{(R^*_{t-1})^{\rho_{R^*}}(\bar{R}^*)^{1-\rho_{R^*}}} \exp(-\varepsilon_{R^*t}) \tag{125}$$

$$P^*_{Ot} = \alpha_{O^*} \frac{MC^*_t}{(O^*_{t-1})^{\rho_{O^*}}(\bar{O}^*)^{1-\rho_{O^*}}} \exp(-\varepsilon_{O^*t}) \tag{126}$$

Prices of materials and oil react negatively to supply shocks ε_{R^*t} and ε_{O^*t}, respectively, and react positively to growth in real marginal costs in foreign economies. Such a decomposition of oil and material price dynamics allows for more accurate identification of the sources of the foreign business cycle.

The foreign central bank sets the rate according to the Taylor rule:

$$i^*_t = \rho_{i^*} i^*_{t-1} + (1-\rho_{i^*})\left(i^* + k_{\pi^*}\pi^*_t + k_{y^*}\frac{Y^*_t - Y^*}{Y^*} + \varepsilon_{i^*t}\right), \tag{127}$$

where i^*_t is the foreign central bank rate; ρ_{i^*} is the inertia indicator of the foreign rate dynamics; ε_{i^*t} is the foreign rate shock; $k_{\pi^*} > 1$ is the coefficient of response of the foreign rate to foreign inflation; k_{y^*} is the coefficient of the foreign rate response to the output gap abroad.

Foreign demand for domestic firms' products

The foreign economy demands our export products: tradable goods and oil.

Let us assume that the export demand for domestic oil X_{rOt} is an ad hoc function of foreign output Y^*_t with constant elasticity:

$$\frac{X_{rOt} - \bar{X}_{rO}}{\bar{X}_{rO}} = k_{XO}\frac{Y^*_t - \bar{Y}^*}{\bar{Y}^*} + u_{Ext} + u_{ExOt} + z_{Ext}, \tag{128}$$

where k_{XO} is the elasticity of demand for domestic oil in terms of output volume abroad; u_{Ext} and u_{ExOt} are AR(1) processes that define changes in foreign demand for all domestic exports and domestic oil exports, respectively:

$$u_{Ext} = \rho_{Ex} u_{Ext-1} + \varepsilon_{Ext} \qquad \rho_{Ex} \in [0,1] \tag{129a}$$

$$u_{ExOt} = \rho_{ExO} u_{ExOt-1} + \varepsilon_{ExOt}, \qquad \rho_{ExO} \in [0,1] \tag{129b}$$

where ε_{Ext} is a temporary shock to demand for all domestic exports; ε_{ExOt} is a temporary shock to demand for domestic oil exports.

z_{Ext} is the quasi-I(1) component of the permanent export shock:

$$z_{Ext} = 0.9999 z_{Ext-1} + \upsilon_{Ext}, \tag{130}$$

where υ_{Ext} is the permanent export shock.

In the market for tradable goods, domestic goods compete with foreign goods, and demand for domestic exports follows from the task of optimally distributing foreign demand among monopolistic competitors:

$$X_{rHt} = \omega_H \exp(u_{Ext} + u_{ExHt} + z_{Ext}) Y_t^* \left(\frac{P^*_{rH^*t}}{P_t^*}\right)^{-\delta^*}, \tag{131}$$

where ω_H is the share of global demand accounted for by domestic tradable goods in the long term; δ^* is the price elasticity of demand for domestic exports; u_{ExHt} is the AR(1) process that determines changes in foreign demand for exports of domestic tradable goods. In order to ensure that sectoral components u_{ExOt} and u_{ExHt} do not affect overall export demand, we will impose a restriction on them:

$$\gamma_{rO} u_{ExOt} + (1 - \gamma_{rO}) u_{ExHt} = 0, \tag{132}$$

where γ_{rO} is the share of oil in total exports.

In this case, shock ε_{ExOt} redistributes the foreign economy's demand between the domestic oil sector and the domestic tradable goods sector. Shocks ε_{Ext} and υ_{Ext} symmetrically affect both sectors of domestic exports simultaneously.

For the purposes of model identification, we will define the nominal exports and imports of region r (in units of foreign currency):

$$NEx_{rt} = \frac{P_{Ot}}{S_t} X_{rOt} + P^*_{rHt} X_{rHt} \tag{133}$$

$$NIm_{rt} = Y_{rFt} MC_t^* P_t^* + \frac{\varphi - 1}{\varphi} P^*_{Rt} P_t^* R_{rt}. \tag{134}$$

General equilibrium

General equilibrium requires that, at any given time, supply and demand in each market segment be equal to each other. The equilibrium condition is written for all markets in each region r.

In the goods market k of region r, the volume of demand from households, firms, the government, and the foreign economy equals the volume of supply. For non-tradable industries, demand is generated within the region:

$$Y_{rNt} = C_{rNt} + I_{rNt} + G_{rNt} \tag{135}$$

$$Y_{rSt} = C_{rSt} + I_{rSt} + G_{rSt} \tag{136}$$

The volume of foreign tradable goods produced is also determined by the demand of domestic agents in region r:

$$Y_{rFt} = C_{rFt} + I_{rFt} + G_{rFt} \tag{137}$$

Demand for domestic tradable goods of region r is created in all regions q and abroad:

$$Y_{rHt} = \sum_q \lambda_{rq} \frac{\lambda_q}{\lambda_r} \left[(C_{qHt} + I_{qHt} + G_{qHt}) \left(\frac{P_{rHst}}{P_{qHt}}\right)^{-\delta_R} \right] + X_{rHt} \tag{138}$$

Demand for oil is only foreign:

$$Y_{rOt} = X_{rOt} \tag{139}$$

The above describes the condition of equilibrium in the labour market: the labour supply from households of group j in region r is equal to the demand for labour from households of this group j from firms in all regions q and all sectors m:

$$L_{rjt} = \sum_q \sum_m L_{rqmjt} \tag{40}$$

Equilibrium in the financial market means that, given the interest rates set by domestic and foreign monetary authorities, as well as the current volume of securities issued by fiscal and monetary authorities, Ricardian households ensure an optimal level of expenditure through transactions with foreign securities.

Let us define real gross regional product (GRP) as the sum of the added values of all sectors $m \in \{H, N, S, O\}$, as well as in the sectors of imports of goods F and materials R:

$$Y_{rt} = \sum_m Y_{rmt} - R_{rt} + \frac{Y_{rFt}+R_{rt}}{\varphi} \tag{140}$$

The real gross domestic product is the sum of the GRP of all regions:

$$Y_t = \sum_r Y_{rt} \tag{141}$$

The balance of payments equation for the region follows from the budget constraint of households of type n in region r:

$$\frac{X_{rOt}P_{Ot}}{S_t} + X_{rHt}P^*_{rH^*t} - Y_{rFt}MC^*_t P^*_t - \frac{\varphi-1}{\varphi}R_{rt}P^*_{Rt}P^*_t - \frac{P^*_t Adj_{B^*t} + B^*_{rnt-1}(1+i^*_{rnt-1}) - B^*_{rnt}}{1-\lambda_n} - \frac{1}{S_t}\frac{B_{rnt-1}(1+i_{rnt-1}) - B_{rnt}}{1-\lambda_n} = 0 \tag{142}$$

We have defined B_{rnt} as the debt of households to the Central Bank, which the monetary authorities can manipulate at their discretion. From (142) it can be seen that $\frac{B_{rnt}}{S_t}$ has the economic meaning of the Central Bank's currency reserves. Due to imperfect international capital mobility, the second monetary policy instrument will be effective (see Shulgin, 2014). At the same time, we do not set rules for this instrument (the systematic component of currency reserves), leaving monetary authorities the possibility to pursue discretionary exchange rate policy.

Aggregation

Having defined the general equilibrium at the level of region r, we obtain the dynamics of variables for the country as a whole by aggregating regional variables:

$$x_t = \sum_r \lambda_r x_{rt}, \tag{143}$$

where $x_t \in \{Y_t, C_t, \pi_{ct}\pi_{Rt}, W_t, L_t, B_t, T_t, G_t, T_{Lt}, T_{0t}, i_t, P_{ct}, MC_{Rt}, B_{nt}, B^*_{nt}, A_{nt}, rp_{nt}, R_t, I_t, K_t, i_{nt}, BD_t, BD_{0t}\}$

For the purpose of identifying information about the behaviour of heterogeneous households, firms, and the state of goods markets, we will conduct regional aggregation of variables of the type:

$$x_{jt} = \sum_r \frac{\lambda_r \lambda_{rj}}{\lambda_j} x_{rjt}, \tag{144}$$

where $x_{jt} \in \{C_{jt}, W_{jt}, A_{jt}\}$

$$x_{kt} = \sum_r \lambda_r x_{rkt}, \tag{145}$$

where $x_{kt} \in \{C_{kt}, G_{kt}, I_{kt}, \pi_{kt}\}$

$$x_{mt} = \sum_r \lambda_r x_{rmt}, \tag{146}$$

where $x_{mt} \in \{Y_{mt}, X_{mt}, MC_{mt}\}$

Certain regional variables are also obtained by aggregation:

$$x_{rt} = \sum_j \lambda_{rj} x_{rjt}, \tag{147}$$

where $x_{rt} \in \{B^*_{rt}, C_{rt}, B_{rt}, A_{jt}, rp_{rt}\}$

$$x_{rt} = \sum_m x_{rmt}, \tag{148}$$

where $x_{mt} \in \{R_{rt}, I_{rt}, L_{rt}, K_{rt}\}$

Consumption of goods of type k in region r is aggregated by household types j:

$$C_{rkt} = \sum_j \lambda_{rj} x_{rkjt} \tag{149}$$

Investments in goods of type k in region r are the sum across sectors m:

$$I_{rkt} = \sum_m I_{rmkt}. \tag{150}$$

Balanced growth path

Due to the presence of exogenous growth rates of labour resources n and TFP g, we will determine the properties of the balanced growth path (bgp). Let us determine the long-term (on bgp) rate of output growth and, accordingly, all variables in the long term proportional to output: capital, materials, consumption, government spending, investment.

To do this, we will perform a logarithm and take the derivative of the production function (62) with respect to time:

$$g_Y = g + \alpha_L(n + g_L) + \alpha_K g_K + \alpha_R g_R, \tag{151}$$

where we denote g_x as the growth rate of variable x_t. When determining the growth rates of variables, we omit the indices of the region, industry, and household, as in the bgp the regional, industrial, and agent structure of the economy does not change, so the growth rates do not depend on these indices.

Since output, capital, and materials in the bgp grow at the same growth rate, we have:

$$g_Y = g_K = g_R = n + g_L + \frac{g}{\alpha_L} \tag{152}$$

The demand for labour (58) determines the rate of growth in real wages:

$$g_w = g_Y - n - g_L = \frac{g}{\alpha_L} \tag{153}$$

From the condition of labour supply in the long term (39), it follows that:

$$g_w = \sigma(g_Y - n) + \kappa g_L \tag{154}$$

By solving (153) and (154) together, we obtain the long-term growth rate of hours worked per worker:

$$g_L = \frac{g}{\alpha_L}\frac{1-\sigma}{\sigma+\kappa} \tag{155}$$

The long-term growth rate of output on bgp:

$$g_Y = \frac{g}{\alpha_L}\left(1 + \frac{1-\sigma}{\sigma+\kappa}\right) + n \tag{156}$$

The same applies to the foreign economy:

$$g^*_Y = \frac{g^*}{\alpha^*_L}\left(1 + \frac{1-\sigma^*}{\sigma^*+\kappa^*}\right) + n^* \tag{157}$$

We assume that the price level also has an exogenous long-term growth rate π for domestic prices and π^* for foreign prices.[16] Thus, the long-term growth rate of the model's nominal variables consists of the growth rate of the real and price components. For example, the growth rate of nominal taxes is $g_T = g_Y + \pi$; the growth rate of nominal imports and exports in foreign currency units: $g_{Im} = g_{Ex} = g_Y + \pi^*$.

The properties of the balanced growth path generally correspond to Kaldor's stylised facts. An unconventional property of bgp is the non-zero growth rate of hours worked per worker g_L on bgp. This conclusion arises from the properties of the household's instantaneous utility

[16] We ignore the impact of the long-term price growth rate on the business cycle.

function and the resulting properties of the long-term labour supply function. The traditional solution for households ($g_L = 0$) corresponds to the case of unit elasticity of intertemporal substitution $\sigma = 1$, which is standard in the literature on economic growth theory. In our case, the medium-term adjustment to long-term equilibrium is more important than the properties of the long-term equilibrium itself, so we do not fix this parameter and allow it to adjust to quarterly data.

Parameterisation

Parameterisation consists of two steps. In the first step, we calibrate those parameters that are difficult to identify in quarterly statistics but can be calibrated based on relevant regional, sectoral, agent international, and aggregated data. For these purposes, we use various sources of information, ranging from statistical compilations to the works of other authors, including foreign ones. In the second step, we gather prior information about the parameters, which are then estimated using the Bayesian method on quarterly statistics. The parameterisation of the model is based on the multidimensional model of the posterior distribution found.

Data

For the Bayesian estimation of the model parameters, we use regional, sectoral, agent, all-Russian, and international series of observed variables. Table 1 shows the observed variables, formulas linking observed and model variables, deterministic trend coefficients for each, and the source of information. Most aggregated country-wide observed series are traditionally used to identify shocks in practice-oriented DSGE models, so we will focus on the information about three types of heterogeneity used in the model. All regional variables are calculated by aggregating the series of Russian regions included in the so-called Volga-Vyatka macroregion (VVMR): Republic of Mari El, Republic of Mordovia, Republic of Tatarstan, Udmurt Republic, Chuvash Republic, Kirov Region, Nizhny Novgorod Region, Penza Region, Samara Region, Saratov Region, Ulyanovsk Region. Gross regional product (GRP) is not calculated by Rosstat on a quarterly basis, so we use the methodology and results obtained through the temporal disaggregation methodology of Boyko et al. (2020).

To evaluate the model, we use a series of logarithms of the relative price levels in the VVMR relative to Russia as a whole RP_{VVobs}, which is calculated with an accuracy of a constant based on two series: CPI inflation in the Russian Federation π_{obs} and CPI inflation in the VVMR π_{VVobs}.

Let us present several empirical facts about the ranks of the VVMR in comparison with their aggregated counterparts for the country. The correlation of seasonally adjusted inflation series of Russia and the VVMR in the range of 2014–2023 is $Cor(\pi_{ct}, \pi_{cVVt}) = 0.99$. This extremely high correlation indicates the decisive role of common shocks in explaining inflation, the limited impact of the regional business cycle on regional inflation, and strong regional spill-over effects.

The GRP of the VVMR grew at an average rate of 1.78% per annum between 2014 and 2025, while the average GDP growth rate of the Russian Federation for the same period was 1.35% per annum. This difference is most likely not related to the difference in the long-term growth rates of the two series. For example, between 2014 and 2021, Russia's GDP growth rate was 1.27%, while the GRP of the VVMR grew at a rate of 0.89%. This difference in GRP growth rates in the VVMR at different time intervals is most likely related to the regional component of the business cycle: either regional shocks or the peculiarities of the sectoral and household structure of the microregion. In the model, we postulate the equality of long-term growth rates (both deterministic and stochastic components) of the Russian Federation's GDP and GRP of the VVMR, and explain the emerging discrepancies primarily by heterogeneity shocks. This does not contradict the presented empirical facts.

The difference in the dynamics of real wages of the VVMR and the Russian Federation behaves similarly to output. For the period 2014–2025, the growth rates are close, although not identical: for real wages of the VVMR grew by 2.75%, while in Russia they grew by 2.38%. During the period of accelerated growth from 2022 to 2025, real wages grew by 22.4% in the VVMR and by 17.1% in the Russian Federation. This variation indicates a significant contribution to the dynamics of real wages by regional shocks and/or regional characteristics of the VVMR, and is consistent with the model's assumption of equal long-term growth rates for real wages in the VVMR and the Russian Federation.

When calculating the logarithm of the consumption volume of hand-to-mouth households per capita, we used a methodology similar to that used in Shulgin et al. (2025).

To assess the parameters of foreign economies, we used data from the US (prices, interest rates), the G20 (output) and information from commodity exchanges. In our opinion, using data on production in the G20 rather than the US is justified, as it provides much more information about Russia's foreign trade. The inconsistency with the monetary policy rule that links the interest rate (US) to the output gap (G20) is attributed to shocks in foreign monetary policy.[17]

Table 1. Quarterly series of observed variables of the model

Designation	Observed variable	Constraint equation	Trend	Source
All-Russian (aggregated) variables				
π_{obs}	Inflation by CPI, QoQ, SA	$\pi_{obs} = 0.01 + \pi_{ct}$	0	Rosstat
Y_{obs}	Logarithm of GDP, 2016 prices, SA	$Y_{obs} = a_{0Y} + g_Y t + \ln Y_t$	g_Y	Rosstat
G_{obs}	Logarithm of government spendign, 2016 prices, SA	$G_{obs} = a_{0G} + g_Y t + \ln G_t$	g_Y	Rosstat
I_{obs}	Logarithm of GFCF, 2016 prices, SA	$I_{obs} = a_{0I} + g_Y t + \ln I_t$	g_Y	Rosstat
w_{obs}	Logarithm of real wages, SA	$w_{obs} = a_{0w} + g_w t + \ln w_t$	g_w	Rosstat

[17] This inconsistency does not appear to be critical if there is no intention to use the developed DSGE model for scenario analysis related to the global economy.

BD/Y_{obs}	Ratio of non-oil and gas budget deficit to GDP, SA	$BD/Y_{obs} = a_{0BD} + \frac{BD_t}{P_{ct}Y_t}$	0	Rosstat, authors' calculations
NEx_{obs}	Logarithm of nominal exports, billion USD	$NEx_{obs} = a_{0NEx} + (g_Y + \pi^*)t + \ln NEx_t$	$g_Y + \pi^*$	Balance of payments, Bank of Russia
NIm_{obs}	Logarithm of nominal exports, billion USD	$NIm_{obs} = a_{0NEx} - a_{0NIm} + (g_Y + \pi^*)t + \ln NIm_t$	$g_Y + \pi^*$	Balance of payments, Bank of Russia
L_{obs}	Logarithm of hours worked per employee, SA	$L_{obs} = a_{0L} + g_L t + \ln L_t + me_{Lt}$	g_L	Rosstat
N_{obs}	Employed = Labour force - unemployed, SA	$N_{obs} = a_{0N} + nt + \ln N_t$	n	Rosstat
i_{kobs}	Key rate, adjusted to the quarter	$i_{kobs} = 0.01 + i_{kt}$	0	Bank of Russia
i_{obs}	Credit (loan) rate, adjusted to the quarter	$i_{obs} = a_{0i} + i_t$	0	Bank of Russia
Q_{obs}	Real US dollar exchange rate	$Q_{obs} = a_{0Q} + \ln Q_t$	0	Bank of Russia, authors' calculations
Regional variables (VV = Volga-Vyatka macroregion (VVMR))				
π_{VVobs}	CPI inflation in the VVMR, QoQ, SA	$\pi_{VVobs} = 0.01 + \pi_{cVVt}$	0	Rosstat, authors' calculations
RP_{VVobs}	Logarithm of the level of relative prices of the VVMR compared to Russia as a whole	$RP_{VVobs} = a_{0RP_{VV}} + \ln\left(\frac{P_{cVVt}}{P_{ct}}\right)$	0	Rosstat, authors' calculations
Y_{VVobs}	Logarithm of GRP in the VVMR, 2016 prices, SA	$Y_{VVobs} = a_{0Y_{VV}} + g_Y t + \ln Y_{VVt}$	g_Y	Calculation methodology based on Boyko et al. (2020)
G_{VVobs}	Logarithm of government spending in the VVMR, 2016 prices, SA	$G_{VVobs} = a_{0G_{VV}} + g_Y t + \ln G_{VVt}$	g_Y	Rosstat, authors' calculations
w_{VVobs}	Logarithm of real wages in the VVMR, SA	$w_{VVobs} = a_{0w_{VV}} + g_w t + \ln w_{VVt}$	g_w	Rosstat, authors' calculations

i_{VVobs}	Credit (loan) rate in the VVMR, adjusted to the quarter	$i_{VVobs} = a_{0i_{VV}} + i_{VVt}$	0	Bank of Russia, authors' calculations
Sectoral variables				
Y_{Nobs}	Logarithm of production volume in the services sector, SA	$Y_{Nobs} = a_{0Y_N} + g_Y t + \ln Y_{Nt}$	g_Y	Rosstat
Y_{Oobs}	Logarithm of production volume in the extraction sector, SA	$Y_{Oobs} = a_{0Y_O} + g_Y t + \ln Y_{Ot}$	g_Y	Rosstat
π_{Nobs}	Services inflation, QoQ, SA	$\pi_{Nobs} = 0.01 + \pi_{Nt}$	0	Rosstat
Agent variables				
C_{hobs}	Logarithm of hand-to-mouth household consumption volume per person, SA	$C_{hobs} = a_{0C_h} + (g_Y - n)t + \ln C_{ht}$	$g_Y - n$	RLMS, HBS, Rosstat, authors' calculations
Foreign variables				
Y^*_{obs}	Logarithm of G20 GDP, SA	$Y^*_{obs} = a_{0Y^*} + g^*_Y t + \ln Y^*_t$	g^*_Y	World Bank
π^*_{obs}	Inflation in the United States based on CPI, SA	$\pi^*_{obs} = a_{0\pi^*} + \pi^*_t$	0	Fed
i^*_{obs}	Upper bound of the federal funds rate range in the United States	$i^*_{obs} = a_{0i^*} + i^*_t$	0	Fed
P^*_{Onobs}	Logarithm of oil price (Urals), USD/barrel	$P^*_{Onobs} = a_{0PO^*} + \pi^* t + \ln(P^*_{Ot} P^*_t)$	π^*	IMF
P^*_{Rnobs}	Logarithm of the non-energy commodities index	$P^*_{Rnobs} = a_{0PR^*} + +\pi^* t + \ln(P^*_{Rt} P^*_t)$	π^*	IMF

Source: authors' calculations.

Figure P1 in the Appendix shows the dynamics of the main variables in the model (data + forecast)

Parameter calibration

A number of parameters are poorly [or not at all] identified in quarterly data. In this case, we look for alternative sources of information for their identification. Table 2 contains information on the calibration of such model parameters.

Table 2. Calibrated parameters

Designation	Parameter name	Value	Source/formula/comment
All-Russian parameters			

β	Discount factor	0.995	Time discount of 2% on an annualised basis
δ_K	Capital retirement rate	0.025	Capital depreciation of 10% per annum
δ_L	Elasticity of substitution between labour of different household groups	0.6	Substitution of labour of large household groups is quite low
φ	Elasticity of substitution between differentiated goods	6	Monopolistic markup of 20%: $\frac{1}{\varphi-1}$
φ_L	Elasticity of substitution between differentiated household labour	6	Monopolistic markup of 20%: $\frac{1}{\varphi_L-1}$
τ	Labour income tax	0.3	Corresponds to government spending of 30% of GDP
ρ_{ik}	Autoregression coefficient of the key rate in the Taylor rule	0.6	Poorly identifiable together with other parameters of the Taylor rule
$\psi_{\Delta B^*}$	Cost function parameter related to the deviation of the current level of foreign debt from the previous level	0.1	Low adjustment costs
α_L	Share of wages in costs	0.66	Taking into account the monopolistic markup, it corresponds to the share of labour income in GDP of about 55%.
α_R	Share of payment for imported materials in costs	0.05	Share of imported materials in the domestic economy is somewhat higher than abroad ($\alpha_R^* = 0.025$)
α_K	Share of capital payment in costs	0.29	$\alpha_K = 1 - \alpha_L - \alpha_R$
ψ_i	Semi-elasticity of risk premium by region's asset volume	0.0001	Minimum value ensuring the stability of the region's financial market
Foreign parameters			
φ^*	Foreign elasticity of substitution between differentiated goods	6	Monopolistic markup of 20%: $\frac{1}{\varphi-1}$
n^*	Foreign population growth rate	0.002	Corresponds to a population growth rate of 0.8% per annum
κ^*	Value inverse to the elasticity of labour supply according to Frisch	1	
γ_{B^*}	Ratio of foreign debt of domestic households to exports in a steady state	4	Corresponds to the value of foreign debts of domestic households equal to annual exports
ρ_{i^*}	Parameter of inertia of foreign rate dynamics	0.8	Poorly identifiable in the data. Corresponds to estimates by other authors
Regional parameters (VV = Volga-Vyatka macroregion (VVMR))			
λ_{VV}	Share of the population in the VVMR	0.138	Rosstat, authors' calculations
λ_{RU}	Share of the population in other regions	0.862	$\lambda_{RU} = 1 - \lambda_{VV}$
GRP_{VV}	GRP per capita in the VVMR, thousand rubles, 2021, per quarter	160	Rosstat, authors' calculations
GRP_{RU}	GRP per capita in other regions, thousand rubles, 2021, per quarter	213	Rosstat, authors' calculations
k_{CVVn}	Ratio of average per capita consumption of households of group n to average per capita consumption of households of group h in the VVMR	2.6	RLMS, HBS, authors' calculations

k_{CRUn}	Ratio of average per capita consumption of households of group n to average per capita consumption of households of group h in other regions of Russia	2.8	RLMS, HBS, authors' calculations
γ_{OVV}	Share of oil in total exports of the VVMR	0.66	$\gamma_{Or} = \gamma_O$, Federal Customs Service
γ_{ORU}	Share of oil in total exports of other regions of Russia	0.66	$\gamma_{Or} = \gamma_O$, Federal Customs Service
γ_{GVV}	Ratio of government spending to GRP in the VVMR	0.3	$\gamma_{Gr} = \gamma_G$, Rosstat
γ_{GRU}	Ratio of government spending to GRP in other regions of Russia	0.3	$\gamma_{Gr} = \gamma_G$, Rosstat
γ_{BVV}	Ratio of issued government securities to GRP in the VVMR in a steady state	1	$\gamma_{Br} = \gamma_B$. Corresponds to the ratio of annual public debt to GDP = 1/4
γ_{BRU}	Ratio of issued government securities to GRP in other regions of Russia in a steady state	1	$\gamma_{Br} = \gamma_B$. Corresponds to the ratio of annual public debt to GDP = 1/4
γ_{TRYVV}	Ratio of the volume of transfers to households of group h from the state budget to GRP in the VVMR in a steady state	0.033	$\gamma_{TRYr} = \gamma_{TRY}$, Ministry of Finance
γ_{TRYRU}	Ratio of the volume of transfers to households of group h from the state budget to GRP in other regions of Russia in a steady state	0.033	$\gamma_{TRYr} = \gamma_{TRY}$, Ministry of Finance
$k_{VV^H/_T}$	Share of domestic tradable goods in the total volume of tradable goods in the VVMR	0.6	Balance of payments, authors' calculations
$k_{RU^H/_T}$	Share of domestic tradable goods in the total volume of tradable goods in other regions of Russia	0.6	Balance of payments, authors' calculations
θ_{RU}	Coefficient for determining Calvo parameters for other regions of Russia: $\theta_{RUm} = \theta\theta_{RU}\theta_m$ and $\theta_{WRUj} = \theta_W\theta_{RU}\theta_j$	1	Normalisation
Regional + sectoral parameters			
λ_{VVN}	Share of non-tradable goods (services) in the aggregate demand of the VVMR	0.230	Rosstat, authors' calculations
λ_{VVS}	Share of housing and public utility services in the aggregate demand of the VVMR	0.050	Rosstat, authors' calculations
λ_{VVH}	Share of domestic tradable goods in the aggregate demand of the VVMR	0.432	$\lambda_{VVH} = k_{VV^H/_T}(1 - \lambda_{VVN} - \lambda_{VVS})$
λ_{VVF}	Share of foreign tradable goods in the aggregate demand of the VVMR	0.288	$\lambda_{VVF} = (1 - k_{VV^H/_T})(1 - \lambda_{VVN} - \lambda_{VVS})$
λ_{RUN}	Share of non-tradable goods (services) in the aggregate demand of other regions of Russia	0.230	Rosstat, authors' calculations
λ_{RUS}	Share of housing and public utility services in the aggregate demand of other regions of Russia	0.050	Rosstat, authors' calculations

λ_{RUH}	Share of domestic tradable goods in the aggregate demand of other regions of Russia	0.432	$\lambda_{RUH} = k_{RUH/T}(1 - \lambda_{RUN} - \lambda_{RUS})$
λ_{RUF}	Share of foreign tradable goods in the aggregate demand of other regions of Russia	0.288	$\lambda_{RUF} = (1 - k_{RUH/T})(1 - \lambda_{RUN} - \lambda_{RUS})$
Regional + agent parameters			
λ_{VVh}	Share of households of group h in the VVMR	0.45	RLMS, HBS, authors' calculations
λ_{VVn}	Share of households of group n in the VVMR	0.55	$\lambda_{VVn} = 1 - \lambda_{VVh}$
λ_{RUh}	Share of households of group h in other regions of Russia	0.4	RLMS, HBS, authors' calculations
λ_{RUn}	Share of households of group n in other regions of Russia	0.6	$\lambda_{RUn} = 1 - \lambda_{RUh}$
Sectoral parameters			
$\rho_{\mu S}$	Autoregression coefficient of relative utility prices to CPI	0.95	The gap between utility price index and CPI closes by 19% in one year
θ_H	Coefficient for determining the Calvo parameter in the domestic tradable goods sector: $\theta_{rH} = \theta\theta_r\theta_H$	1	Normalisation
θ_F	Coefficient for determining the Calvo parameter in the foreign tradable goods sector: $\theta_{rF} = \theta\theta_r\theta_F$	1	Normalisation
θ_{H^*}	Coefficient for determining the Calvo parameter in the domestic tradable goods sector abroad: $\theta_{rH^*} = \theta\theta_r\theta_{H^*}$	1	Normalisation
Agent parameters			
λ_h	Share of households of group h in Russia	0.4069	$\lambda_h = \sum_r \lambda_r\lambda_{rh}$
λ_n	Share of households of group n in Russia	0.5931	$\lambda_n = \sum_r \lambda_r\lambda_{rn} = 1 - \lambda_h$
θ_n	Coefficient for determining the Calvo parameter for wages: $\theta_{Wrn} = \theta_W\theta_r\theta_n$	1	Normalisation

Source: authors' calculations.

Bayesian parameter estimation

We use Bayesian methodology: (a) taking into account additional information about parameters and shocks; (b) taking into account a priori information about the properties of the object, such as the transfer of the exchange rate to prices, the elasticity of the exchange rate to the price of oil, the response of inflation to the key rate, etc.; (c) improving the convergence properties (to a multidimensional model) of the posterior density function when using maximisation algorithms.

We will conduct the Bayesian estimation in two stages. At the first stage, at time interval $2010q1–2025q2$, we separately evaluate the parameters of the external sector model, since these parameters are not related to the posterior density calculated on the basis of Russian series. In the second stage, we fix the deep parameters estimated in the first stage and estimate the full model based on both Russian and external series. At the same time, we re-evaluate the dynamic parameters of exogenous shocks of the external sector, as the estimation is carried out over a different time interval ($2014q1 – 2025q4$). Another reason for re-evaluating the dynamic parameters of foreign economy shocks is the correction of identification errors in the part of the foreign economy that interacts with Russia.

A priori parameter distributions

We estimate most standard deviations of shocks using a non-informative (uniform) a priori distribution. The exception is shocks, which are difficult to identify due to a lack of information. For example, TFP shocks ε_A have an effect on the economy similar to that of monopolistic markup shocks ε_μ; short-term labour shocks ε_N compete with permanent labour shocks υ_N; household shocks ε_{Tj} and $\varepsilon_{\beta j}$ compete to explain the dynamics of the only observable agent series: the consumption of household group h. In all cases, we use an inverse gamma distribution: $\Gamma^{-1}(0.005, \infty)$ so that the estimates of the standard deviations of these shocks are different from zero. For all autoregressive shock coefficients, we use beta distribution $\mathrm{B}(0.5, 0.1)$ to improve the joint identification of temporary and permanent shocks (see below).

To estimate the Calvo parameters for goods θ, materials (resources) θ_R and wages θ_W we use the beta distribution $\theta, \theta_R, \theta_W \sim \mathrm{B}(0.8, 0.03)$. The fairly high values of the expected mean of these parameters reflect our confidence in the low transfer of costs to prices, which follows from a number of empirical studies devoted to assessing transmission mechanisms based on Russian data (Andreev, 2019, Napalkov et al., 2021). The distribution of all regional, sectoral, and agent factors affecting the Calvo parameters is assessed using the normal distribution: $\theta_r, \theta_{m_K}, \theta_j \sim N(1, 0.05)$. Exception: prior distribution for sector N: $\theta_N \sim N(1.05, 0.05)$. $E(\theta_N) > 1$ reflects a lower probability of price adjustment in the service sector, as demonstrated in Nyoma et al. (2026). The parameters of the indexation degree are estimated based on the beta distribution $\chi, \chi_R, \chi_W \sim \mathrm{B}(0.5, 0.1)$, which is in line with common practice.

The prior distribution for the parameters of regional bias: $\Omega, \Omega_\mathrm{L} \sim \mathrm{B}(0.7, 0.05)$. This corresponds to the fact that local regional goods make up at least 30% of the domestic tradable goods in the consumption basket for smaller regions.

Prior distributions for the elasticities of substitution between different types of goods within the country and for our goods abroad: $\delta_C, \delta^* \sim N(0.8, 0.1)$. This reflects the assumption of a low degree of substitution between different types of goods in aggregates. The same distribution is used for the elasticity of substitution between labour of households from different regions: $\delta_{LR} \sim N(0.8, 0.1)$. The elasticity of substitution between tradable goods from different regions is assumed to be higher: $\delta_R \sim N(1.5, 0.1)$.

For both parameters of the utility function, we use a normal prior distribution with traditional means found in literature: $\sim N(2.0, 0.35)$ $\kappa \sim N(1.0, 0.2)$. The habit parameter in consumption is estimated with a prior distribution ⍰$\sim \mathrm{B}(0.4, 0.1)$. The parameter of the investment adjustment cost function $\phi_I \sim \mathrm{N}(1, 0.1)$. $E(\phi_I)$ is set experimentally (in the range of 0.5–2.0) to achieve moderate procyclicality of investments. The response coefficient to expected annual inflation in the Taylor rule is estimated using the prior distribution $k_\pi \sim \mathrm{N}(2, 0.1)$. While most studies suggest $E(k_\pi) = 1.5$, for the inflation targeting regime, the expected value of this parameter is typically higher. The prior distribution of the elasticity of lump-sum taxes by the level of real public debt $\psi_G \sim \mathrm{N}(0.1, 0.02)$. We assume that when $\psi_G \approx 0.1$, the public debt stabilisation rule leads to a lengthy adjustment period for public debt, which has a limited effect on the business cycle in the medium term.

The distribution of the cost function parameter associated with the deviation of the current level of foreign debt from the stationary level: $\psi_{B^*} \sim N(0.08, 0.01)$. The choice $E(\psi_{B^*})$ is related to the need to achieve the correct sign of the impulse response functions of inflation in response to balance of payments shocks (see below). We estimate the semi-elasticity of the external premium to the real oil price with a prior distribution: $\xi_O \sim N(0.003, 0.001)$. $E(\xi_O)$ was chosen in such a way as to obtain the elasticity of the foreign currency exchange rate to the price of oil close to the author's calculations of this elasticity based on data after 2022 $(-0.2;\ -0.15)$. We assume that the elasticity of demand for domestic oil in terms of output volume abroad is slightly less than the elasticity of demand for domestic tradable goods abroad with a prior distribution $k_{XO} \sim N(0.5, 0.05)$. We estimate the parameter characterising the degree of stabilisation of foreign oil price trends for the domestic market using the following prior distribution: $k_{SF} \sim N(0.5, 0.05)$. The historical range includes periods for which the degree of oil price stabilisation due to various principles of replenishment/use of stabilisation funds varied significantly, so in this paper we propose to focus on the mean value of 50%.

For all parameters of the observed series x_{obs}: a_{0x}, we use a non-informative prior distribution. To estimate productivity growth rates, we use a prior distribution $g \sim N(0.0025, 0.0005)$ and labour force growth rates $n \sim N(0, 0.001)$, which roughly corresponds to the observed population and GDP growth rates. The assumption of a non-zero discrepancy between real wages growth rates and the growth rates of other observed variables $g_{werr} \sim N(0.002, 0.0005)$ is related to a number of reasons: failure to take human capital into account, inaccuracy in wages measurement, etc.

To estimate the standard deviations of external shocks, we use a uniform prior distribution; $\rho_{A^*}, \rho_{\beta^*} \sim \mathrm{B}(0.6, 0.1)$ for autoregression coefficients; $\rho_{O^*}, \rho_{R^*} \sim \mathrm{B}(0.7, 0.04)$ for autoregression coefficients of oil and materials supply shocks. The expected values of the prior distributions of the autoregression coefficients for foreign shocks are higher than for domestic temporary shocks because, when explaining domestic observed series, we assume the existence of permanent shocks, while all foreign shocks are temporary. We chose the expected values for the shares of factors in total factor income so that they roughly correspond to the contribution of oil and material price dynamics to inflation and output: $\alpha_{R^*} \sim \mathrm{B}(0.03, 0.01)$, $\alpha_{O^*} \sim \mathrm{B}(0.04, 0.01)$. Based on the values accepted in the literature, we determine the prior distributions for: inverse elasticity of intertemporal substitution: $\sigma^* \sim N(2.0, 1.0)$; parameter of external consumption habits: $h^* \sim N(0.5, 0.2)$; the Calvo model parameter: $\theta^* \sim \mathrm{B}(0.75, 0.1)$; the interest rate response to inflation: $k_{\pi^*} \sim N(1.5, 0.1)$; the interest rate response to the output gap: $k_{y^*} \sim N(0.125, 0.02)$. We estimate the constants in the observed variables using a uniform distribution, with the exception of the constants for nominal material prices $a_{0P^*_{Rn}}$ and oil prices $a_{0P^*_{On}}$. Their values were set for technical reasons.[18] The productivity growth rate g^* was estimated based on a uniform distribution.

All prior distributions are summarised in Tables 3 and 4.

Features of assessing permanent shocks

Permanent shocks υ_{xt} in the model are designed to take into account structural shifts that have occurred in the economy. These shocks have a permanent effect on I(1) processes z_{xt} and

[18] To minimise the contribution of initial conditions to the dynamics of nominal oil and material prices.

on real endogenous variables. In the model, we ignore the impact of permanent shocks on the linearised solution of the model found for the balanced growth trajectory.[19]

For all permanent shocks, we have a counterpart in the form of an AR(1) process (temporary shocks) competing with the permanent shock for the explanation of the same data. To avoid identification problems, we assume that the coefficients of AR(1) processes differ significantly from one, for which we use prior B(0.5; 0.1) distribution for all autoregressive coefficients of temporary shocks ρ_x.

For clarity and convenience, we normalise all I(1) processes z_{xt} to zero for the last quarter before Covid19: $2020q1$ To improve identification, we use informative prior distributions when estimating the standard deviations of permanent shocks. For permanent shocks to TFP, labour and real wages, we use a weak inverse gamma distribution with infinite variance: $\sigma(\upsilon_A)\sim\Gamma^{-1}(0.01,\infty)$, $\sigma(\upsilon_N)\sim\Gamma^{-1}(0.002,\infty)$, and $\sigma(\upsilon_W)\sim\Gamma^{-1}(0.02,\infty)$. For permanent fiscal and foreign trade shocks, informative normal distributions are used: $\sigma(\upsilon_G),\sigma(\upsilon_T)\sim N(0.02,0.002)$, $\sigma(\upsilon_{Ex})\sim N(0.02,0.003)$, and $\sigma(\upsilon_{IM})\sim N(0.005,0.002)$.

The expected values for the prior distributions of fiscal shocks were selected so that most of the growth in government spending and taxes (above the deterministic trend) that occurred in 2020–2025 was attributed by the model to permanent shocks υ_G and υ_T. For the same purposes, we imposed constraints (soft tunes) on the variance of permanent fiscal shocks in the period $2022q2$ – $2025q4$ (double standard deviation).

The expected value for prior distributions of foreign trade shocks were selected so that the contribution of permanent shocks υ_{Ex} and υ_{IM} to the dynamics of export and import in 2022–2025 (explaining the decline in foreign trade activity) was about 50%.[20]

We also assumed that monopolistic mark-up shocks υ_W, like fiscal shocks, had a doubled standard deviation in the period $2022q2$ – $2025q4$. Finally, we imposed soft tunes on the variance of TFP shocks υ_A (double standard deviation) during the Covid crisis in $2020q2$ – $2020q3$, as well as in the period $2022q2$ – $2025q4$

Features of assessing common and heterogeneous shocks

A distinctive feature of the model is its ability to work with information about heterogeneous variables. One of the objectives of the study is to understand the value of this information for analysing the business cycle in the country. This objective dictates the need to distinguish between shocks that are common to the country as a whole and shocks that create heterogeneity in regional, sectoral and agent variables. For example, let there be N_r regional series of variables x_{rt}, each of which is determined by regional shocks ε_{xrt} (there are regional spillover effects) and a common ε_{xt}. To identify N_r+1 shocks, we will have N_r independent observable series x_{rt} (the aggregated series is a linear combination of regional series: $x_t=\sum_{r=1}^{N_r}\lambda_r x_{rt}$). In this case, to identify N_r+1 shocks, we impose restrictions on the variances(y) of these shocks.

[19] Similarly, we ignore the increasing error of linearised equations at a significant distance from the stationary point. According to our estimates, these errors are comparable, and if we traditionally accept linearisation errors, then the existence of permanent shocks only increases this type of error, inherent in practically oriented DSGE models estimated using the Bayesian method, to a certain extent.

[20] Experiments with these averages showed that they significantly affect the dynamics of the foreign currency real exchange rate (decomposition into shocks, forecast), but much less affect the dynamics of inflation and GDP (due to the joint assessment of permanent shocks and other elements of DSGE models flexibility).

We use the following identification scheme. Let us assume that each region r consists of a large number (N_{mr}) of micro-regions of equal size, in which shocks occur, creating a regional shock:

$$\varepsilon_{xrt} = \sum_{mr=1}^{N_{mr}} \frac{1}{N_{mr}} \varepsilon_{xmrt} \tag{1}$$

The number of microregions in region r is directly proportional to the size of the population in the region, so that for any pair of regions $r1$ and $r2$, the following holds:

$$\frac{N_{mr1}}{N_{mr2}} = \frac{\lambda_{r1}}{\lambda_{r2}} \tag{2}$$

In each microregion of the country, the following occurs: (a) an idiosyncratic shock ε_{ximrt} with the same variance σ_{xi}^2, which is unknown to us; (b) a region-specific shock ϵ_{xrt} with dispersion $\sigma_{x\epsilon}^2$:

$$\varepsilon_{xmrt} = \varepsilon_{ximrt} + \epsilon_{xrt} \tag{3}$$

If $\sigma_{xi}^2 = 0$, then $\varepsilon_{xrt} = \epsilon_{xrt}$ – the entire shock is determined by the regional component. If $\sigma_{x\epsilon}^2 = 0$, then the equality of variances in all microregions of the country requires that for any pair of regions $r1$ and $r2$, the following condition on variances must be satisfied:

$$\frac{\sigma^2(\varepsilon_{xr1})}{\sigma^2(\varepsilon_{xr2})} = \frac{{}^{1}/_{N_{mr1}}\sigma_{xi}^2}{{}^{1}/_{N_{mr2}}\sigma_{xi}^2} = \frac{N_{mr2}}{N_{mr1}} \tag{4}$$

Condition (4) requires that the smaller the region (i.e. the fewer microregions N_{mr} it contains), the greater the variance of the regional shock. This is because large regions experience many small idiosyncratic shocks, which ensures low variance of their sum. In terms of contribution to the aggregate sum, the larger the region, the proportionally higher the contribution of idiosyncratic shocks ε_{ximrt} of a given region r to the variance of the aggregate variable x_t.

Let us consider the general case: $\sigma_{xi}^2 > 0$ and $\sigma_{x\epsilon}^2 > 0$. By making another assumption that the variances of region-specific shocks $\sigma_{x\epsilon}^2$ are the same for all regions, we can obtain the ratio of regional shock variances:

$$\frac{\sigma^2(\varepsilon_{xr1})}{\sigma^2(\varepsilon_{xr2})} = \frac{{}^{1}/_{N_{mr1}}\sigma_{xi}^2 + \sigma_{x\epsilon}^2}{{}^{1}/_{N_{mr2}}\sigma_{xi}^2 + \sigma_{x\epsilon}^2} \tag{5}$$

If $N_{mr1} = \lambda_r N_{mr}$, where N_{mr} is the total number of microregions in the country, then:

$$\frac{\sigma^2(\varepsilon_{xr1})}{\sigma^2(\varepsilon_{xr2})} = \frac{{}^{1}/_{\lambda_{r1}} + \frac{\sigma_{x\epsilon}^2}{\sigma_{xi}^2} N_{mr}}{{}^{1}/_{\lambda_{r2}} + \frac{\sigma_{x\epsilon}^2}{\sigma_{xi}^2} N_{mr}} \tag{6}$$

Let us denote the ratio of the variances specific to the region and idiosyncratic shocks as $\frac{\sigma_{x\epsilon}^2}{\sigma_{xi}^2} N_{mr} \equiv \zeta r$ and express the ratio of the standard deviations of shocks:

$$\frac{\sigma(\varepsilon_{xr1})}{\sigma(\varepsilon_{xr2})} = \sqrt{\frac{\frac{1}{\lambda_{r1}} + \zeta r}{\frac{1}{\lambda_{r2}} + \zeta r}} \tag{7}$$

Condition (7) is a constraint on the variances of regional shocks. The variance of a regional shock is higher, the lower the share of the region in the country (due to the high variance of idiosyncratic shocks). This relationship weakens if the contribution of region-specific shocks $\sigma_{x\epsilon}^2$ to the total variance of regional shocks increases. At the limit:

$$\lim_{\zeta r \to \infty} \frac{\sigma(\varepsilon_{xr1})}{\sigma(\varepsilon_{xr2})} = 1 \tag{8}$$

For simplicity of identification, we assume that the ratio of region-specific and idiosyncratic shocks ζr is the same for all types of regional shocks.

A similar scheme can be applied for the joint identification of common and agent shocks. If $x_t = \sum_{j=1}^{N_j} \lambda_j x_{jt}$ (where λ_j is the share of group j), each group consists of N_{mj} microgroups, and the shock for each microgroup ε_{xmjt} consists of an idiosyncratic ε_{ximjt} and a component specific to group j ϵ_{xjt}:

$$\varepsilon_{xjt} = \sum_{mj=1}^{N_{mj}} \frac{1}{N_{mj}} \varepsilon_{xmjt}, \quad (9)$$

$$\varepsilon_{xmjt} = \varepsilon_{ximjt} + \epsilon_{xjt}, \quad (10)$$

then, similarly, denoting the ratio of variances $\frac{\sigma_{x\epsilon}^2}{\sigma_{xi}^2} N_{mj} \equiv \zeta j$, we obtain the desired constraint on variances:

$$\frac{\sigma(\varepsilon_{xj1})}{\sigma(\varepsilon_{xj2})} = \sqrt{\frac{\frac{1}{\lambda_{j1}}+\zeta j}{\frac{1}{\lambda_{j2}}+\zeta j}}. \quad (11)$$

The proposed scheme is also suitable for the joint identification of common and sectoral shocks, but it is not used in the current version of the estimation, since we use a limited set of sectoral information as observed sectoral variables and simplistically assume that sectoral dynamics are determined by the corresponding shocks of the sector under consideration. In this case, the problem of joint identification of general and sectoral shocks does not arise, and the estimation is performed using a uniform prior distribution of standard deviations of sectoral shocks.

Technically, for example, for intertemporal preference shock, we assume that for region $r1$, the standard deviation of intertemporal preference shock is $\sigma(\varepsilon_{\beta r1}) = \sigma(\varepsilon_{r\beta})\sqrt{\frac{1}{\lambda_{r1}} + \zeta r}$, where $\sigma(\varepsilon_{r\beta})$ is a value that is the same for all regions, estimated from the data and characterising the variance of regional preference shocks (the common part in Table 4). The parameters ζr and ζj are also estimated based on data with a priori distribution $\zeta r, \zeta j \sim \Gamma^{-1}(1.0, \infty)$. The estimation formula is similar for other regional shocks, as well as for agent shocks.

Results

First, we estimate the parameters of the external sector model independently of the estimation of the other model parameters.[21] The result of the assessment of the external sector model over the interval of $2010q1 - 2025q2$ are presented in Table 3.

The estimates of the shares of oil and materials in foreign factor income were: $\alpha_{O^*} = 0.0224$ and $\alpha_{R^*} = 0.0248$, which is lower than the average share of oil and other commodities in global foreign trade turnover (15–20%). The elasticity of intertemporal substitution is assessed higher than its prior estimates: $\frac{1}{\sigma^*} = \frac{1}{0.973} = 1.028 > \frac{1}{2}$. The parameter of external consumption habits is lower than its prior expectation: $h^* = 0.292 < 0.5$. The Calvo parameter turned out to be quite large $\theta^* = 0.808$, which is usually the case if the model is simple enough and does not contain various mechanisms that increase real price rigidity. The parameters in Taylor rule for

[21] An alternative option is to estimate the parameters of the external sector model together with the other parameters, using the same historical range. The rationale for this estimate is that a model assessed in this way could better correspond to that part of the global economy that interacts with the Russian economy.

the foreign rate: $k_{\pi^*} = 1.4$ and $k_{y^*} = 0.128$ correlate well with the estimates of other authors, despite the fact that a series of outputs from G20 countries is used for identification. The estimated TFP growth rate abroad was $g^* = 0.0055$. Taking into account the estimated parameters, as well as $n^* = 0.002$ and $\kappa^* = 1$ recorded during the estimation, the long-term output growth rate will be: $g_Y^* = \frac{g^*}{\alpha_L^*}\left(1 + \frac{1-\sigma^*}{\sigma^*+\kappa^*}\right) + n^* = 0.00789$, which corresponds to 3.15% per annum.

Table 3. Result of Bayesian parameter estimation for the external sector of the model

		Prior distribution			Posterior distribution	
Designation	Parameter	Type	Mean	Std. dev.	Mode	Std. dev.
$\sigma(\varepsilon_{A^*})$	Standard deviation of foreign productivity shocks	U	–	–	0.0171	0.0028
$\sigma(\varepsilon_{O^*})$	Standard deviation of oil supply shocks	U	–	–	0.1523	0.0149
$\sigma(\varepsilon_{R^*})$	Standard deviation of material supply shocks	U	–	–	0.0552	0.0056
$\sigma(\varepsilon_{i^*})$	Standard deviation of foreign rate shocks	U	–	–	0.0016	0.0002
$\sigma(\varepsilon_{\beta^*})$	Standard deviation of foreign intertemporal preference shocks	U	–	–	0.0220	0.0055
ρ_{A^*}	Autoregressive coefficient of foreign productivity shocks	B	0.6	0.1	0.7679	0.0614
ρ_{O^*}	Autoregressive coefficient of oil supply shocks	B	0.7	0.04	0.7732	0.0290
ρ_{R^*}	Autoregressive coefficient of material supply shocks	B	0.7	0.04	0.7795	0.0295
ρ_{β^*}	Autoregressive coefficient of foreign intertemporal preference shocks	B	0.6	0.1	0.5523	0.0933
α_{R^*}	Share of materials in total factor income abroad	B	0.03	0.01	0.0248	0.0082
α_{O^*}	Share of oil in total factor income abroad	B	0.04	0.01	0.0224	0.0059
σ^*	The inverse elasticity of intertemporal substitution abroad	N	2	1	0.9728	0.3011
θ^*	Calvo model parameter abroad	B	0.75	0.1	0.8082	0.0207
h^*	Habit parameter in consumption abroad	B	0.5	0.2	0.2919	0.1363
k_{π^*}	Coefficient of the foreign rate response to inflation abroad	N	1.5	0.1	1.3995	0.1012
k_{y^*}	Coefficient of the foreign rate response to the output gap abroad	N	0.125	0.02	0.1283	0.0197
a_{0Y^*}	Constant in the observed variable Y^*_{obs}	U	–	–	-5.7231	0.0115
a_{0i^*}	Constant in the observed variable i^*_{obs}	U	–	–	-0.0015	0.0016
$a_{0\pi^*}$	Constant in the observed variable π^*_{obs}	U	–	–	0.0048	0.0015
$a_{0P^*_{Rn}}$	Constant in the observed variable P^*_{Rnobs}	N	0.16	0.02	0.1531	0.0196
$a_{0P^*_{On}}$	Constant in the observed variable P^*_{Onobs}	N	4.05	0.02	4.0569	0.0196
g^*	Growth rate of productivity abroad	U			0.0055	0.0009

Source: authors' calculations.

The results of the full model assessment over the interval $2014q1 - 2025q4$ are presented in Table 4.

Table 4. Result of the Bayesian model parameter estimation

		Prior distribution			Posterior distribution	
Designation	Parameter	Type	Mean	Std. dev.	Mode	Std. dev.
$\sigma(\varepsilon_{ExO})$	Standard deviation of demand shock for domestic oil exports	U	–	–	0.0502	0.0056
$\sigma(\varepsilon_{dN})$	Standard deviation of consumption preference shock for non-tradable goods	U	–	–	0.0280	0.0032
$\sigma(\varepsilon_{\mu_N})$	Standard deviation of monopoly mark-up shock in the industry N	U	–	–	0.0939	0.0312
$\sigma(\varepsilon_{\beta})$	Standard deviation of intertemporal preference shock	U	–	–	0.2006	0.0450
$\sigma(\varepsilon_{\mu})$	Standard deviation of monopolistic mark-up shock	U	–	–	0.0593	0.0107
$\sigma(\varepsilon_{ik})$	Standard deviation of monetary policy shock	U	–	–	0.0062	0.0008
$\sigma(\varepsilon_{W})$	Standard deviation of monopolistic mark-up shock in wages	U	–	–	0.0886	0.0159
$\sigma(\varepsilon_{rp})$	Standard deviation of foreign borrowing/investment rate shock	U	–	–	0.0294	0.0063
$\sigma(\varepsilon_{I})$	Standard deviation of investment efficiency shock	U	–	–	0.0408	0.0059
$\sigma(\varepsilon_{G})$	Standard deviation of government spending shock	U	–	–	0.0256	0.0039
$\sigma(\varepsilon_{T})$	Standard deviation of lump-sum tax shock	U	–	–	0.0092	0.0037
$\sigma(\varepsilon_{Ex})$	Standard deviation of demand shock for all domestic export	U	–	–	0.0552	0.0072
$\sigma(\varepsilon_{IM})$	Standard deviation of import preference shock	U	–	–	0.0712	0.0082
$\sigma(\varepsilon_{i})$	Standard deviation of borrowing/investment rate shock for households	N	0.002	0.004	0.0041	0.0004
$\sigma(\varepsilon_{L})$	Standard deviation of labour share shock in the production function	Γ^{-1}	0.002	∞	0.0006	0.0001
$\sigma(\varepsilon_{A})$	Standard deviation of TFP shock	Γ^{-1}	0.005	∞	0.0020	0.0007
$\sigma(\varepsilon_{N})$	Standard deviation of labour force shock	Γ^{-1}	0.005	∞	0.0019	0.0002

$\sigma(\upsilon_A)$	Standard deviation of permanent TFP shock	Γ^{-1}	0.01	∞	0.0077	0.0011
$\sigma(\upsilon_G)$	Standard deviation of permanent government spending shock	N	0.02	0.002	0.0140	0.0024
$\sigma(\upsilon_T)$	Standard deviation of permanent lump-sum tax shock	N	0.02	0.002	0.0152	0.0021
$\sigma(\upsilon_{Ex})$	Standard deviation of permanent demand shock for all domestic exports	N	0.02	0.003	0.0190	0.0030
$\sigma(\upsilon_{IM})$	Standard deviation of permanent import shock	N	0.005	0.002	0.0065	0.0023
$\sigma(\upsilon_N)$	Standard deviation of the permanent labour force shock	Γ^{-1}	0.002	∞	0.0024	0.0005
$\sigma(\upsilon_W)$	Standard deviation of permanent monopolistic mark-up shock in wages	Γ^{-1}	0.02	∞	0.0106	0.0055
$\sigma(\varepsilon_{A^*})$	Standard deviation of foreign productivity shocks	U	–	–	0.0177	0.0019
$\sigma(\varepsilon_{O^*})$	Standard deviation of oil supply shocks	U	–	–	0.1437	0.0151
$\sigma(\varepsilon_{R^*})$	Standard deviation of material (resource) supply shocks	U	–	–	0.0513	0.0054
$\sigma(\varepsilon_{i^*})$	Standard deviation of foreign rate shocks	U	–	–	0.0019	0.0002
$\sigma(\varepsilon_{\beta^*})$	Standard deviation of foreign intertemporal preference shocks	U	–	–	0.0259	0.0027
$\sigma(me_{Lt}$	Standard deviation of labour supply measurement error	Γ^{-1}	0.005	∞	0.0022	0.0008
$\sigma(\varepsilon_{r\beta})$	Standard deviation of regional shocks to intertemporal preference (common part)	U	–	–	0.0514	0.0106
$\sigma(\varepsilon_{r\mu})$	Standard deviation of regional shocks to monopolistic mark-up (common part)	U	–	–	0.0037	0.0006
$\sigma(\varepsilon_{rW})$	Standard deviation of regional shocks to monopolistic mark-up in wages (common part)	U	–	–	0.0231	0.0034
$\sigma(\varepsilon_{rG})$	Standard deviation of regional shocks to government spending (common part)	U	–	–	0.0028	0.0004
$\sigma(\varepsilon_{ri})$	Standard deviation of regional shocks borrowing/investment rate of households (common part)	N	0.0002 45	0.0001	0.0002	6.6e-08

$\rho_{\beta VV}$	Autoregression coefficient of intertemporal preference shocks in VVMR	B	0.5	0.1	0.6057	0.0848
$\rho_{\mu VV}$	Autoregression coefficient of monopolistic mark-up shocks in VVMR	B	0.5	0.1	0.4101	0.0845
ρ_{WVV}	Autoregression coefficient of monopolistic mark-up shocks in wages in VVMR	B	0.5	0.1	0.3950	0.0895
ρ_{iVV}	Autoregression coefficient of borrowing/investment rate shocks of households in VVMR	B	0.5	0.1	0.5101	0.0155
ρ_{GVV}	Autoregression coefficient of government spending shocks in VVMR	B	0.5	0.1	0.6928	0.0638
$\rho_{\beta RU}$	Autoregression coefficient of intertemporal preference shocks in other regions of Russia	B	0.5	0.1	0.4750	0.1007
$\rho_{\mu RU}$	Autoregression coefficient of monopolistic mark-up shocks in other regions of Russia	B	0.5	0.1	0.4472	0.0766
ρ_{WRU}	Autoregression coefficient of monopolistic mark-up shocks in wages in other regions of Russia	B	0.5	0.1	0.4526	0.0841
ρ_{iRU}	Autoregression coefficient of borrowing/investment rate shocks of households in other regions of Russia	B	0.5	0.1	0.5120	0.0165
ρ_{GRU}	Autoregression coefficient of government spending shocks in other regions of Russia	B	0.5	0.1	0.6310	0.1027
Ω	Regional consumption bias parameter	B	0.7	0.05	0.7078	0.0497
Ω_L	Regional bias parameter in the labour market	B	0.7	0.05	0.7318	0.0465
δ_R	Substitution elasticity between domestic tradable goods from different regions	*N*	1.5	0.1	1.4949	0.0990
δ_{LR}	Substitution elasticity between labour of one household group from different regions	*N*	0.8	0.1	0.8178	0.0991
θ_{VV}	Coefficient for determining Calvo parameters in VVMR: $\theta_{VVm_K} = \theta\theta_{VV}\theta_{m_K}$, $\theta_{Rrj} = \theta_R\theta_r$ and $\theta_{WVVj} = \theta_W\theta_{VV}\theta_j$	*N*	1	0.05	1.0042	0.0061
$a_{0Y_{VV}}$	Constant in the observed variable Y_{VVobs}	*U*	–	–	2.4167	0.0415
$a_{0w_{VV}}$	Constant in the observed variable w_{VVobs}	*U*	–	–	-4.8979	0.0750
$a_{0G_{VV}}$	Constant in the observed variable G_{VVobs}	*U*	–	–	-3.9563	0.0556
$a_{0i_{VV}}$	Constant in the observed variable i_{VVobs}	*U*	–	–	0.0209	0.0014
a_{0C_h}	Constant in the observed variable C_{obsh}	*U*	–	–	4.4621	0.0736
a_{0Y_N}	Constant in the observed variable Y_{Nobs}	*U*	–	–	5.8241	0.0797

a_{0Y_O}	Constant in the observed variable Y_{Oobs}	U	–	–	4.4880	0.0686
$\sigma(\varepsilon_{j\beta})$	Standard deviation of intertemporal preference shock of household groups	Γ^{-1}	0.005	∞	0.0023	0.0009
$\sigma(\varepsilon_{jT})$	Standard deviation of lump-sum tax shocks of household groups	Γ^{-1}	0.005	∞	0.0401	0.0040
θ_h	Coefficient for determining Calvo parameters for household group h $\theta_{Wrh} = \theta_W\theta_r\theta_h$	N	1	0.05	1.0608	0.0321
ρ_{ExO}	Autoregressive coefficient of oil export shocks	B	0.5	0.1	0.7406	0.0524
ρ_{dN}	Autoregression coefficient of preference shocks for non-tradable goods consumption	B	0.5	0.1	0.6809	0.0551
$\rho_{\mu N}$	Autoregression coefficient of monopolistic mark-up shocks in non-tradable goods	B	0.5	0.1	0.4947	0.1244
θ_N	Coefficient for determining the Calvo parameter in the non-tradable goods sector: $\theta_{rN} = \theta\theta_r\theta_N$	N	1.05	0.05	1.2052	0.0369
ρ_β	Autoregression coefficient of intertemporal preference shocks	B	0.5	0.1	0.4674	0.0907
ρ_μ	Autoregression coefficient of monopolistic mark-up shocks	B	0.85	0.02	0.8095	0.0234
ρ_W	Autoregression coefficient of monopolistic mark-up shocks in wages	B	0.5	0.1	0.4818	0.0855
ρ_{rp}	Autoregression coefficient of foreign borrowing/investment rate shocks	B	0.5	0.1	0.5441	0.0941
ρ_I	Autoregression coefficient of investment efficiency shocks	B	0.5	0.1	0.8377	0.0439
ρ_G	Autoregression coefficient of government spending shocks	B	0.8	0.03	0.8234	0.0329
ρ_T	Autoregression coefficient of tax shocks	B	0.8	0.03	0.7793	0.0323
ρ_{Ex}	Autoregression coefficient of demand shocks for all domestic exports	B	0.5	0.1	0.7406	0.0561
ρ_{IM}	Autoregression coefficient of import preference shocks	B	0.5	0.1	0.6883	0.0532
ρ_i	Autoregression coefficient of borrowing/investment rate shocks of households	B	0.5	0.1	0.5016	0.0080
ρ_N	Autoregression coefficient of labour force shocks	B	0.5	0.1	0.5808	0.0976
ρ_L	Autoregression coefficient of labour share shocks in the production function	B	0.5	0.1	0.4970	0.0110
ρ_A	Autoregression coefficient of TFP shocks	B	0.5	0.1	0.4986	0.0171
θ	Coefficient for determining Calvo parameters $\theta_{rm_K} = \theta\theta_r\theta_{m_K}$ and	B	0.8	0.02	0.7627	0.0261
θ_W	Coefficient for determining Calvo parameters $\theta_{Wrj} = \theta_W\theta_r\theta_j$ and	B	0.8	0.03	0.7859	0.0192
θ_R	Coefficient for determining Calvo parameters in the differentiated materials production sector $\theta_{Rrj} = \theta_R\theta_r$	B	0.8	0.03	0.7907	0.0310
χ	Degrees of price indexation of goods to	B	0.5	0.1	0.1093	0.0323

	previous inflation					
χ_W	Degrees of wages indexation to previous inflation	B	0.5	0.1	0.2685	0.0767
χ_R	Degrees of material price indexation to previous inflation	B	0.5	0.1	0.5021	0.1063
h	Parameter of external consumption habits	B	0.4	0.1	0.5081	0.0774
ψ_{B^*}	Cost function parameter related to the deviation of the current level of foreign debt from the stationary level	N	0.08	0.01	0.0757	0.0100
σ	The parameter inverse to the elasticity of intertemporal substitution of consumption	N	2	0.35	1.2758	0.0213
κ	The parameter inverse to the elasticity of labour supply according to Frisch	N	1	0.2	1.0833	0.1915
δ^*	The price elasticity of demand for domestic exports	N	0.8	0.1	0.8335	0.0971
δ_C	Elasticity of substitution between different types of goods	N	0.8	0.1	0.8846	0.0962
ϕ_I	Parameter of the investment adjustment cost function	N	1	0.1	1.0615	0.0941
k_{SF}	Parameter characterising the degree of stabilisation of the foreign oil price trends for the domestic market	N	0.5	0.05	0.5287	0.0345
k_{XO}	Elasticity of demand for domestic oil in terms of output volume abroad	N	0.5	0.05	0.5004	0.0499
k_π	Response coefficient [annualised] of the key rate to expected annual inflation in the Taylor rule	N	2	0.1	1.7384	0.0947
ψ_G	Elasticity of lump-sum taxes by the level of real public debt	N	0.1	0.02	0.0972	0.0203
ξ_O	Semi-elasticity of the external premium to the real oil price	N	0.003	0.001	0.0030	0.0010
a_{0Y}	Constant in the observed variable Y_{obs}	U	–	–	4.9351	0.0413
a_{0I}	Constant in the observed variable I_{obs}	U	–	–	5.2568	0.0384
a_{0G}	Constant in the observed variable G_{obs}	U	–	–	-4.1817	0.0556
a_{0N}	Constant in the observed variable N_{obs}	U	–	–	11.233	0.0074
a_{0L}	Constant in the observed variable L_{obs}	U	–	–	4.0920	0.0670
a_{0w}	Constant in the observed variable w_{obs}	U	–	–	-5.1104	0.0710
a_{0NEx}	Constant in the observed variable NEx_{obs}	U	–	–	7.5468	0.0690
a_{0NIm}	Constant in the observed variable NIm_{obs}	U	–	–	0.1210	0.0391
a_{0i}	Constant in the observed variable i_{obs}	U	–	–	0.0206	0.0013
a_{0S}	Constant in the observed variable S_{obs}	U	–	–	3.6766	0.1478
g	Long-term TFP growth rate	N	0.003	0.0005	0.0026	0.0004
n	Long-term labour force growth rate	N	0	0.001	-0.0001	0.0002
g_{werr}	Inconsistency in the formula for long-term real wages growth rate	N	0.002	0.0005	0.0021	0.0005
ζr	Ratio of the variances specific to the region and idiosyncratic shocks	Γ^{-1}	1	∞	0.5294	0.1022
ζj	Ratio of the variances specific to the household group and idiosyncratic shocks	Γ^{-1}	1	∞	0.5190	0.2488

Source: authors' calculations.

For all 110 parameters assessed in the second stage, the maximum of the posterior function was found. For many parameters, the prior and posterior distributions turned out to be very similar. This indicates problems with identifying these parameters, despite which we estimate them using the Bayesian method, trying to find a balance between the speed of convergence to modes and reliance on available data. Let's discuss the most fundamental results of the assessment.

Estimation of the inverse elasticity of intertemporal substitution: $\sigma = 1.28$. In addition to its traditional influence on real rigidity in the model and the persistence of variable responses to shocks, this parameter affects the long-term level of interest rates and the long-term growth rate of virtually all real variables. Since the trend of the observed real variables is assessed together with other structural parameters, this means that σ is estimated on a broader range of data than in the traditional case of filtered series. When $\sigma > 1$, the impact of the TFP growth rate on the long-term real rate is: $\frac{\partial \bar{r}}{\partial g} \cong \sigma > 1$. The growth rate of employment per worker is negative: $g_L = \frac{g}{\alpha_L}\frac{1-\sigma}{\sigma+\kappa} < 0$ and is negatively dependent on the TFP growth rate of TFP g: $\frac{\partial g_L}{\partial g} = \frac{1}{\alpha_L}\frac{1-\sigma}{\sigma+\kappa} < 0$. This makes a negative contribution to the estimate of the long-term [deterministic] GDP growth rate: in annual terms $g_L = g\frac{1-\sigma}{\sigma+\kappa} \cdot 4 \cdot 100\% = -0.12\%$.

The estimates of Calvo pricing parameters turned out to be lower than their prior expectations (0.8): $\theta = 0.763$, $\theta_W = 0.786$, $\theta_R = 0.791$. At the same time, all Calvo parameter factors were not lower than one: $\theta_{VV} = 1.00$, $\theta_h = 1.061$, $\theta_N = 1.205$. The mode for the posterior distribution of θ_N was even higher than the prior expectation of this parameter (1.05). This means that the Calvo parameter for the non-tradable goods sector is higher than for the tradable goods sector: $\theta\theta_N = 0.763 \cdot 1.205 = 0.919 > \theta\theta_H = \theta\theta_F = 0.763 \cdot 1$. The degrees of indexation to the previous quarter inflation $\chi = 0.109$, $\chi_W = 0.268$, $\chi_R = 0.502$ were also estimated lower than their prior expectation (0.5). Low Calvo and indexation parameters mean faster than previously assumed price adjustment to changes in marginal costs, which, all else being equal, leads to a higher transfer of the exchange rate to prices and greater monetary policy strength (see below). It also means that the prior distributions used for these parameters are restrictive [from below] in terms of estimating the speed of marginal cost pass-through to prices.

We estimated the TFP growth rate: $g = 0.0026$, and the labour force growth rate $n = -0.0001$. This allowed us to calculate the long-term growth rate of output: $g_Y = \frac{g}{\alpha_L}\left(1 + \frac{1-\sigma}{\sigma+\kappa}\right) + n = 0.003366$, which corresponds to 1.35% in annual terms. The estimate of the wage growth rate discrepancy was $g_{werr} = 0.00215$, which corresponds to the real wage growth rate in the observed data $g_w = \frac{g}{\alpha_L} + g_{werr} = 0.00612$ or 2.49% in annual terms. At the same time, according to the theoretical model, the real wages growth rate should have been $g_w = \frac{g}{\alpha_L} = 0.00397$ or 1.59% in annual terms.[22]

Model adequacy checks

We will perform four types of model adequacy checks:

1. Main elasticities of the model
2. Impulse response functions
3. Decomposition of variables into shocks
4. Forecast quality assessment

[22] In this work, we attribute this discrepancy to factors not accounted for in the model, methodology and measurement error.

Main elasticities of the model

We will calculate the main elasticities, semi-elasticities and sensitivities of the model variables when various structural business cycle shocks occur. We will summarise them in Table 5.

We measure the strength of monetary policy in terms of the inflation response to an interest rate change. For each percentage point change in the key rate caused by a monetary policy shock ε_{ikt}, expected inflation in the coming year falls by 0.15 pp with the maximum effect at the moment of the shock (a 0.225 pp decline in sequential inflation). The immediate output response to a rate shock can be calculated by multiplying the strength of the monetary policy shock by the corresponding real rigidity indicator: a 1 pp change in the key rate will lead to a decrease in output at the moment of the shock $-0.225 \cdot 0.76 = -0.17\%$. It should be noted that the strength of monetary policy in foreign economies is estimated to be higher: a 1 pp increase in foreign interest rates due to a monetary policy shock abroad ε_{i^*t} leads to a 0.6 pp decrease in expected inflation in the coming year.

One of the most important characteristics of the model is the exchange rate pass-through to prices. We measure this pass-through by the price response to an immediate jump in the exchange rate caused by an external premium shock ε_{rpt}. A 1% jump in the foreign currency exchange rate leads to an increase in expected inflation in the coming year by 0.15%. Note that we measure the response of prices to an immediate exchange rate jump, assuming that in the long run the entire exchange rate increase is passed through to prices: a 100% pass-through. At the same time, when analysing long-term equilibrium, it is more appropriate to discuss the 'pass-through of prices to the exchange rate': in the long term, the foreign exchange rate changes by the same percentage as the long-term price level has risen in response to certain shocks (provided that these shocks have not changed the level of foreign prices).

Low values of monetary policy strength and exchange rate pass-through to prices (e.g. compared to foreign economies) were obtained by setting appropriate restrictive prior distributions on the parameters of nominal price rigidity. Historically, measuring the strength of monetary policy has yielded a slightly higher inflation response to monetary policy instrument (e.g. in Napalkov et al., 2021, this indicator was about -0.7). When parameterising the model, we took into account the latest trends towards a decrease in the strength of monetary policy due to a whole range of factors, from the increased anchoring of expectations after the introduction of IT to a decreased efficiency of the monetary policy transmission mechanism in the context of fiscal stimulus to the economy. The model estimates that pass-through about 15% of exchange rate movements are passed on to prices, which is a bit higher than the 10% pass-through confirmed empirically (see Andreev, 2019, Eliseev et al., 2023).

An important indicator of the interaction between the domestic economy and the outside world is the elasticity of the foreign exchange rate to the global oil price. A 1% increase in the oil price leads to a 0.18% strengthening of the ruble. This indicator is less than in most studies, where it is estimated based on data covering the period before the transition to inflation targeting (for example, in Polbin et al. (2019), it is estimated at 0.26, and in Polbin (2017) at 0.33). However, the elasticity of the ruble exchange rate to oil prices is higher than for the period 2015–2022 (around 0.1), reflecting the introduction of significant restrictions on cross-border capital flows in 2022, as well as a modification of the fiscal rule. This has increased the dependence of the Russian economy on oil price fluctuations.

The model pays considerable attention to the response of inflation to labour market shocks. The model predicts a significant transfer of real wages growth to prices. For example, a 1% increase in real wages due to labour market shocks leads to an increase in expected inflation in the coming year by 0.51 pp if there is a shock to the monopolistic wages mark-up ε_{Wt}, by 0.68 pp if there is a labour force shock ε_{Nt} , and by 3.75 pp if there is a labour demand shock ε_{Lt}. Such significant values of the transfer of real wages to prices indicate the need for qualitative identification of the trend and sources of the business cycle in the labour market.

Table 5. Main properties of the model

Group	[Semi]elasticity/ sensitivity	Respons e	To what	Shock	Value
Monetary policy strength	Sensitivity of current inflation to the key rate during a monetary policy shock	P_t	i_{kt}	ε_{ikt}	-0.225
	Sensitivity of annual inflation to the key rate during a monetary policy shock	$P_{t+3}/_4$	i_{kt}	ε_{ikt}	-0.149
Exchange rate pass-through into prices	Elasticity of current prices to the exchange rate during an external premium shock	$4P_t$	S_t	ε_{rpt}	0.222
	Semi-elasticity of annual inflation to the exchange rate during an external premium shock	P_{t+3}	S_t	ε_{rpt}	0.154
Real price rigidity (based on current prices)	Output elasticity to changes in current prices with an intertemporal preference shock	Y_t	$4P_t$	$\varepsilon_{\beta t}$	1.166
	Output elasticity to changes in current prices with an investment efficiency shock	Y_t	$4P_t$	ε_{It}	1.436
	Output elasticity to changes in current prices with a government spending shock	Y_t	$4P_t$	ε_{Gt}	1.665
	Output elasticity to changes in current prices with a tax shock	Y_t	$4P_t$	ε_{Tt}	1.334
	Output elasticity to changes in current prices with a monetary policy shock	Y_t	$4P_t$	ε_{ikt}	0.759
	Output elasticity to changes in current prices with a borrowing/investment rate shock to households	Y_t	$4P_t$	ε_{it}	-6.404
Real price rigidity (based on annual inflation)	Output elasticity to annual inflation with an intertemporal preference shock	Y_t	P_{t+3}	$\varepsilon_{\beta t}$	1.479
	Output elasticity to annual inflation with an investment efficiency shock	Y_t	P_{t+3}	ε_{It}	1.710
	Output elasticity to annual inflation with a government spending shock	Y_t	P_{t+3}	ε_{Gt}	1.953
	Output elasticity to annual inflation with a tax shock	Y_t	P_{t+3}	ε_{Tt}	1.553
	Output elasticity to annual inflation with a monetary policy shock	Y_t	P_{t+3}	ε_{ikt}	1.147
	Output elasticity to annual inflation with a borrowing/investment rate shock to households	Y_t	P_{t+3}	ε_{it}	-5.126
Balance of payments shocks	Elasticity of the current price level to changes in the nominal trade balance with an export shock	$4P_t$	NTB_t	ε_{Ext}	-0.095
	Elasticity of the current price level to	$4P_t$	NTB_t	ε_{Imt}	-0.043

	changes in the nominal trade balance with an import shock				
	Semi-elasticity of annual inflation to changes in the nominal trade balance with an export shock	P_{t+3}	NTB_t	ε_{Ext}	-0.088
	Semi-elasticity of annual inflation to changes in the nominal trade balance with an import shock	P_{t+3}	NTB_t	ε_{Imt}	-0.053
	Elasticity of the long-term domestic price level to the long-term foreign price level with a foreign intertemporal preference shock	$P_{t+\infty}$	$P^*_{t+\infty}$	ε_{β^*t}	1.319
	Elasticity of the long-term domestic price level to the long-term foreign price level with a TFP shock abroad	$P_{t+\infty}$	$P^*_{t+\infty}$	ε_{A^*t}	0.759
	Sensitivity of the domestic rate to the foreign rate with a foreign monetary policy shock	i_{kt}	i^*_t	ε_{i^*t}	0.388
	Elasticity of domestic output to foreign output with a materials supply shock	Y_t	Y^*_t	ε_{R^*t}	0.375
	Elasticity of domestic output to foreign output with a TFP shock abroad	Y_t	Y^*_t	ε_{A^*t}	0.167
	Elasticity of domestic output to foreign output a foreign intertemporal preference shock	Y_t	Y^*_t	ε_{β^*t}	0.241
Changes in global oil prices	Elasticity of foreign currency exchange rate to oil price with a global oil supply shock	S_t	P^*_{Ot}	ε_{O^*t}	-0.178
	Elasticity of domestic output to oil price with a global oil supply shock	Y_t	P^*_{Ot}	ε_{O^*t}	0.011
	Semi-elasticity of annual inflation to oil price with a global oil supply shock	P_{t+3}	P^*_{Ot}	ε_{O^*t}	-0.044
Characteristics of foreign economy	Elasticity of foreign prices to foreign rate with a monetary policy shock (strength of monetary policy abroad)	$4P^*_t$	i^*_t	ε_{i^*t}	-0.878
	Semi-elasticity of foreign annual inflation to foreign rate with a monetary policy shock (strength of monetary policy abroad)	P^*_{t+3}	i^*_t	ε_{i^*t}	-0.583
	Foreign output elasticity to changes in current prices abroad with an intertemporal preference shock (real rigidity)	Y^*_t	$4P^*_t$	ε_{β^*t}	1.233
	Foreign output elasticity to changes in current prices with a foreign monetary policy shock (real rigidity)	Y^*_t	$4P^*_t$	ε_{i^*t}	0.763
	Foreign output elasticity to changes in annual inflation with an intertemporal preference shock (real rigidity)	Y^*_t	P^*_{t+3}	ε_{β^*t}	2.831
	Foreign output elasticity to annual inflation abroad with a foreign monetary policy shock (real rigidity)	Y^*_t	P^*_{t+3}	ε_{i^*t}	1.149
Real price rigidity (supply shocks,	Output elasticity to changes in current prices with a TFP shock	Y_t	$4P_t$	ε_{At}	-0.020
	Output elasticity to changes in current prices with a monopolistic mark-up shock	Y_t	$4P_t$	$\varepsilon_{\mu t}$	-0.093

based on current prices)	Output elasticity to changes in current prices with a monopolistic mark-up shock in wages	Y_t	$4P_t$	ε_{Wt}	-0.115
Real price rigidity (supply shocks, based on annual inflation)	Output elasticity to annual inflation with a TFP shock	Y_t	P_{t+3}	ε_{At}	-0.038
	Output elasticity to annual inflation with a monopolistic mark-up shock	Y_t	P_{t+3}	$\varepsilon_{\mu t}$	-0.143
	Output elasticity to annual inflation with a monopolistic mark-up shock in wages	Y_t	P_{t+3}	ε_{Wt}	-0.142
Current inflation response	Elasticity of the current price level to real wages with a monopolistic mark-up shock in wages	$4P_t$	w_t	ε_{Wt}	0.628
	Elasticity of the current price level to government spending with a government spending shock	$4P_t$	G_t	ε_{Gt}	0.124
	Elasticity of the current price level to nominal lump-sum taxes with a lump-sum tax shock	$4P_t$	T_t	ε_{Tt}	-0.024
	Elasticity of the current price level to investments with an investment efficiency shock	$4P_t$	I_t	ε_{Tt}	0.092
	Elasticity of the current price level to the real price of materials with a material supply shock	$4P_t$	P^*_{Rt}	ε_{R^*t}	0.032
	Elasticity of the current price level to the real oil price with a global oil supply shock	$4P_t$	P^*_{Ot}	ε_{O^*t}	-0.048
Annual inflation response	Semi-elasticity of annual inflation to real wages with a monopolistic mark-up shock in wages	P_{t+3}	w_t	ε_{Wt}	0.507
	Semi-elasticity of annual inflation to government spending with a government spending shock	P_{t+3}	G_t	ε_{Gt}	0.106
	Semi-elasticity of annual inflation to nominal lump-sum taxes with a lump-sum tax shock	P_{t+3}	T_t	ε_{Tt}	-0.021
	Semi-elasticity of annual inflation to investments with an investment efficiency shock	P_{t+3}	I_t	ε_{Tt}	0.077
	Semi-elasticity of annual inflation to the real price of materials with a material supply shock	P_{t+3}	P^*_{Rt}	ε_{R^*t}	0.026
	Semi-elasticity of annual inflation to the real oil price with a global oil supply shock	P_{t+3}	P^*_{Ot}	ε_{O^*t}	-0.044
Response to labour market shocks	Elasticity of the current price level to demand for labour with a labour share shock in the production function	$4P_t$	L_t	ε_{Lt}	0.237
	Elasticity of the current price level to the labour force with a labour force shock	$4P_t$	N_t	ε_{Nt}	-0.110
	Semi-elasticity of annual inflation to demand for labour with a labour share shock in the production function	P_{t+3}	L_t	ε_{Lt}	0.487
	Semi-elasticity of annual inflation to the labour force with a labour force shock	P_{t+3}	N_t	ε_{Nt}	-0.391

	Elasticity of output to the current price level with a labour share shock in the production function (real rigidity)	Y_t	$4P_t$	ε_{Lt}	0.047
	Elasticity of output to the current price level with a labour force shock (real rigidity)	Y_t	$4P_t$	ε_{Nt}	-0.824
	Elasticity of output to annual inflation with a labour share shock in the production function (real rigidity)	Y_t	P_{t+3}	ε_{Lt}	0.092
	Output elasticity to annual inflation with a labour force shock (real rigidity)	Y_t	P_{t+3}	ε_{Nt}	-0.928
	Elasticity of the current price level to real wages with a monopolistic mark-up shock in wages	$4P_t$	w_t	ε_{Wt}	0.628
	Elasticity of the current price level to real wages with a labour share shock in the production function	$4P_t$	w_t	ε_{Lt}	7.300
	Elasticity of the current price level to real wages with a labour force shock	$4P_t$	w_t	ε_{Nt}	0.763
	Semi-elasticity of annual inflation to real wages with a monopolistic mark-up shock in wages	P_{t+3}	w_t	ε_{Wt}	0.507
	Semi-elasticity of annual inflation to real wages with a labour share shock in the production function	P_{t+3}	w_t	ε_{Lt}	3.747
	Semi-elasticity of annual inflation to real wages with a labour force shock	P_{t+3}	w_t	ε_{Nt}	0.678
	Elasticity of real wages to labour supply with a monopolistic mark-up shock in wages	w_t	L_t	ε_{Wt}	-7.368
	Elasticity of real wages to labour supply with a labour share shock in the production function	w_t	L_t	ε_{Lt}	0.032
	Elasticity of real wages to labour supply with a labour force shock	w_t	L_t	ε_{Nt}	0.166
	Elasticity of real wages to labour force with a labour force shock	w_t	N_t	ε_{Nt}	-0.144
	Elasticity of demand for labour to labour force with a labour force shock	L_t	N_t	ε_{Nt}	-0.868
	Elasticity of output to labour force with a labour force shock	Y_t	N_t	ε_{Nt}	0.091

Source: authors' calculations.

Impulse response functions

In addition to the calculated elasticities, we present the IRF of the main variables to the main shocks. Figure 2 shows the response of the interest rate (annualised = AR), inflation, GDP and its demand components, and real wages to the key rate shock ε_{ikt}. An increase in the key rate reduces inflation, output, real wages and virtually all components of output. The exception is the trade balance, which grows due to a decline in imports.

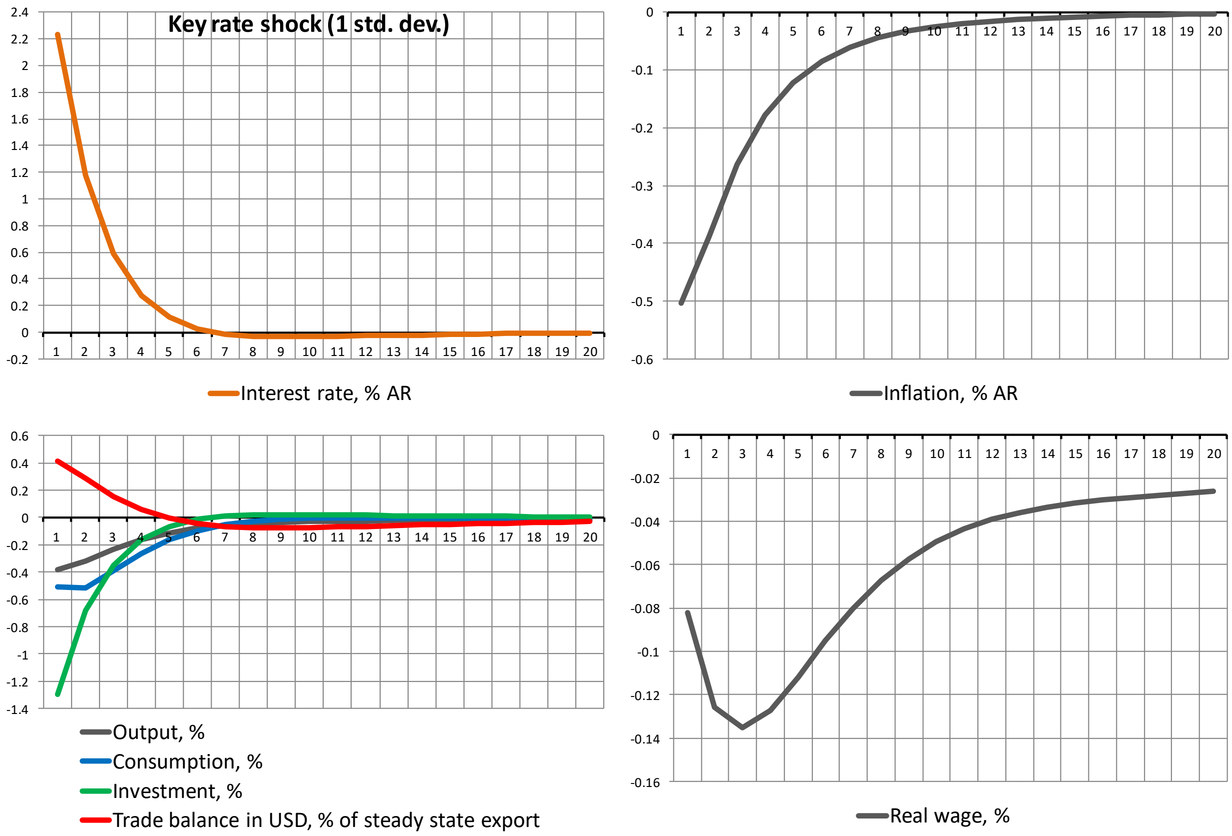

Figure 2. IRF on key rate shock ε_{ikt} (1 std. dev.)

Figure 3 shows the response of variables to a government spending shock ε_{Gt}, which stimulates output, causes inflation and raises the key rate. The growth in government spending crowds out other types of spending: private consumption, investment and external demand. The effect of government spending on output occurs mainly due to an increase in the labour supply of households. A decline in private consumption increases the marginal utility of an additional unit of private consumption, making leisure a more expensive good. Households increase the labour supply in the market, which reduces the equilibrium wage and allows firms to produce more goods. The instantaneous elasticity of output with respect to government spending is $\frac{dY_t/Y_t}{dG_t/G_t} = 0.206$.

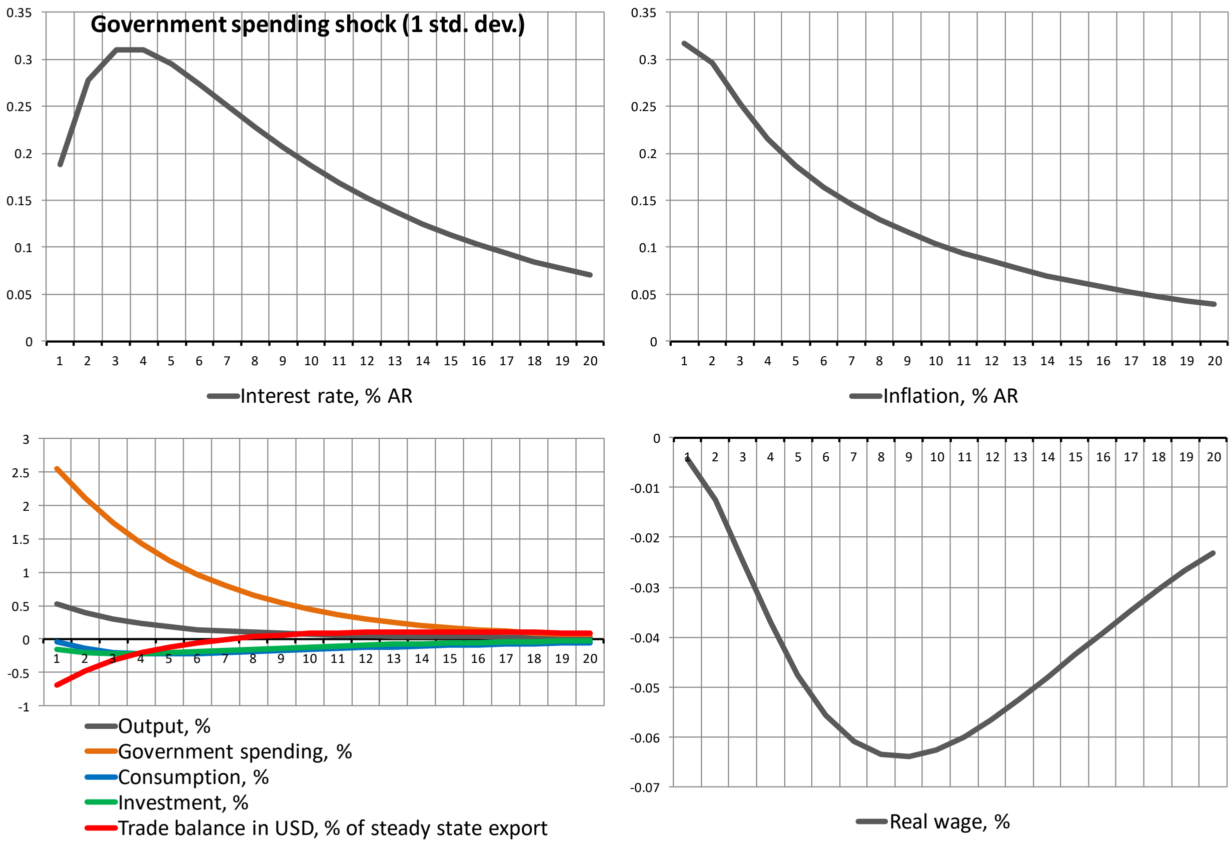

Figure 3. IRF on government spending shock ε_{Gt} (1 std. dev.)

Figure 4 shows the response of variables to a shock in global oil supply. An increase in oil supply reduces its price, weakens the domestic currency and has a negative impact on all components of aggregate demand: trade balance, private consumption and investment. This causes moderate and fairly persistent inflation growth through the exchange rate channel. The pro-inflationary nature of falling oil prices is a phenomenon that emerged after the Russian Federation introduced significant restrictions on cross-border capital flows in 2022. Most models estimated on data prior to 2022 suggest that the impact of oil price fluctuations on inflation is minimal. Figures 5 and 6 show the comparative responses of the domestic and foreign economies to global oil and material supply shocks.

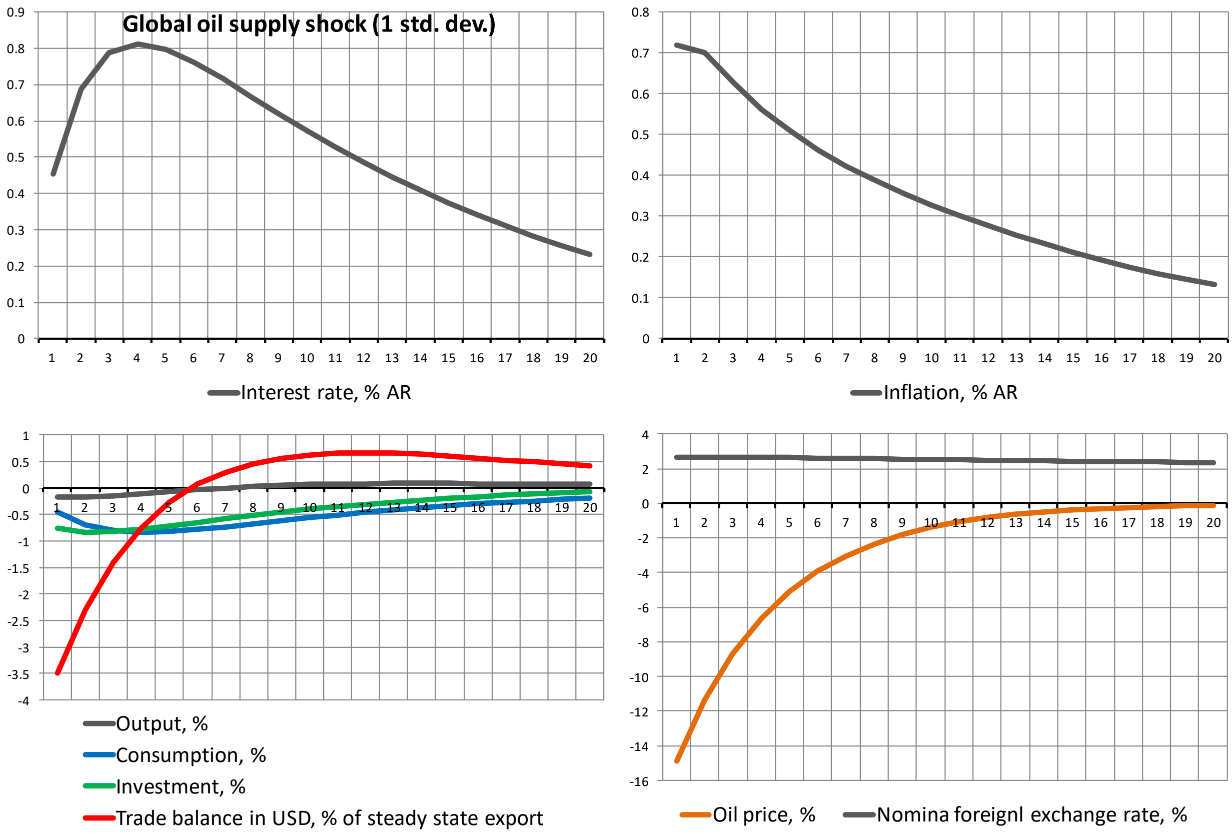

Figure 4. IRF on global oil supply shock ε_{O^*t} (1 std. dev.)

The oil supply shock has a positive effect on foreign economy: it expands the possibilities for the production of goods, reduces marginal costs and inflation. The domestic economy, on the contrary, suffers from the decline in oil prices, as it derives income from oil exports.

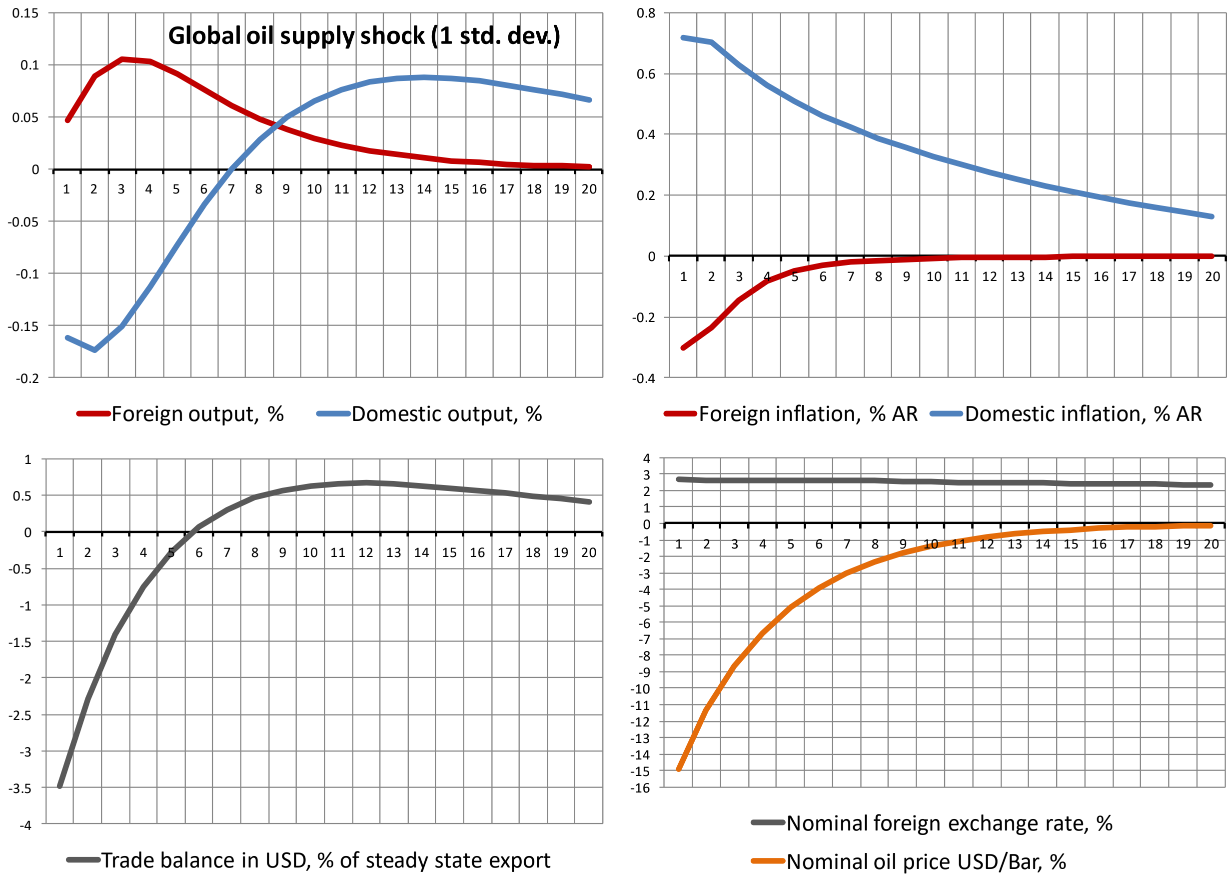

Figure 5. IRF on global oil supply shock ε_{O^*t} (1 std. dev.). Comparison of domestic and foreign economy response

The shock to global material supply has a more symmetrical impact on the domestic and global economies. For foreign economies, the expansion of material supply leads to an increase in production capacity. For the domestic economy, lower import prices strengthen the national currency and reduce marginal costs (import purchase prices). For both economies, this is a positive supply shock that reduces inflation and increases production. The inflationary response in the domestic economy is stronger and more prolonged due to higher nominal price rigidity. The real price rigidity in response to resource and oil shocks (the same applies to other supply and demand shocks) is higher in the foreign economy than in the domestic economy. At the same time, the integral price rigidity (nominal and real) of the domestic economy is slightly higher. This is explained by the fact that the domestic economic model introduces additional mechanisms of nominal rigidity (wages and resource prices) compared to the foreign economic model. Increased real price rigidity abroad partially compensates for reduced nominal price rigidity.

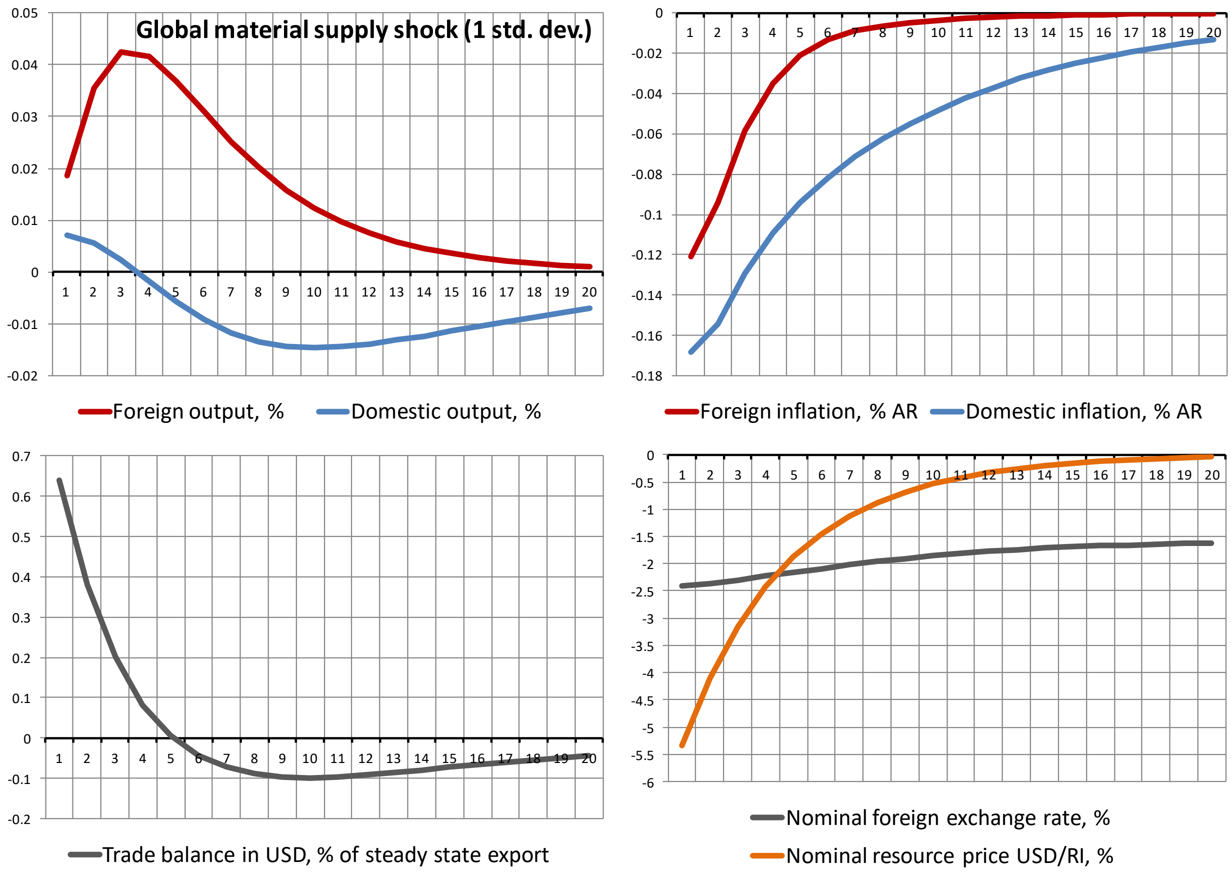

Figure 6. IRF on global material supply shock ε_{R^*t} (1 std. dev.). Comparison of domestic and foreign economy response

As further calculations will show, balance of payments shocks have a significant impact on domestic output dynamics. Figure 7 shows the economy's response to an export shock. Export growth stimulates output, and through the trade balance channel, strengthens the national currency. In the regime of restricted cross-border capital flows, the effect of the export shock on inflation through the exchange rate channel will dominate over the aggregate demand channel, and inflation will slow down.

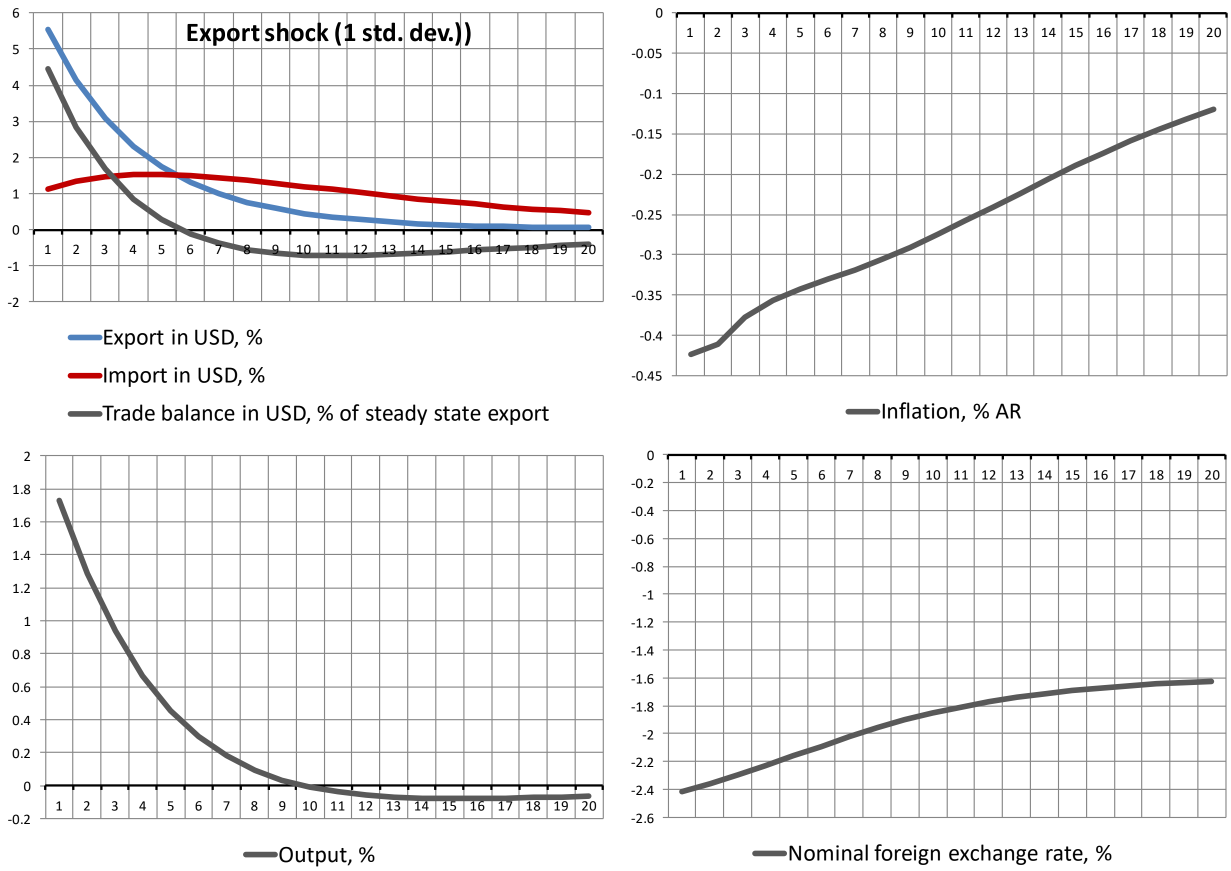

Figure 7. IRF on export shock ε_{Ext} (1 std. dev.)

Import shock has the opposite effect on the economy: demand is discouraged (we assume that demand for imports grows due to the effect of switching demand from domestic goods to foreign goods), the national currency weakens and inflation accelerates. Comparing Figures 7 and 8, we see that imports quickly adjust to changes in exports, while exports react weakly to changes in imports. This is because exports depend mainly on the state of the global economy, while imports react to the domestic economy reacting to these shocks.

Another specific of the response to shocks that provoke fluctuations in the balance of payments (Figures 5–8) is the effect of the return of accumulated net international assets, which changes the sign of the balance of payments response to any shock approximately five quarters after the shock. During this period, the costs of adjusting net international assets B_t^* to their stationary level reverse the dynamics of the trade balance through the real foreign exchange rate channel.

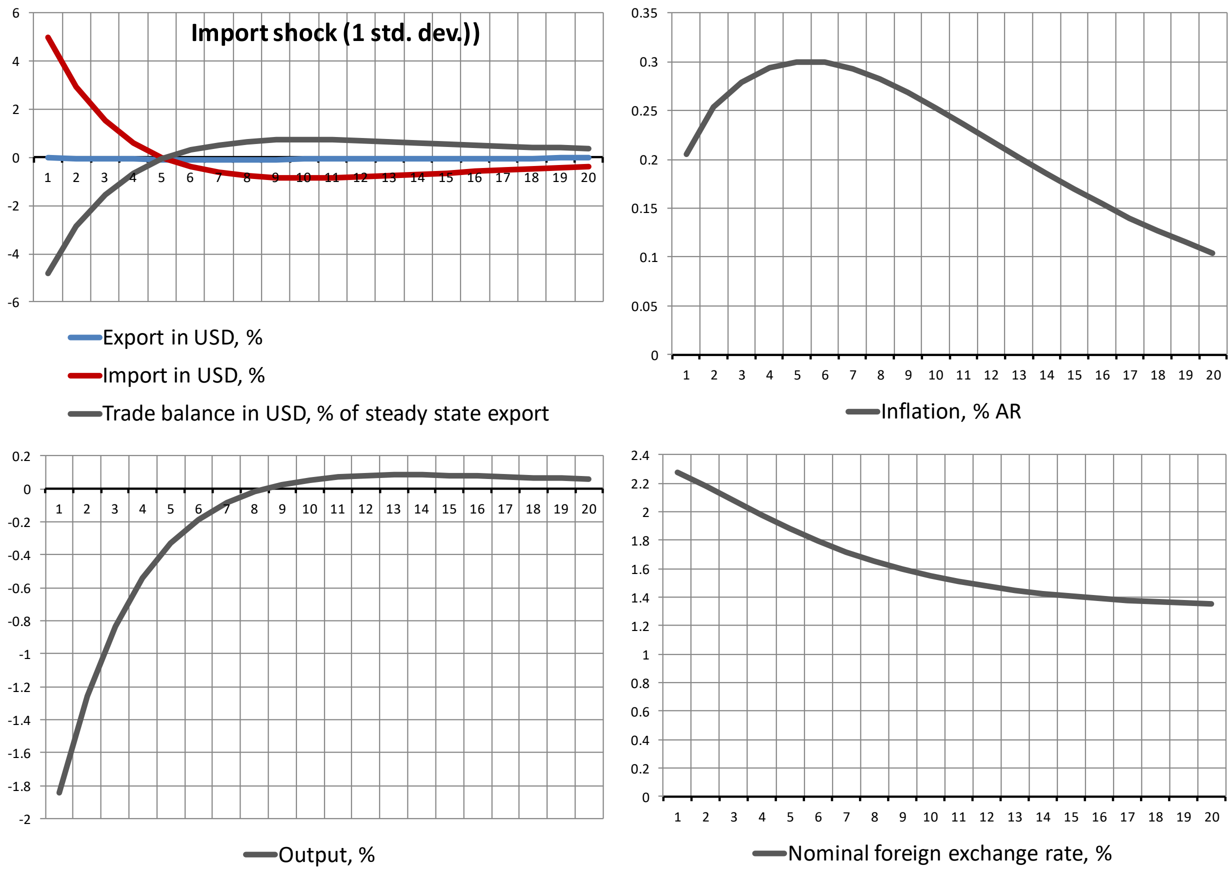

Figure 8. IRF on import shock ε_{Imt} (1 std. dev.)

Using the example of intertemporal preference shock, let us consider the difference in response to general and regional shocks. Figure 9 shows a general intertemporal preference shock $\varepsilon_{\beta t}$. An increase in private consumption leads to an increase in output, real wages, inflation and interest rates. Private consumption partially displaces other types of consumption: investment and net exports.

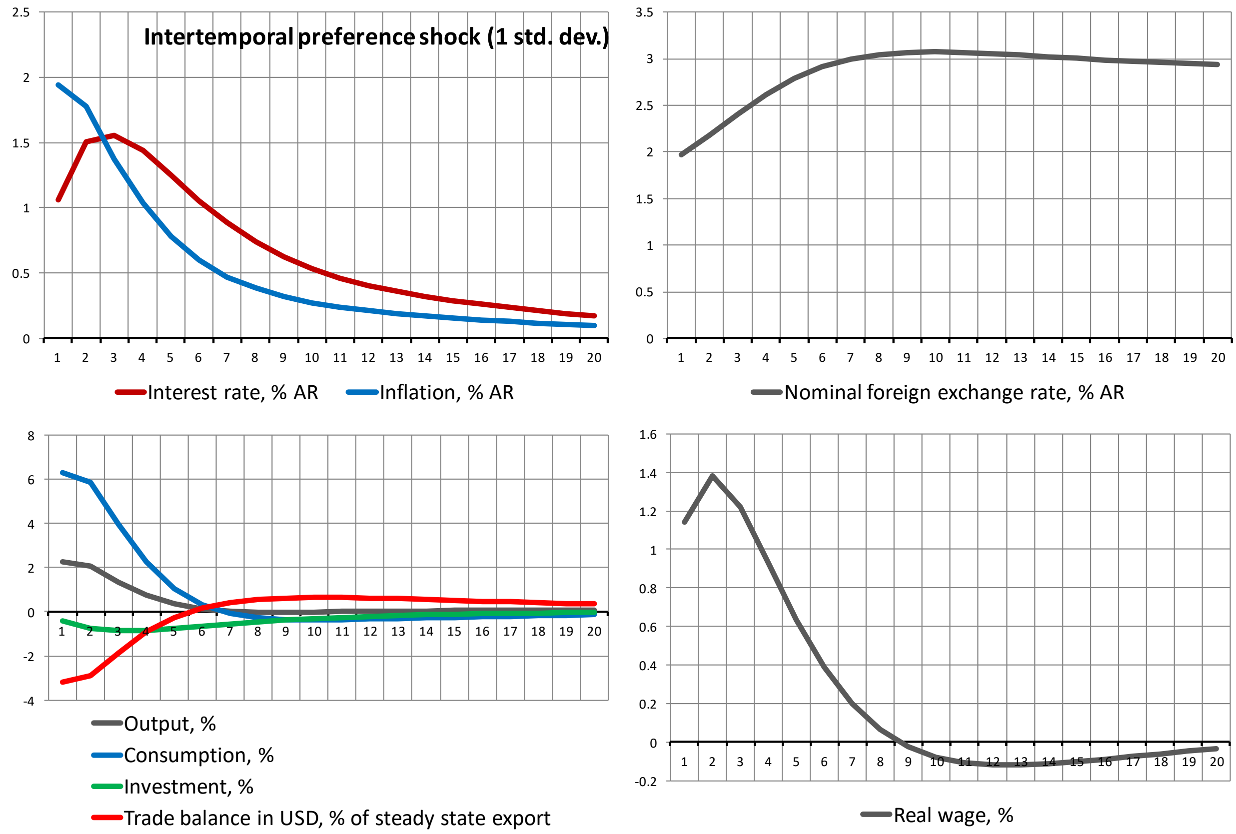

Figure 9. IRF on intertemporal preference shock $\varepsilon_{\beta t}$ (1 std. dev.)

Figure 10 shows the regional shock of intertemporal preferences $\varepsilon_{\beta VVt}$ for a small region (VV = Volga-Vyatka MR). In general, the regional shock generates responses of aggregate variables very similar to the responses to a similar general shock (the only difference is in the autoregression coefficient of shocks). Fig. 10 shows the main properties of regional responses: inflation in the region itself reacts more strongly than in other regions, but regional differences are much less pronounced than differences in regional output and real wages.

Another important property of regional shocks is the reversal of relative price trends. For example, a shock accelerates inflation in the Volga-Vyatka MR more than in the rest of Russia, so relative prices $\frac{P_{cVVt}}{P_{cRUt}}$ rise. However, competitive forces cause prices to reverse and move in the opposite direction. In addition to competitive forces, the regional model incorporates the dynamics of regional interest rates in response to the volume of accumulated assets (debts) of households. Increased consumption of households in the Volga-Vyatka MR contributes to debt accumulation and a long-term increase in interest rates in the region. Therefore, at some point in time, relative prices move into the negative zone: from this point on, the effect of accumulated wealth dominates over other business cycle effects. In the long term (beyond the five-year interval shown), the relative prices of the two regions will even out.

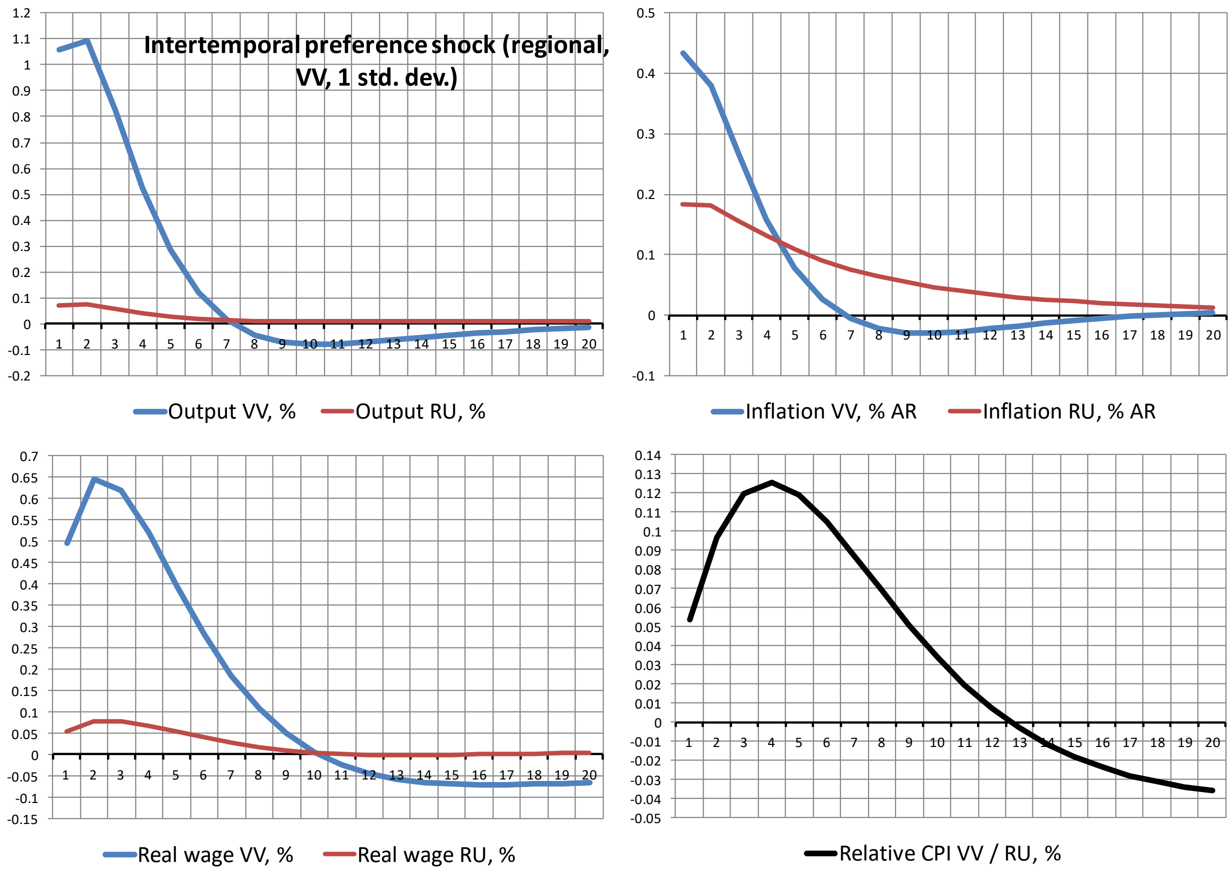

Figure 10. IRF on intertemporal preference shock $\varepsilon_{\beta VVt}$ (regional, VV, 1 std. dev.) VV = VVMB; RU = Other regions of Russia

Using the example of total factor productivity (TFP) shock, let us compare the economy response to temporary and permanent shocks. Figure 11 shows a temporary TFP shock, while Figure 12 shows a permanent TFP shock. A temporary TFP shock (with an autoregression coefficient of around 0.5) traditionally has a positive effect on output and investment and contributes to a reduction in marginal costs and inflation. At the same time, the declined labour demand pulled down real wages, and consequently, consumption. However, consumption, and subsequently real wages a few quarters later, move into the positive area, which is associated with a positive effect on output and a decline in inflation and interest rates.

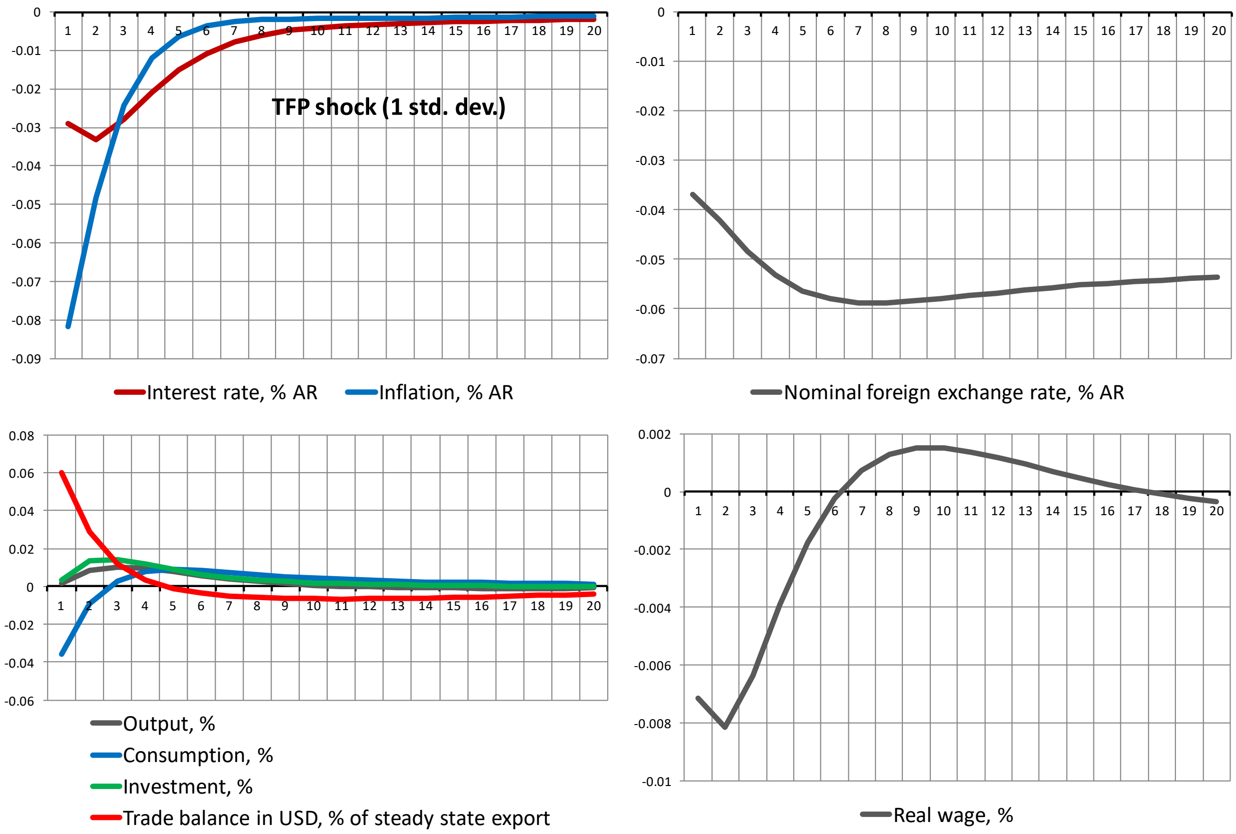

Figure 11. IRF on productivity shock (TFP) ε_{At} (1 std. dev.)

The permanent TFP shock (Fig. 12) shows the impact of permanent productivity shocks υ_{At} on long-term equilibrium and medium-term adjustment. With long-term TFP growth, wages and consumption do not fall into the negative area that is consistent with the properties of deterministic growth rates: an increase in productivity growth rates also increases the long-term growth rate of real wages. With a long-term TFP level increase of 0.77%, long-term output growth is 0.52%. This means that the demand for labour slightly declines, which for firms is associated with long-term growth in real wages, and for households with increased consumption and a reduction in the marginal effect of a unit of labour. The inflationary process caused by a permanent TFP shock is more persistent than with a temporary TFP shock.

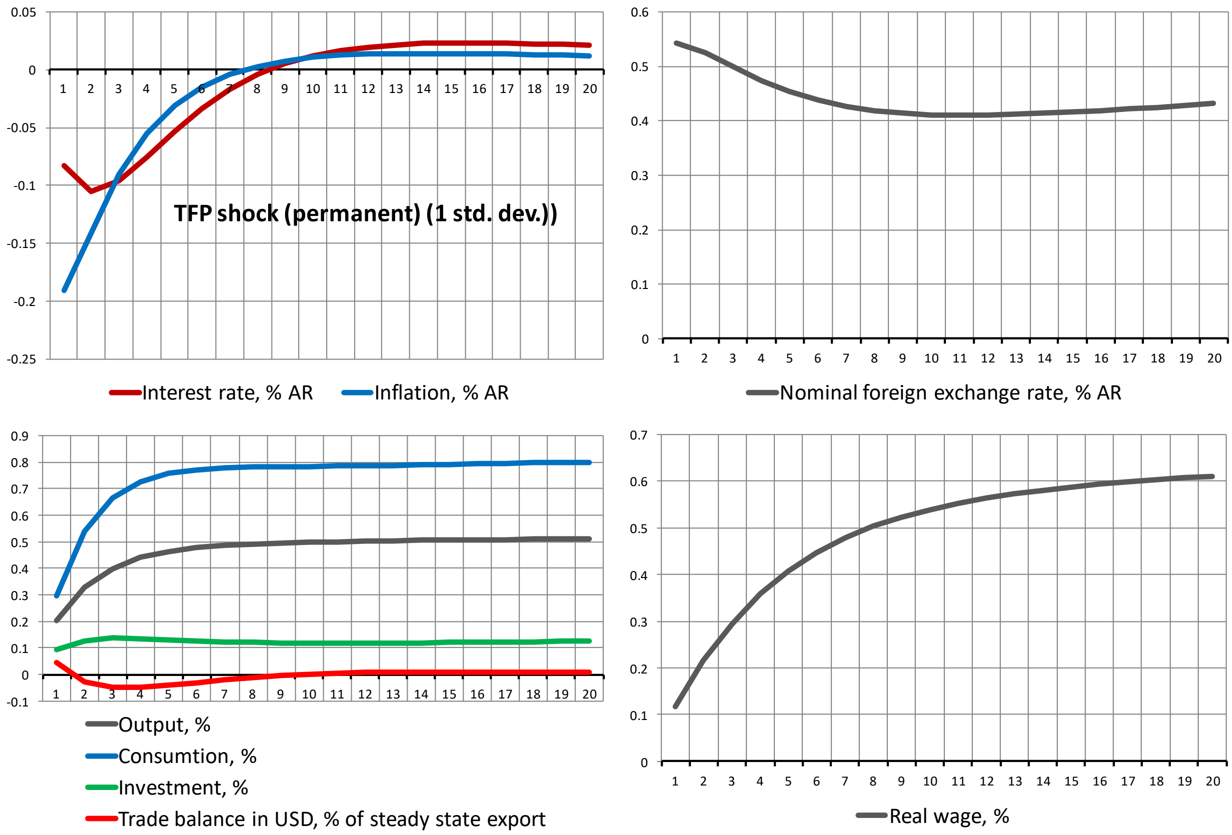

Figure 12. IRF on permanent productivity shock (TFP) υ_{At} (1 std. dev.)

Finally, we need to discuss the property of permanent demand shocks to influence long-term output. Figure 13 shows the response to a permanent government spending shock υ_{Gt}. It can be seen that output grows alongside government spending in the long term with elasticity $\frac{d\bar{Y}/\bar{Y}}{d\bar{G}/\bar{G}} = 0.175$. This is due to the fact that the growth of government spending in the long term leads to a crowding out of private consumption, which affects the incentives for households to offer their labour services to firms. These incentives increase due to the marginal effect of a unit of labour effort for households. For firms, an increase in the share of labour in production is associated with a decrease in real wages. Thus, the effect of permanent demand shocks (in our case, shocks to government spending and exports) is related to the redistribution of output between consumption and other types of expenditures. This redistribution affects the incentives for households to offer their labour services to firms and has an effect on long-term output.

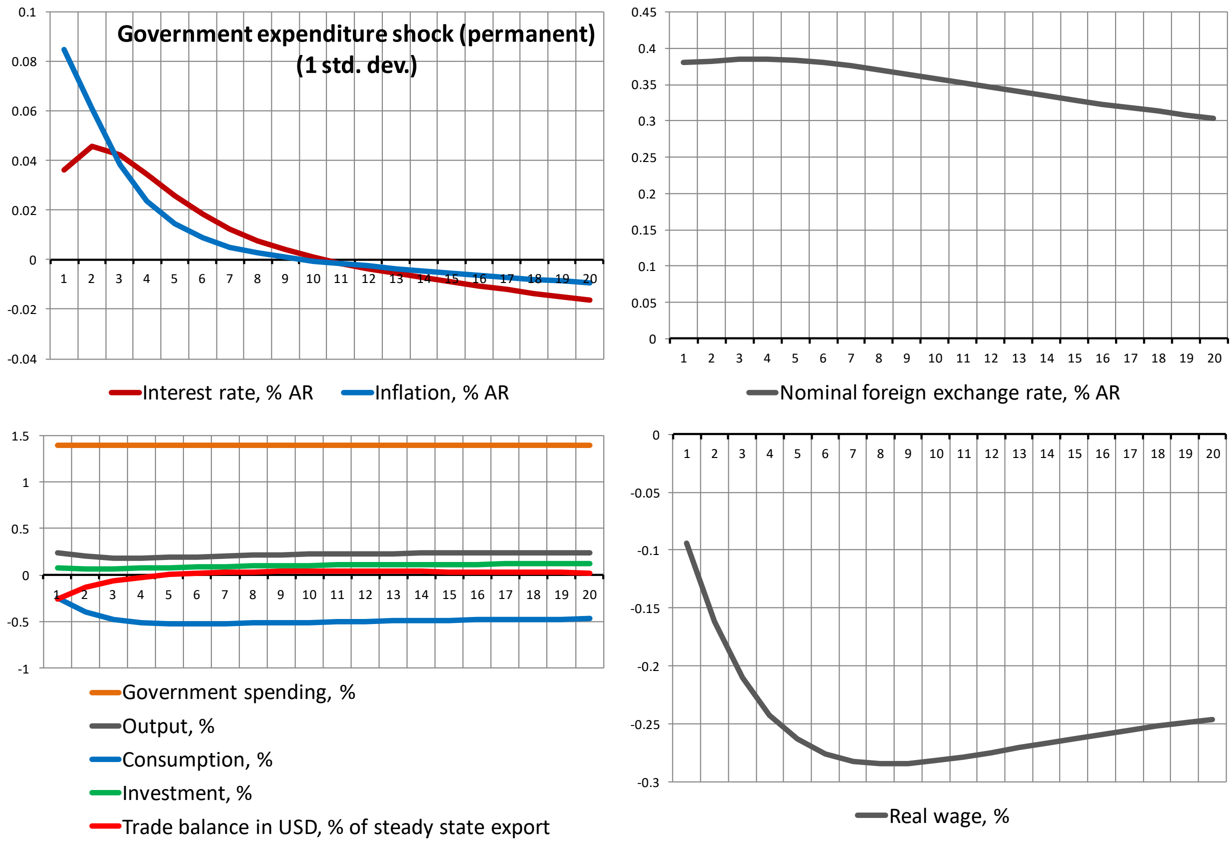

Figure 13. IRF on permanent government spending shock υ_{Gt} (1 std. dev.)

The appendix provides responses to the other shocks used in the assessment.

Decomposition of variable dynamics into structural shocks

Let us consider the decomposition into shocks of the main variables of the model. For ease of interpretation, we have divided all shocks into groups corresponding to groups of macroeconomic factors. We identify the following groups:

- Demand ($\varepsilon_{\beta t}, \varepsilon_{It}, \varepsilon_{\beta RUt}, \varepsilon_{\beta nt}, \varepsilon_{\beta ht}, \varepsilon_{dNt}$)
- Supply ($\varepsilon_{At},\ \varepsilon_{\mu t}, \varepsilon_{\mu RUt}, \varepsilon_{\mu Nt}$)
- Labour market ($\varepsilon_{Wt},\ \varepsilon_{Nt}, \varepsilon_{WRUt}$)
- Fiscal policy ($\varepsilon_{Gt}, \varepsilon_{Tt}, \varepsilon_{GRUt}, \varepsilon_{Tnt}, \varepsilon_{Tht}$)
- Trade balance ($\varepsilon_{Ext}, \varepsilon_{IMt}$)
- Risk premium [external] (ε_{rpt})
- Foreign shocks ($\varepsilon_{\beta^* t}, \varepsilon_{A^* t}, \varepsilon_{i^* t}$)
- Global prices ($\varepsilon_{R^* t}$)
- Oil ($\varepsilon_{O^* t}$)
- Monetary conditions (MC) ($\varepsilon_{it}, \varepsilon_{iRUt}$)
- Monetary policy (ε_{ikt})
- Permanent shocks ($\upsilon_{At}, \upsilon_{Gt}, \upsilon_{Tt}, \upsilon_{Ext}, \upsilon_{IMt}, \upsilon_{Wt}, \upsilon_{Nt}$)
- Regional shocks of the Volga-Vyatka MR ($\varepsilon_{\beta VVt}, \varepsilon_{\mu VVt}, \varepsilon_{WVVt}, \varepsilon_{GVVt}, \varepsilon_{iVVt}$).

Figures 14–17 and 20–25, along with the historical decomposition (2014–2025), present the decompositions of forecasts of endogenous variables up to 2031. In addition to historical dynamics, this forecast is based on available information about the dynamics of variables at the beginning of 2026 (in the short-term forecast). We do not calculate any scenario, do not assume any exogenously determined behaviour of variables in the future, but only show how the economy of Russia and the Volga-Vyatka MR should develop based on the laws of shock transmission embedded in it (in the absence of new shocks).

Figure 14 shows the decomposition of inflation (% AR) in deviations from the target (4%). The model was parameterised for use in medium-term forecasting, so the main focus was on providing the most adequate explanation of events starting from 2022. For earlier events, the decomposition of variables into shocks may be inaccurate because the transmission mechanism differed from the current one.[23]

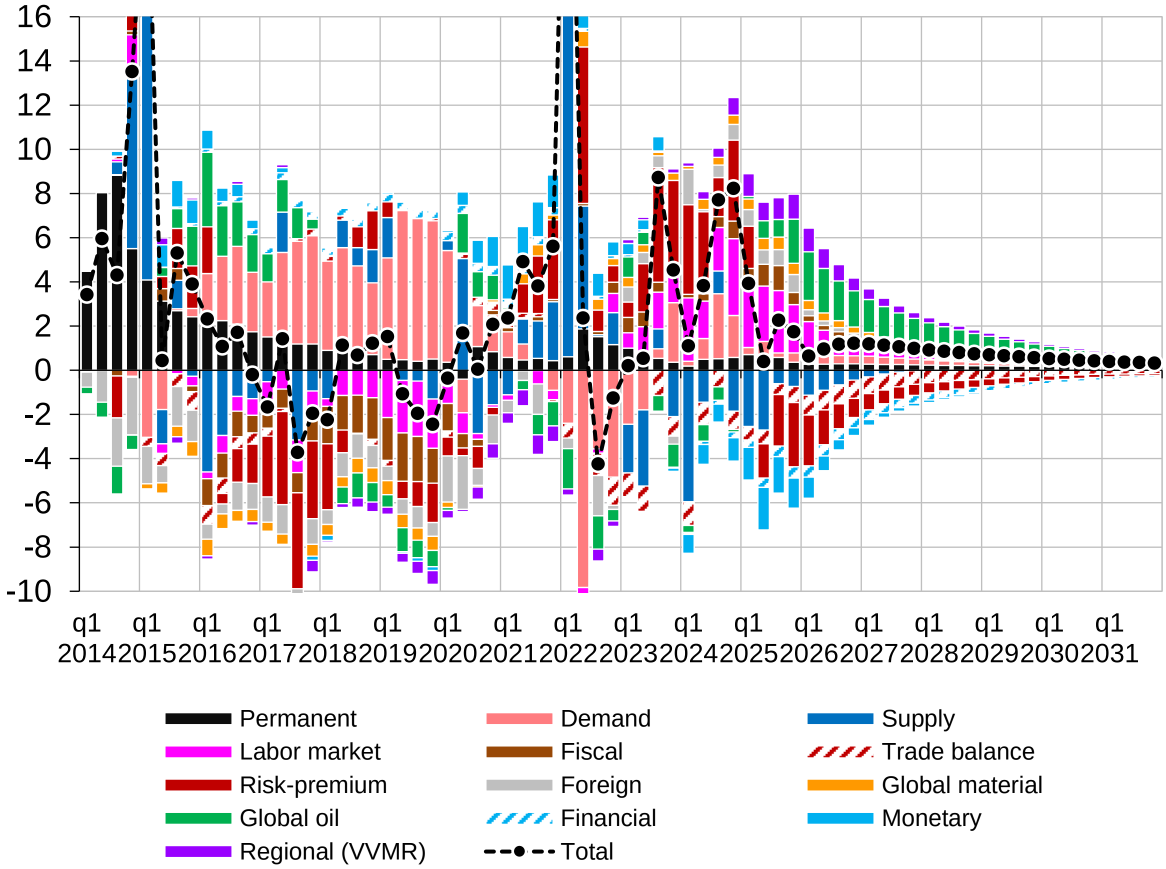

Figure 14. Decomposition of the inflation in Russia, pp QoQSAAR in deviations from the target (4%).

Note: actual (2014–2025), forecast (2026–2031).

During the Covid-19 pandemic, inflation remained at a moderate level, mainly because negative supply shocks were counterbalanced by shocks explaining a decrease in economic activity, particularly from abroad. In 2021, inflationary trends intensified abroad due to supply chain disruptions, and similar processes occurring in Russia, which resulted in surge in inflation in 2021 that the model adequately attributed to these supply shocks.

[23] For example, the model explains the acceleration of inflation in 2015 largely by latent supply shocks. The contribution of external premium shocks appears to be small, which somewhat contradicts the perceptions of the causes of inflation during the 2015 geopolitical crisis. The main explanation lies in the much more significant pass-through of exchange rate movements to prices that was observed before the Central Bank switched to inflation targeting.

The sharp rise in inflation at the start of 2022 was almost entirely due to supply shocks (like monopolistic mark-ups and productivity) as well as an external premium shock that largely explained the sudden rise of foreign exchange rate. The slowdown in inflation seen in 2023 can be attributed to negative demand shocks and the rapid disappearance of pro-inflationary factors (faster than the model had anticipated), which had a stronger impact on inflation than the pro-inflationary supply and labour market shocks. By late 2023, the acceleration of inflation at the end of 2023 was mainly attributed to external premium shocks, which were intensified by labour market and fiscal policy shocks. A tight monetary policy, monopolistic mark-up shocks and a weak trade balance weakened the inflationary impulse. In 2024–2025, labour market shocks emerged as the primary driver of inflation in Russia, while the easing of inflationary process by the end of 2025 was largely due to the strengthening of the ruble under the influence of a mix of shocks, including monetary policy, trade balance and external premium shocks (geopolitics).

The role of permanent, regional, sectoral and agent shocks in explaining inflation turned out to be minor (see Table 9 for more details).

Figure 15 illustrates the decomposition of Russia's output. The decline in output during the Covid-19 pandemic in 2020 was largely due to both domestic and external demand factors. While fiscal policy offered some moderate support to the economy, it was really the soft monetary policy that drove the main heating up of the economy towards the end of 2021. The 2022–2024 episode is characterised by significant counter-directional shocks associated with the structural restructuring of the economy. The output decline in 2022 is mainly interpreted as a demand shock (private and investment).

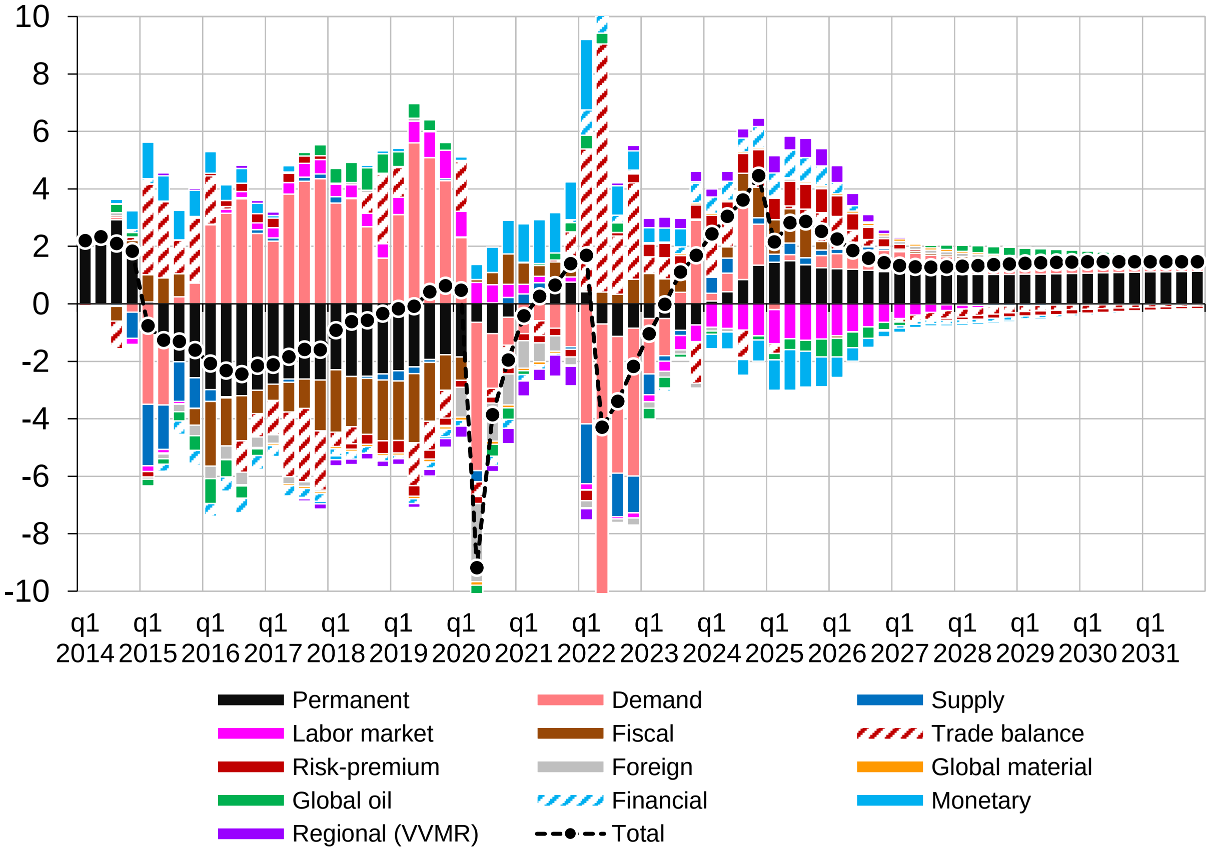

Figure 15. Decomposition of Russia's output (GDP), in % deviation from the deterministic trend (1.35% per year).

Note: actual (2014–2025), forecast (2026–2031).

Supply shock also contributed to the GDP decline. In 2022, part of the GDP decline was attributed to permanent shocks. The Russian economy found some support in the form of reduced imports and a shift in consumer demand preference towards domestic goods. The decomposition demonstrates the divergent impact of fiscal and monetary policies. At the same time, an assessment of the contribution of monetary conditions reflecting the state of the financial market suggests that the negative impulse for the economy from monetary policy was largely offset by the softness of MC. According to the forecast for 2024–2025, the most significant negative contribution to output will come from the labour market, which, together with tight monetary policy and weak external demand, will be the main factor in the economy's transition to recession. Fiscal policy could support the economy, which in the current forecast assumes rapid budget consolidation.

Figure 16 shows the decomposition of real wages. In wages dynamics, permanent shocks reduce the long-term level of wages relative to the deterministic trend (2.6% per year). With significant wages growth in 2023–2024, this could create a notable wages gap, which will have a considerable effect on both the acceleration of inflation and the negative GDP gap.

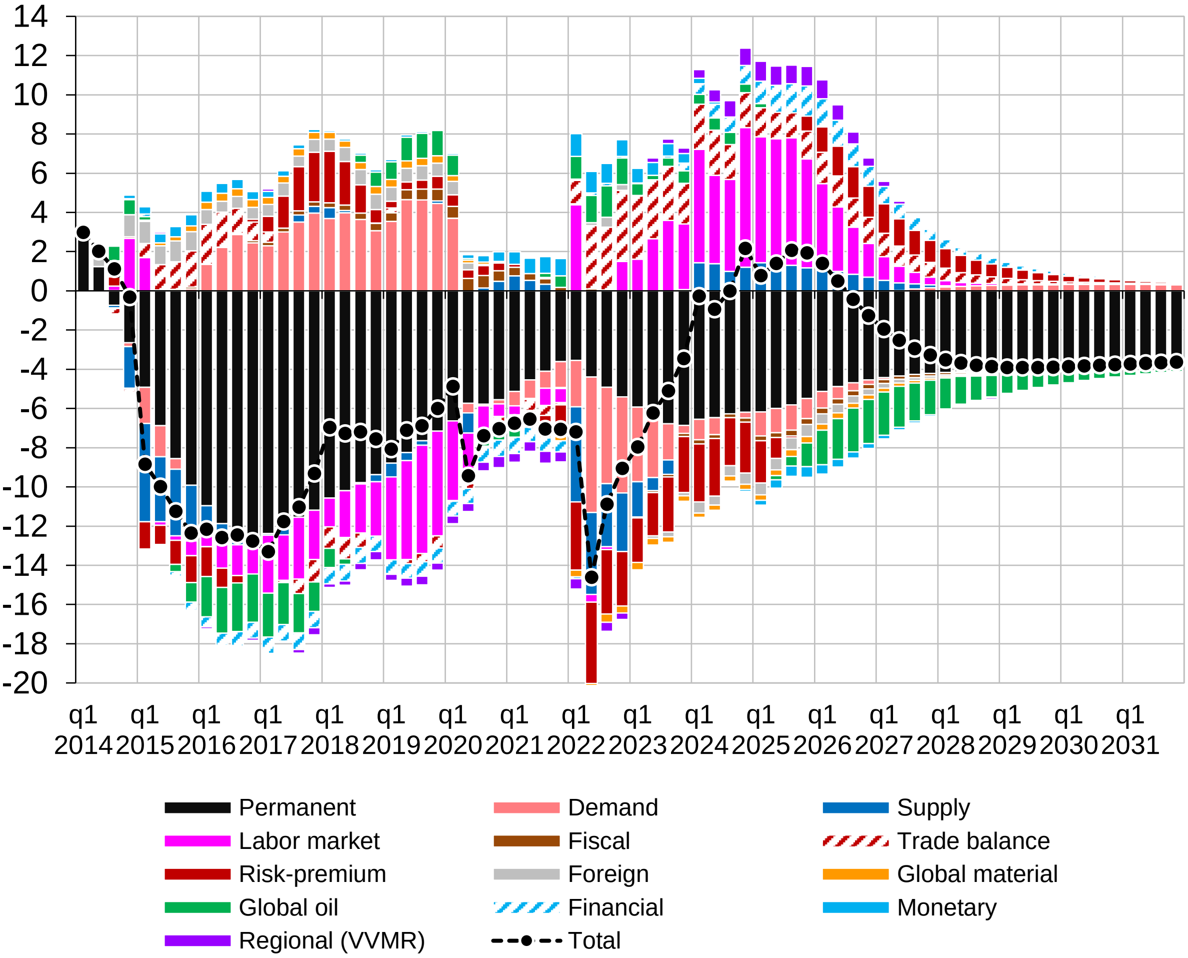

Figure 16. Decomposition of real wages in Russia, in % deviation from the deterministic trend (2.6% per year).

Note: actual (2014–2025), forecast (2026–2031).

Most shocks that impact output tend to have a similar effect on real wages. However, there is the most notable exception: factors (shocks) that originate within the labour market itself.

Since labour is a cost for producers, these specific shocks create divergent dynamics for output and wages.

The floating exchange rate regime causes notable fluctuations in both the nominal and real exchange rates of the ruble against the dollar (and other global currencies). Figure 17 shows the decomposition of the dynamics of the real dollar exchange rate into shocks. A significant portion of the fluctuations in the real exchange rate can be attributed to oil supply shocks, external shocks and latent external premium shocks.

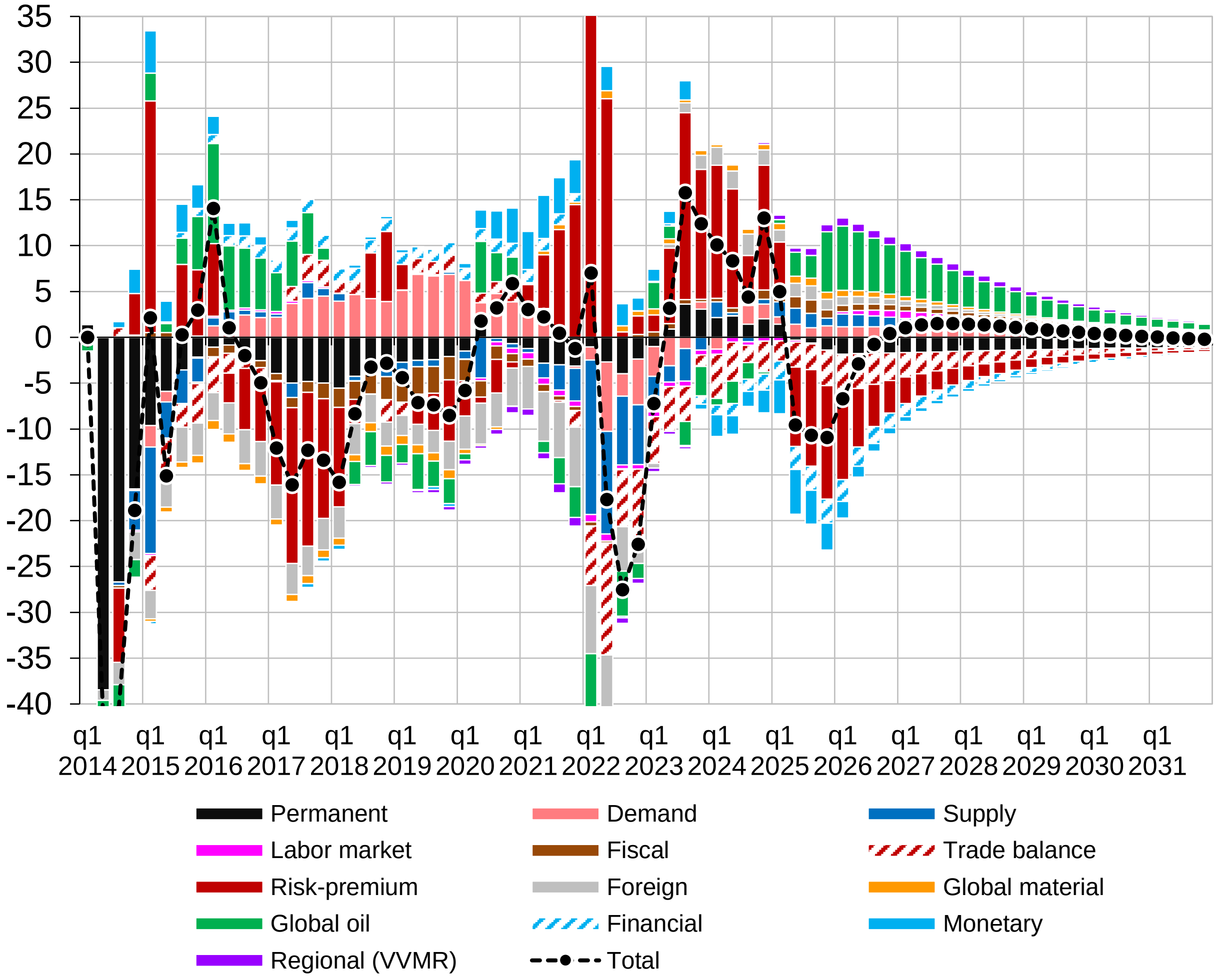

Figure 17. Decomposition of the real dollar exchange rate, as a percentage of the stationary level.
Note: actual (2014–2025), forecast (2026–2031).

Following the introduction of restrictions on cross-border capital flows and the imposition of sanctions on Russian exports and imports in 2022, balance of payments shocks became the most significant factor affecting the exchange rate. The ruble's significant strengthening in 2022 can be attributed to these actions, which also led to a significant slowdown in inflation (bringing it below the target). According to our model, this inflation drop was primarily due to the impact of latent external premium shocks that took place in the first half of 2022. In 2024, most structural shocks should have contributed to the strengthening of the ruble (high key rate, import shocks and expensive oil). Why this did not happen, the model explains by the action of latent external premium shocks (geopolitics).[24] In contrast, in 2025, the model, on the contrary, indicates an excessively strong ruble (up to 11% to the equilibrium level).

Forecast quality assessment

As part of our quality assurance for the model, let us engage in some forecasting exercises. We will start with ‘in-sample’ forecast calculations, a forecast option where we utilise a model that has been estimated on the full dataset to predict variables within the estimation period. We will then compare these model forecasts (also In-sample), which we will take as a reference point. Next, we will perform the same procedure for out-of-sample forecasts, which involve making forecasts for periods that are not included in the model estimation period. To work with Out-of-sample forecasts, we will reassess the model over several periods and compare the forecasts we generate with those from our reference models, also obtained by the Out-of-sample method.

As reference models, we will consider two simple statistical models: an autoregressive (AR1) model with a deterministic trend and a Random walk (RW) model that also features a deterministic trend. In both models, we link the dynamics of each of the considered variables exclusively to the previous dynamics of this variable. The deterministic trends for all models will be estimated along with the dynamic parameters of the models.

We will measure the accuracy of our forecasts using the RMSE metric.

In-sample forecasts

We will make In-sample forecast calculations for three models (MARS = estimated DGSE model; AR1 model; RW model) for the period $2019Q1 - 2025Q4$ (28 quarters), included in the model estimation period. The assessment results are presented in Table 6:

Table 6. RMSE for In-sample forecasts of the three models: MARS – estimated DSGE model; AR1 model, RW model

	RMSE, 1 quarter			RMSE, 2 quarters			RMSE, 4 quarters			RMSE, 8 quarters		
Variable	**MARS**	**AR1**	**RW**	**MARS**	**AR1**	**RW**	**MARS**	**AR1**	**RW**	**MARS**	**AR1**	**RW**
Inflation	**0.016**	0.016	0.018	0.017	**0.016**	0.021	**0.016**	0.016	0.022	0.017	**0.016**	0.024
GDP	**0.019**	0.022	0.021	**0.023**	0.024	0.027	0.028	**0.025**	0.032	0.029	**0.025**	0.031
Real wages	0.021	0.020	**0.019**	0.029	0.025	**0.024**	0.040	**0.029**	0.035	0.061	**0.031**	0.066
Investment	0.034	0.033	**0.032**	0.051	**0.042**	0.044	0.068	**0.054**	0.066	0.081	**0.055**	0.096
Nominal exchange rate of foreign currency	0.081	**0.080**	0.080	0.117	**0.102**	0.119	0.151	**0.107**	0.155	0.218	**0.106**	0.224
Key rate	0.006	0.005	**0.004**	0.010	0.008	**0.007**	0.013	**0.010**	0.010	0.015	**0.010**	0.013
Inflation, Volga-Vyatka MR	**0.016**	0.017	0.019	0.017	**0.017**	0.022	**0.017**	0.017	0.023	0.018	**0.017**	0.025
GRP, Volga-Vyatka MR	0.018	0.019	**0.017**	0.026	0.022	**0.021**	0.036	**0.027**	0.028	0.045	**0.029**	0.038
Real wages, Volga-Vyatka MR	0.020	0.019	**0.017**	0.032	0.027	**0.026**	0.051	**0.037**	0.041	0.071	**0.043**	0.077
Inflation abroad	**0.004**	0.004	0.004	**0.004**	0.005	0.005	**0.004**	0.005	0.006	**0.005**	0.005	0.008

[24] Since 2022, the most realistic interpretation of external premium shocks is related to the intensity of sanctions pressure on Russia and problems with cross-border payments.

Foreign output	**0.015**	0.018	0.017	**0.019**	0.021	0.023	**0.020**	0.022	0.026	**0.020**	0.021	0.028

Source: authors' calculations.

Note: For each model, calculations are based on 28 quarters $2019Q1–2025Q4$. Exception: investments and foreign output, 26 quarters $2019Q1–2025Q2$.

The estimated model in In-sample forecasting generally lags behind the AR1 model (especially over 4 and 8 quarter intervals), although it generally outperforms the RW model. The main reason for this discrepancy is that we are trying to forecast variables where the identification of the long-term trend is difficult. In this case, the AR1 model, which identifies a single trend that best describes the historical data, may gain an additional advantage over models with a changing trend estimate (MARS, RW).

Let us consider the forecasting results in more detail.

Figure 18 shows forecasts based on three models. From the perspective of forecasting inflation, the advantage of the AR1 model over the developed structural model is based on the fact that the average inflation for the selected historical range exceeded the target (4%). For example, in the period 2022–2025, the model explains the sustained inflation above the target by a series of pro-inflationary shocks of a short-term nature. This generates a series of errors in the structural model, while the AR1 model, where the average inflation level is above the target, tends to be more accurate in its forecasts.

The key rate being higher than what the MARS Taylor rule estimated, which was necessary to curb pro-inflationary risks during 2022–2025, also contributed to the historical rate being above the real neutral rate (of 4% per annum) embedded in the structural model. This also gave an advantage to the AR1 model, which allows the long-term trend to be freely adjusted to the data.

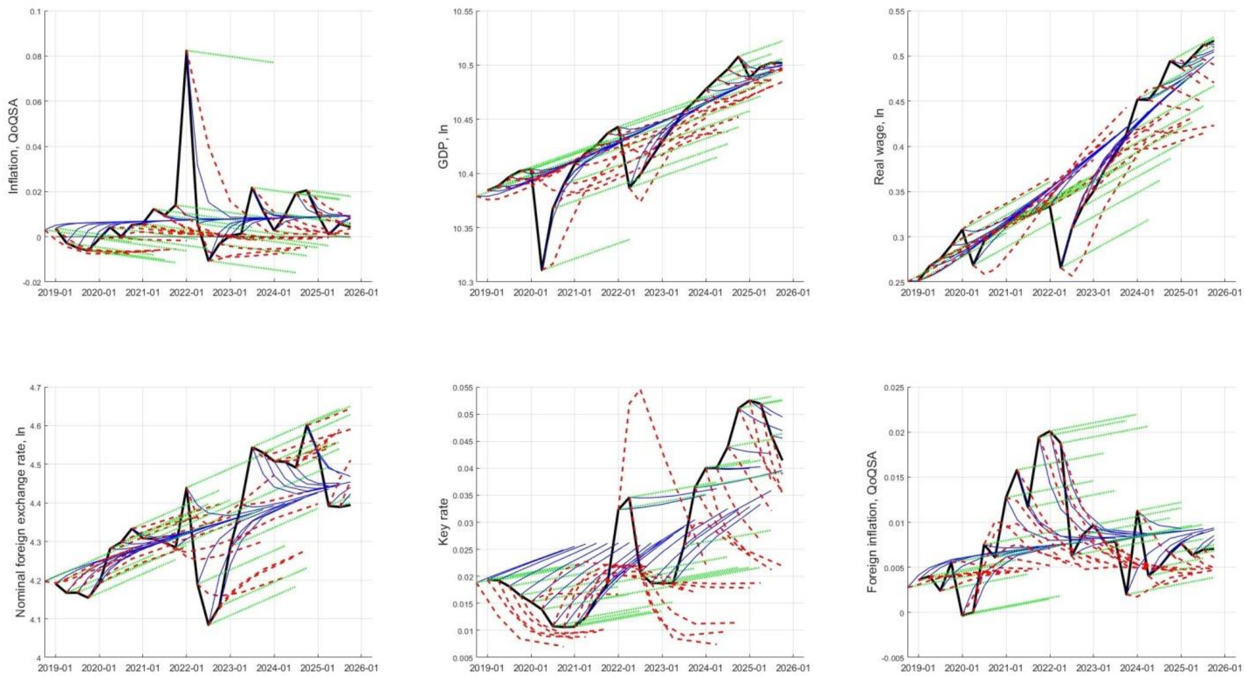

Figure 18. In-sample forecasts based on three models: MARS (red); AR1 (blue); RW (green).

Note: Actual (black)

Errors in forecasting output and real wages are associated with estimates of the trend in the data series. The trend in the AR1 model is estimated without restrictions based on GDP dynamics. The trend in the MARS model is limited by structural relations and is determined based on several observed series: GDP, GRP, investments, consumption, government spending, real wages, nominal exports and imports plus regional series. The deterministic trends of all observed variables in the structural model are interrelated, which prevents the free fitting of deterministic trends to the data.

Similar considerations apply to virtually all series (see Fig. P21 in Appendix 3).

In-sample forecasting allows us to see the limits of the developed model: the difficulties in linking observed trends with theoretically justified trends of real variables. Permanent shocks were introduced, among other things, to partially mitigate this problem,[25] however the accuracy of identifying these shocks in real time limits their ability to improve the quality of structural model forecasts.

Out-of-sample forecasts

Let us consider the properties of Out-of-sample forecasts of the structural MARS model compared to the reference models discussed above. Table 7 presents the results of calculating Out-of-sample forecasts for three models for the period $2020Q1–2025Q4$ (24 quarters) for the same MARS, AR1 and RW models.[26] In each case, we re-estimate all models based on the then available information, starting from $2014Q1$.

[25] Additionally, permanent shocks in the model are mainly used to describe the structural transformation of the economy after 2022.

[26] Exception: investments and foreign output, 22 quarters ($2020q1–2025q2$).

Table 7. RMSE for Out-of-sample forecasts of the three models: MARS – estimated DSGE model; AR1 model, RW model.

	RMSE, 1 quarter			RMSE, 2 quarters			RMSE, 4 quarters			RMSE, 8 quarters		
Variable	**MARS**	**AR1**	**RW**	**MARS**	**AR1**	**RW**	**MARS**	**AR1**	**RW**	**MARS**	**AR1**	**RW**
Inflation	**0.020**	0.021	0.023	**0.022**	0.023	0.027	**0.020**	0.023	0.028	**0.021**	0.028	0.034
GDP	**0.024**	0.026	0.028	**0.029**	0.031	0.036	**0.031**	0.034	0.043	0.038	**0.031**	0.041
Real wages	0.028	0.025	**0.023**	0.039	0.033	**0.031**	0.053	**0.043**	0.046	0.058	**0.055**	0.071
Investment	0.058	0.043	**0.041**	0.090	0.061	**0.057**	0.127	**0.090**	0.092	0.166	**0.129**	0.137
Nominal exchange rate of foreign currency	**0.087**	0.099	0.094	**0.125**	0.133	0.142	0.155	**0.154**	0.183	**0.174**	0.187	0.201
Key rate	0.007	0.007	**0.006**	0.013	0.011	**0.009**	0.017	0.015	**0.013**	0.020	0.022	**0.017**
Inflation, Volga-Vyatka MR	**0.021**	0.022	0.024	**0.022**	0.024	0.029	**0.021**	0.023	0.029	**0.022**	0.029	0.034
GRP, Volga-Vyatka MR	0.026	**0.023**	0.023	0.037	0.029	**0.028**	0.048	0.041	**0.040**	0.059	0.057	**0.056**
Real wages, Volga-Vyatka MR	0.027	0.025	**0.022**	0.046	0.038	**0.034**	0.075	0.058	**0.057**	0.092	**0.080**	0.099
Inflation abroad	**0.004**	0.005	0.005	**0.005**	0.006	0.006	**0.005**	0.007	0.008	**0.006**	0.009	0.011
Foreign output	**0.020**	0.024	0.023	**0.025**	0.034	0.032	**0.025**	0.042	0.039	**0.025**	0.053	0.046

Source: authors' calculations.

Note: RMSE calculated based on 24 quarters $2020Q1 - 2025Q4$. Exception: investments and foreign output, 22 quarters $2020Q1 - 2025Q2$.

The results of the calculations show that statistical models no longer have an advantage over the DSGE (MARS) model. When forecasting inflation in the Russian Federation, VVMR, abroad, GDP, foreign output, nominal exchange rate of foreign currencies, MARS has an advantage over statistical models at virtually all horizons. Key variables, real wages, investment, GRP and wages in VVMR are forecasted worse by the structural model than by statistical models.

Let us consider the nature of DSGE model errors in more detail. Figure 19 (and Figure P22 in Appendix 3) shows Out-of-sample forecasts based on three models. In this case, the DSGE model's focus on a 4% target gives it an advantage over statistical models. Inflation trend estimates made in different quarters show very different results, resulting in the inflation forecast error (overall, regional, foreign) for the AR1 and RW models higher than for the DSGE for all forecast horizons.

The Taylor rule in 2020 and 2022–2025 does not predict interest rate dynamics well, which gives more flexible statistical models an advantage in forecasting the key rate. This provides an opportunity to consider improving the monetary policy rule.

Real wages in Russia, wages and VVMR GRP, as well as investments, are highly dependent on the dynamic properties of shocks. The period 2022–2025, with shocks that are more persistent than in the historical range, has significantly reduced the predictive power of structural models for these variables. For the dynamics of wages, this property of shocks has been enhanced by the forecast of a shift in the long-term wage trend (downwards) due to permanent fiscal shocks.

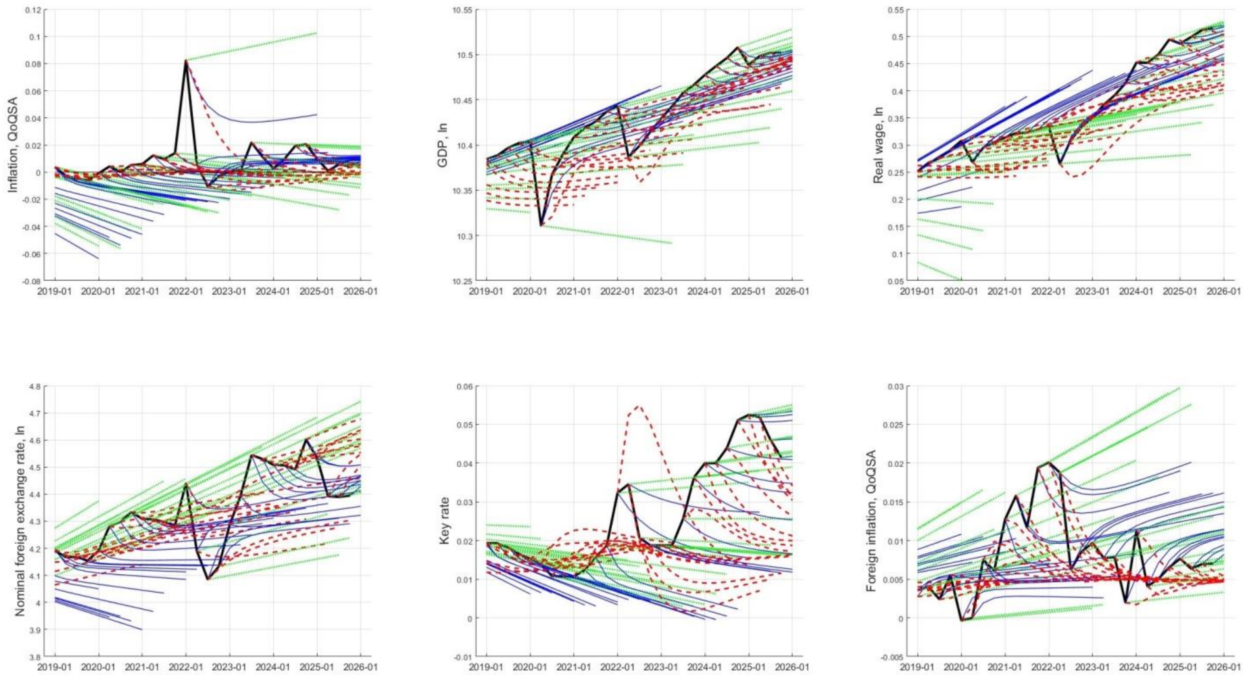

Figure 19. Out-of-sample forecasts based on three models: MARS (red); AR1 (blue); RW (green).
Note: Actual (black)

Permanent shocks are designed to address the problem of underestimating the persistence of shocks, but their real-time assessment is challenging (information about them is gradually incorporated into the data), and their composition is limited. For example, the data show that permanent shocks to investment and regional output could improve the MARS forecasting properties for the period 2022–2025.

The calculations demonstrate well-known problems in macroeconomic forecasting during periods of structural shifts. However, the structural model reinforced by permanent shocks generally allows for a more adequate macroeconomic picture to be formed, and there is no doubt about the usefulness of structural modelling in a turbulent environment: the DSGE model allows for gradual and controlled correction of identification and forecasting errors by adjusting the structure of the economy.

Advanced analytics based on the estimated DSGE model

In this section, we will perform several advanced analytics exercises that assist in making monetary policy decisions. First, we will discuss the permanent components of variables: output, real wages and interest rates. Then we will discuss the decomposition of regional variables. Finally, we will ask about the contribution of shocks that explain heterogeneity and their role for monetary policy.

Permanent components of variables

We assume that, in addition to deterministic trends in endogenous variables, the model contains stochastic trends that are determined by permanent shocks υ_t. Permanent shocks determine the I(1) components of endogenous variables. For the purposes of macroeconomic analysis, it makes sense to calculate separately the dynamics of all endogenous variables, which takes into account both deterministic and stochastic trends, but does not take into account the effect of all temporary shocks ε_t responsible for the business cycle. For this type of calculation, we further use the term 'permanent' (Canova, 2024), and the permanent variables are asymptotes for the corresponding endogenous variables. The deviation of the actual values of the variables from their permanent component is decomposed into business cycle shocks ε_t and is the temporary (transitory) component of the variables.

Permanent GDP and real wages

To characterise the dynamics of permanent output, we first return to the calculation of the deterministic trend. The long-term GDP growth rate according to (156) (as percentage per annum) is:

$$g_Y = \frac{g}{\alpha_L}\left(1 + \frac{1-\sigma}{\sigma+\kappa}\right) + n = \left[\frac{0.00262}{0.66}\left(1 + \frac{1-1.276}{1.276+1.083}\right) - 0.00014\right] \cdot 4 \cdot 100\% = 1.346\%$$

This value was obtained as a result of a Bayesian estimation of long-term TFP growth rates, utility function parameters of households and labour force trends. The main structural constraint of the estimation is the assumption of a general trend in GDP, investment, aggregate consumption and government spending.

Permanent shocks also play a significant role in GDP dynamics (see Fig. 15). In Fig. 20, we will examine in more detail the contribution of each of the seven permanent shocks to Russia's GDP dynamics. Until 2022, the main contribution comes from permanent export shocks and TFP. After the Covid-19 pandemic, the main driver will be permanent government spending and TFP shocks (each contributing up to +1% to long-term GDP). Since 2022, there has been a change in the structure of the economy, which in terms of long-term GDP the model has broken down into positive shocks to government spending and taxes (up to +2.5% of GDP), long-term growth in wages mark-up (up to -2% of GDP), a decrease in TFP (up to -1.5% of GDP) and a decrease in imports (up to +0.5% of GDP).

By the end of 2025, the estimated total impact on GDP of the permanent shocks that occurred after 2022 was close to zero. Restrictions on the estimation of permanent shock variances imposed during model estimation contributed to this result (see the section on the specifics of permanent shock estimation).

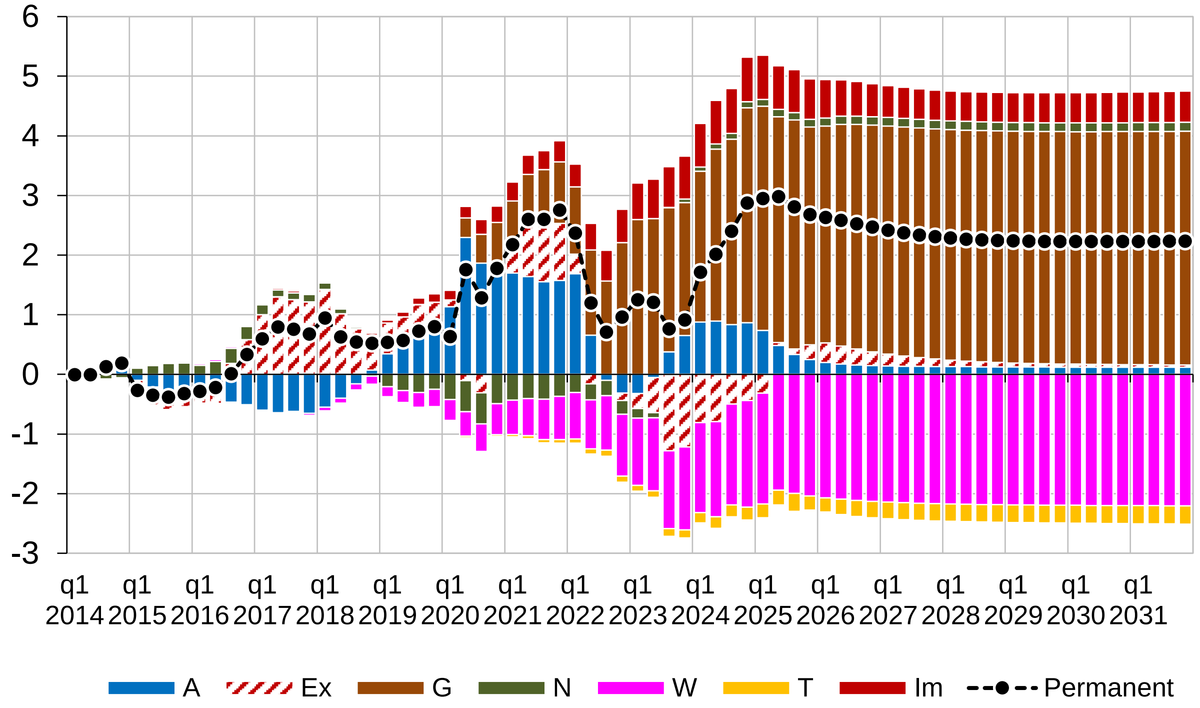

Figure 20. The contribution of permanent shocks to the dynamics of Russia's GDP, in % deviation from the deterministic trend (1.35% per year).
Note: actual (2014–2025), forecast (2026–2031).

The trend deterministic growth rate of real wages is calculated using formula (153), but in our assessment we allowed for the possibility of long-term real wages growth rates deviating from the structurally justified level by an amount g_{werr}, which we also estimated using the Bayesian method. As a result, the general calculation formula (in per cent per annum) is as follows:

$$g_w = \frac{g}{\alpha_L} + g_{werr} = \left[\frac{0.00262}{0.66} + 0.00215\right] \cdot 4 \cdot 100\% = 2.448\%$$

The discrepancy g_{werr} allows us to leave unexplained the reason why, according to statistics, real wages grow at a long-term rate higher than that required by the standard neoclassical model of economic growth (about 1.6% per year).

Permanent shocks also have a significant impact on real wages. Figure 21 shows the contribution of the dynamics of these shocks to the permanent component of real wages.

The most fundamental result is the significant negative contribution of permanent government spending shocks to long-term wages (up to -2.5% after 2022). This leads to a significant deviation of actual real wages (Fig. 16) from their trend level. The negative long-term response of real wages to the growth of government spending is the basis for the positive effect of the permanent demand shock (government spending) on long-term output. This effect manifests through the crowding-out effect of private consumption and the growth of the marginal utility of one unit of labour for households. This stimulates households to increase the labour supply (reduce the required wage), and firms to increase production volume in the long term.

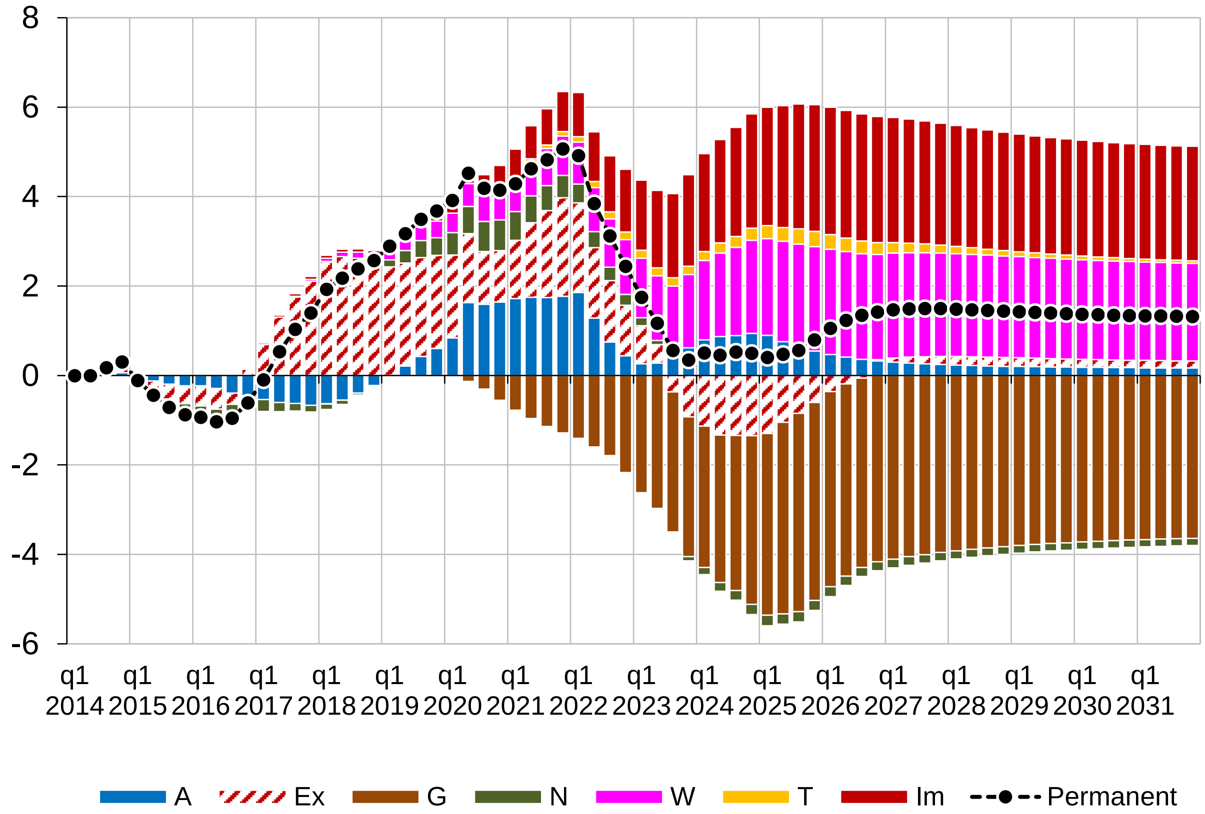

Figure 21. The contribution of permanent shocks to the dynamics of Russia's real wages, in % deviation from the deterministic trend (2.45% per year).

Note: actual (2014–2025), forecast (2026–2031).

Permanent and natural rates

To calculate the permanent component of the rate, we use the same approach as for permanent output: the deterministic part (natural rate) plus the contribution of permanent shocks. In the model, the real rate in the deterministic stationary state (natural rate) is at a level that satisfies the Euler equation (15) for consumption, taking into account the growing output and labour force. In equation (16), we defined the growth rate-adjusted labour force n and output g_Y subjective intertemporal discount:

$$\beta_{adj} = \beta \exp\big(-n - \sigma(g_Y - n)\big) = 0.995 \cdot \exp[0.00014 - 1.276(0.003366 + 0.00014)] = 0.9907$$

According to (17), the natural real interest rate corresponding to the balanced growth path and optimal behaviour of households (long-term natural rate) will be:

$$r = \frac{1 - \beta_{adj}}{\beta_{adj}} \cdot 4 \cdot 100\% = 3.756\%$$

As we noted above, the adjusted intertemporal discount takes into account that, in addition to the subjective discount of the future, there are two other effects. The effect of household size, which reduces the debt burden (if any) on each household member, as well as the effect of the trend decline in the marginal utility of consumption Λ_C, which requires compensation for households in the form of a higher real rate.[27]

We find the nominal long-term natural rate by adding the expected long-term inflation (target 4%) to the real rate:

$$i = r + 4\% = 7.756\%$$

In the developed model, the Taylor rule for the key rate is based on the long-term natural rate, and the contribution of permanent factors to the key rate is calculated through their impact on expected inflation. Figure 22 shows the dynamics of the contribution of permanent shocks to the key rate.

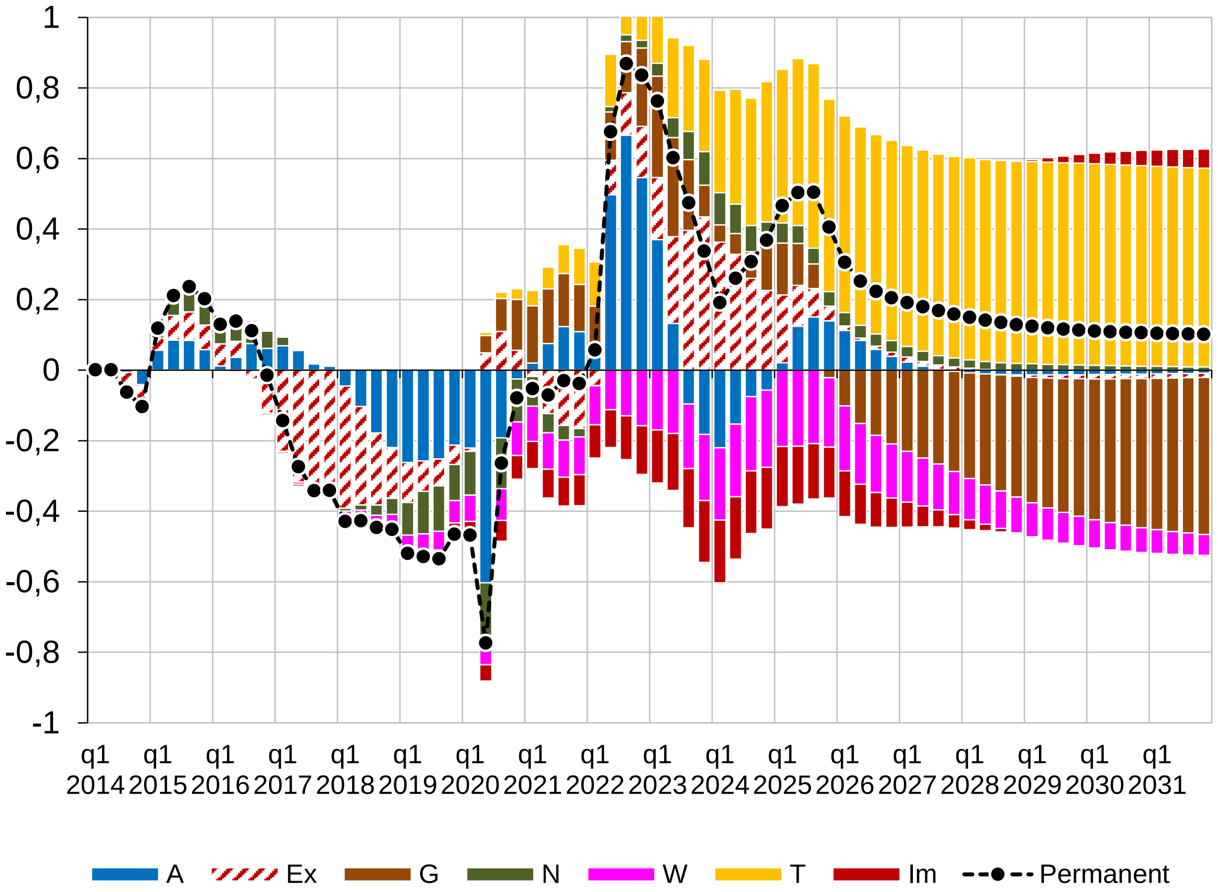

Figure 22. The contribution of permanent shocks to the dynamics of the key rate in % deviation from the stationary level (7.76% per annum).
Note: actual (2014–2025), forecast (2026–2031).

In 2022, the permanent component of the rate goes up, as the permanent decline in exports through the exchange rate channel, as well as TFP through the cost channel, somewhat accelerated inflation, which according to the Taylor rule required an increase in the key rate. The impact of permanent shocks on inflation is not very large, and their response is not as persistent as the response of real variables. The model predicts that balanced expansion of the state budget under the influence of permanent fiscal shocks will create a slight pro-inflationary background (+0.1 pp AR) for the next few years. If it is necessary to keep inflation on target, this will require

[27] We ignore the impact of the habit effect in consumption on β_{adj} and assume that households realise that the level of 'neighbour's consumption' in the long term also grows along with their level.

monetary authorities to keep the key rate slightly above the long-term natural level, which can be interpreted as an increase in the medium-term level of the neutral [to inflation] rate.

In the spirit of the Aguiar and Gopinath (2007) formula ‘trend generates cycle’, note that permanent shocks influence the high-frequency component of inflation and rates, although their contribution to the dynamics of these variables is relatively small (see Table 9).

Regional macroeconomic analysis

For preliminary meetings of the Board of Directors (BoD) of the Bank of Russia each regional branch prepares a medium-term forecast of the dynamics of regional and aggregate macroeconomic variables. The spillover effects between sectors and regions make processes in any macro-region connected with the rest of the Russian Federation, thus integrating the macro-region into the overall economic structure. The developed DSGE model was initially tailored to solve this task, so below we will only focus on some features of forecasting regional macro-series.

Figure 23 shows the decomposition of inflation in the Volga-Vyatka MR into structural shocks along with a forecast up to 2031. If we compare this decomposition with a similar decomposition for Russia (Figure 14), we can see that the qualitative conclusions about the contribution of various factors to inflation dynamics do not change. The role of regional shocks in explaining regional inflation also turns out to be small (although slightly higher than for headline nationwide inflation). Firstly, the actual inflation of the region and the country as a whole differs insignificantly, which does not give significant reason to assume the action of any regional or sectoral shocks. Secondly, other types of heterogeneity characteristic of the Volga-Vyatka MR partially help explain the observed deviations of regional inflation from nationwide: a lower percentage of extractive industries and a higher percentage of the nhtm group. The proximity of headline regional and nationwide inflation increases the significance of analysing regional inflationary trends, which can be partially extrapolated to the country as a whole. In the question of which inflation is more important for decision-making: regional or aggregated, there is no ambiguity: regional monetary authorities should always focus on Russian inflation, using the features of the regional business cycle only as additional information for developing monetary policy decisions for the country as a whole.

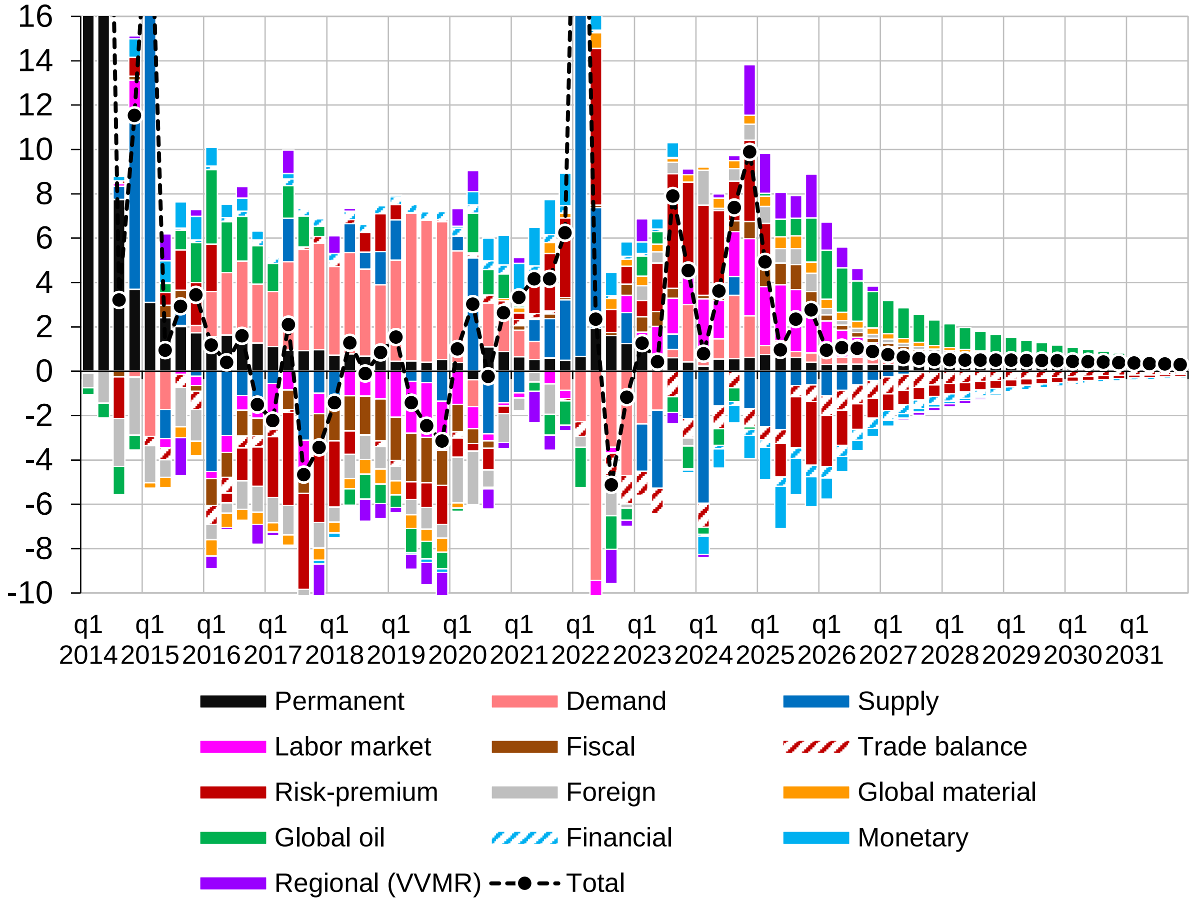

Figure 23. Decomposition of inflation in the Volga-Vyatka MR, pp QoQSAAR in deviations from the target (4%).

Note: actual (2014–2025), forecast (2026–2031).

Next, consider the dynamics of the gross regional product of the Volga-Vyatka MR. Figure 24 presents the decomposition of GRP into structural shocks and a forecast up to 2031. The contribution of regional shocks to the dynamics of GRP turned out to be much more significant than for regional inflation. A similar conclusion can be made for real wages (Figure 25). After 2022, regional output and regional wages grow stronger than in Russia as a whole, which the model attributes to the action of regional shocks (mainly on the demand side $\varepsilon_{\beta VVt}$).[28] Just as for Russia's GDP (Figure 15), in regional output, the negative contribution of tight monetary policy to output is largely offset by the positive contribution of relatively low (for the observed key rate) credit (loan) rates. This indirectly[29] indicates a decrease in the efficiency of monetary policy after 2022 under conditions of fiscal dominance.

[28] The chosen method of calculating the regional series of government spending does not allow for explicit processing of the a priori representation of the public sector's contribution to the dynamics of GRP and real wages in the VVMR, so the model attributes it to latent shocks of private regional demand.

[29] This mechanism of linking the efficiency of monetary policy with fiscal policy is not explicitly present in the model.

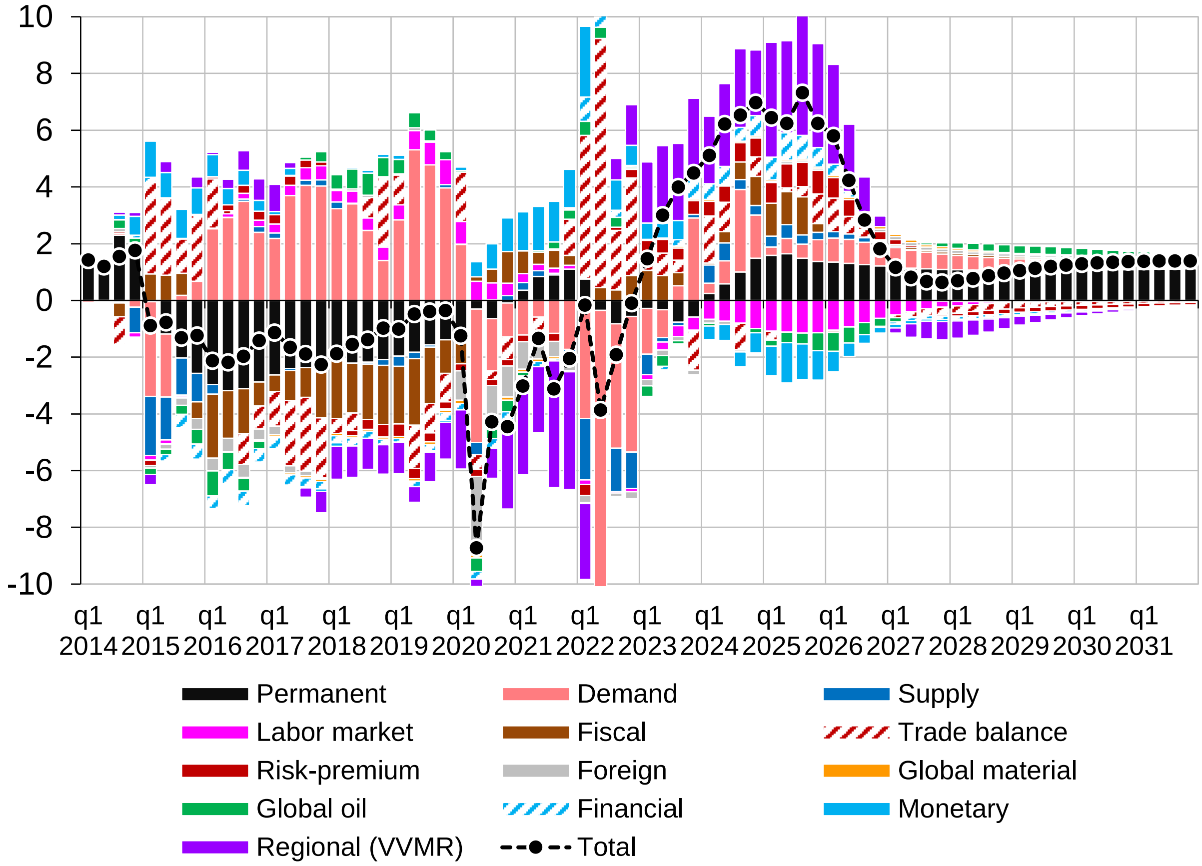

Figure 24. Decomposition of the output of the Volga-Vyatka MR, in % deviation from the deterministic trend (1.35% per year).

Note: actual (2014–2025), forecast (2026–2031).

The labour market of the Volga-Vyatka MR turned out to be more strained (in terms of the gap between supply and demand) compared to Russia as a whole (fig. 16). The model is able to account for this to the extent that this tension is transformed into higher real wages. The main explanation for the dynamics of regional and nationwide wages is related to shocks that create an inverse correlation between output and real wages: shocks of monopolistic mark-up on wages and labour force shocks. Shocks of monopolistic mark-up on wages account for the growth in real wages that does not follow from the behaviour laws of households based on utility maximisation. These shocks in the forecast make a significant contribution to the recession that may follow after the factors that created overheating cease to operate.

Stronger growth in real wages and output does not have a noticeable effect on regional inflation because, firstly, tradable goods distributed in the region are produced not only in this region but also in other regions; secondly, the high wages of the region's households are spent not only in this region but also in other regions. That is, two spill-over effects (through the goods market and the labour market) help explain the weak transmission of the high output and wage gap into regional inflation, in other words, the small slope of the regional Phillips curve.

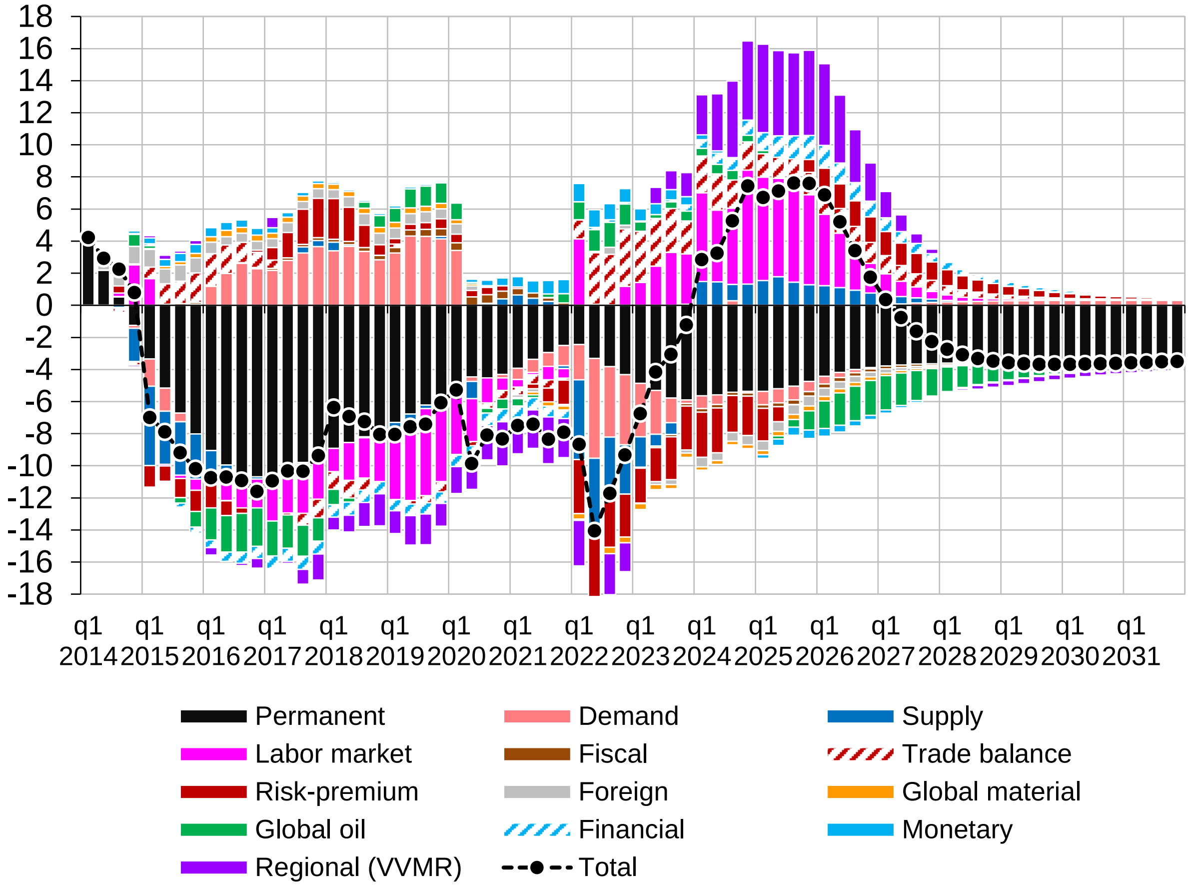

Figure 25. Decomposition of real wages of the Volga-Vyatka MR, in % deviation from the deterministic trend (2.45% per year).

Note: actual (2014–2025), forecast (2026–2031).

Contribution of heterogeneity shocks to the dynamics of variables

We will calculate the decomposition of the variances of key variables into groups of shocks for the estimated parameters of the model, as well as the variances of structural shocks. This time, we will focus on shocks creating heterogeneity: regional, sectoral and agent. We will extend the remaining groups of shocks to internal general, permanent, foreign exchange market, external and monetary policy shocks.

Table 9. Contribution of shock groups to the variance of Russia's inflation, %

Quarters	1	4	8	12	16	20
Internal general	78.4	77.9	75.8	74.5	74.0	73.7
Permanent	0.4	0.3	0.3	0.3	0.3	0.3
Foreign exchange market	12.4	13.0	14.5	15.3	15.8	15.9
External	4.5	4.9	5.6	6.0	6.2	6.3
Monetary policy	0.7	0.6	0.6	0.6	0.6	0.6
Regional	1.7	1.9	1.9	1.9	1.9	1.9
Sectoral	1.9	1.3	1.2	1.2	1.2	1.2
Agent (household)	0.0	0.0	0.0	0.0	0.0	0.0
Sum total	100.0	100.0	100.0	100.0	100.0	100.0
Total heterogeneity shocks	3.7	3.2	3.2	3.1	3.1	3.1

Source: authors' calculations.

The main contribution to inflation dynamics is made by domestic common shocks from 78% for the 1st quarter to 73% in the 20th quarter. Heterogeneity shocks, like other groups of shocks, have an insignificant impact on headline nationwide inflation. In our opinion, the contribution of domestic factors in this calculation is exaggerated. First, the model was evaluated to better describe the dynamics at the end of the range, which led to the underestimation of the exchange rate pass-through effect into prices at the beginning of the range; second, additional domestic shocks were needed to explain the dynamics of real variables around the agreed trends. The contribution of monetary policy shocks to inflation is, first, somewhat underestimated due to the fact that the strength of monetary policy at the beginning of the range is somewhat underestimated (price rigidity is overestimated to better match the business cycle dynamics after 2020); second, it does not fully reflect the contribution of stabilising monetary policy to the business cycle, as it does not take into account the predictable dynamics of the key rate; third, it is underestimated for the period 2022–2025 (see, for example, Fig. 14 for illustration).

Table 10 shows the calculation of the contributions of structural shock groups to the variance of regional inflation.

Table 10. Contribution of shock groups to the variance of inflation in the Volga-Vyatka MR, %

Quarters	1	4	8	12	16	20
Internal general	76.8	76.8	74.7	73.4	72.9	72.6
Permanent	0.4	0.4	0.3	0.3	0.4	0.4
Foreign exchange market	12.3	13.0	14.5	15.4	15.8	16.0
External	4.5	4.9	5.6	6.0	6.2	6.3
Monetary policy	0.7	0.6	0.6	0.6	0.6	0.6
Regional	3.6	3.0	3.0	3.0	3.0	3.0
Sectoral	1.8	1.3	1.2	1.1	1.1	1.1
Agent (household)	0.0	0.0	0.0	0.0	0.0	0.0
Sum total	100.0	100.0	100.0	100.0	100.0	100.0
Total heterogeneity shocks	5.5	4.3	4.2	4.2	4.2	4.2

Source: authors' calculations.

The contribution of regional shocks has increased slightly but still remains at a fairly low level compared to overall shocks that once again confirms our previous reasoning that regional inflation is mostly determined by common factors.

Tables 11 and 12 present the contributions of shock groups to the variance of Russia's GDP and the GRP of the Volga-Vyatka MR, respectively. From the output perspective, foreign exchange market shocks start to play a significant role: the situation with exports, imports and the exchange rate. The role of heterogeneity shocks is relatively small and is around 4%.

Table 11. Contribution of shock groups to the variance of Russia's GDP, %

Quarters	1	4	8	12	16	20
Internal general	42.7	47.4	46.1	44.0	42.0	40.1
Permanent	4.9	8.5	13.3	17.4	21.0	24.2
Foreign exchange market	46.1	37.7	34.6	32.8	31.5	30.2
External	1.1	1.1	1.0	1.0	1.1	1.1
Monetary policy	1.1	1.0	1.0	0.9	0.9	0.9

Regional	3.5	3.6	3.3	3.1	2.9	2.8
Sectoral	0.5	0.6	0.6	0.6	0.6	0.6
Agent (household)	0.2	0.1	0.1	0.1	0.1	0.1
Sum total	100.0	100.0	100.0	100.0	100.0	100.0
Total heterogeneity shocks	4.2	4.3	4.0	3.8	3.6	3.5

Source: authors' calculations.

In the regional output, the contribution of heterogeneity shocks (primarily regional) is higher: 9-10%. At the same time, in 2023–2025, the role of regional shocks in explaining the overheating of the Volga-Vyatka MR economy is very significant. All this may indicate that in 2022, prerequisites arose for increasing the role of heterogeneity in explaining regional and aggregated variable dynamics. However, the presented model does not fully capture this contribution. The main slice of heterogeneity significant for the study does not run along the lines of competition, pricing and the presence of foreign competition, but rather by the degree of connection with the fiscal block, which is not disclosed in the model. Developing the model in this direction is a promising area for its further improvement.

Table 12. Contribution of shock groups to the variance of GRP of the Volga-Vyatka MR, %

Quarters	1	4	8	12	16	20
Internal general	39.4	43.0	41.8	40.0	38.2	36.7
Permanent	4.7	7.9	12.5	16.4	19.9	23.0
Foreign exchange market	44.7	35.8	33.1	31.5	30.2	29.1
External	1.0	1.0	1.0	1.0	1.0	1.0
Monetary policy	1.0	1.0	0.9	0.9	0.8	0.8
Regional	8.5	10.7	10.0	9.6	9.2	8.8
Sectoral	0.4	0.5	0.6	0.6	0.5	0.5
Agent (household)	0.2	0.1	0.1	0.1	0.1	0.1
Sum total	100.0	100.0	100.0	100.0	100.0	100.0
Total heterogeneity shocks	9.1	11.3	10.7	10.2	9.8	9.4

Source: authors' calculations.

Finally, let us consider the role of shock groups in explaining the variance of the key rate during the period under review.

The contribution of heterogeneity shocks to explaining the variance of the key rate is also small (about 2.5%). Permanent shocks do not usually make a significant contribution to the dynamics of the rate, although their contribution became significantly higher in the period after 2022.

Appendix 4 provides decompositions of real wages in Russia and the Volga-Vyatka MR, as well as the real foreign currency exchange rate.

Table 13. Contribution of shock groups to the variance of the key rate, %

Quarters	1	4	8	12	16	20
Internal general	49.0	65.8	62.6	59.7	58.3	57.8
Permanent	0.2	0.3	0.3	0.3	0.3	0.3
Foreign exchange market	8.4	14.0	18.3	20.8	21.9	22.4

External	3.1	5.5	7.4	8.5	9.0	9.2
Monetary policy	37.5	12.1	8.9	8.2	7.9	7.8
Regional	1.2	1.8	2.0	2.0	2.0	2.0
Sectoral	0.7	0.6	0.5	0.5	0.5	0.5
Agent (household)	0.0	0.0	0.0	0.0	0.0	0.0
Sum total	100.0	100.0	100.0	100.0	100.0	100.0
Total heterogeneity shocks	1.8	2.4	2.5	2.5	2.5	2.4

Source: authors' calculations.

Conclusion

This work is devoted to the development of a KANK-type DSGE model with three types of heterogeneity: agent (households), regional and sectoral. The main contribution of this work is seen in practical terms: the developed model can be applied in several modes, depending on the type of heterogeneity that is the focus of the researcher. We have shown the results of the model evaluation for a regional focus: when regional macroeconomic analysis is required to support monetary policy. We have made this practical case the main one since the model operates in a two-region mode – the focus region (in the study, the Volga-Vyatka MR) and the rest of Russia. We evaluated the model using the Bayesian method and conducted a series of model adequacy checks: we calculated the key elasticities and sensitivities of the model when certain shocks occur; we analysed the main impulse response functions; demonstrated the decomposition of variables into shocks for historical and forecast horizons; performed in-sample and out-of-sample forecasting exercises; and decomposed the variance of variables into shocks. All these tests confirm the adequacy of the model and its applicability for medium-term forecasting when developing monetary policy decisions.

The main drawbacks of the model lie in the excessive rigidity of the structure of deterministic long-term growth. The assumption of a single deterministic trend, although it gives the analysis theoretical consistency, reduces the predictive properties of the model and also reduces the model's ability to explain macroeconomic dynamics at the beginning of the historical range (2014–2019).

We performed several useful exercises in advanced analytics: we calculated the dynamics of the trend (permanent) output and real wages, and also showed how the natural (long-term) interest rate is formed. We demonstrated the model's capabilities in conducting regional macroeconomic analysis. Finally, we calculated the contribution of heterogeneity shocks to the variance of inflation, output and interest rates for the period 2014–2023. On the one hand, the results suggest that the role of these shocks is not very significant, but starting from 2022, their role is growing, and the use of the developed model to account for various aspects of heterogeneity in the development of monetary policy may be useful. To reflect better the contribution of heterogeneity to macroeconomic dynamics after 2022, it makes sense to differentiate industries according to the intensity of their interaction with public demand, including through its investment component.

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Appendices

Appendix 1. Data and forecast

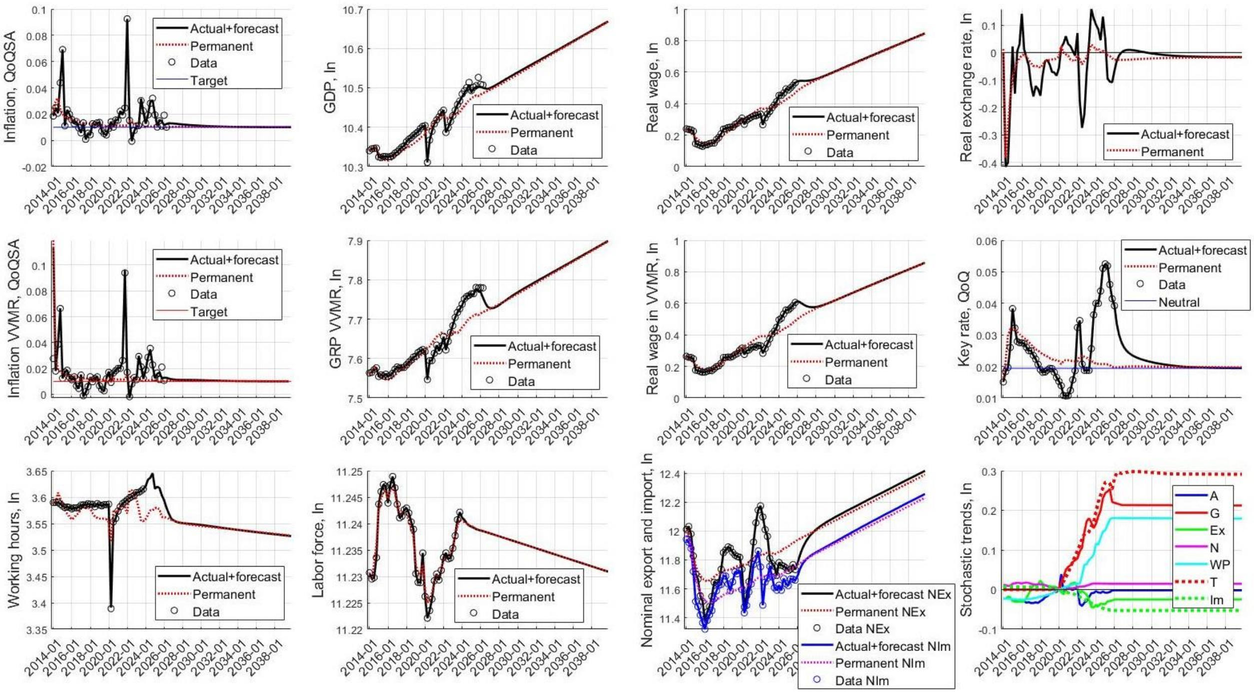

Figure A1. Data and forecast
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Appendix 2. Impulse response functions

IRF on other foreign shocks

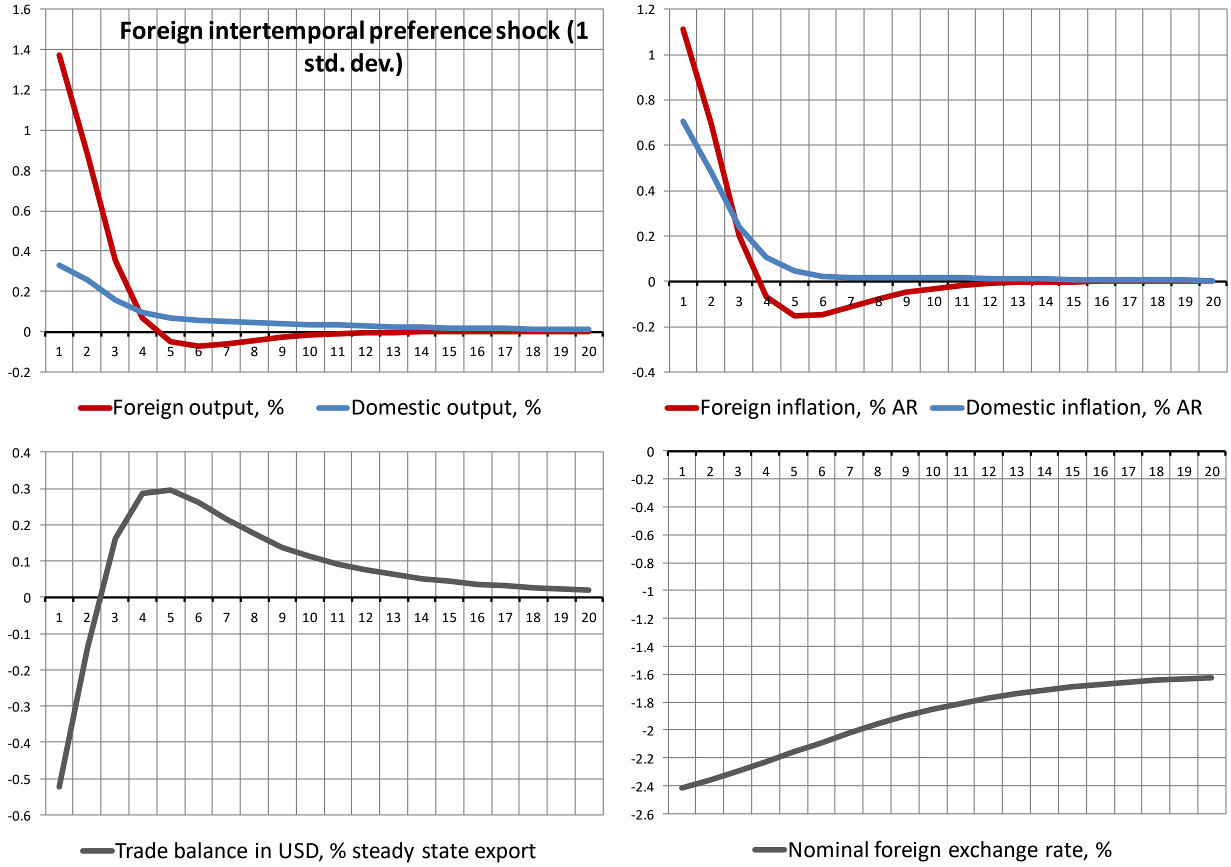

Figure A1. IRF on foreign intertemporal preference shock $\varepsilon_{\beta^* t}$ (1 std. dev.)

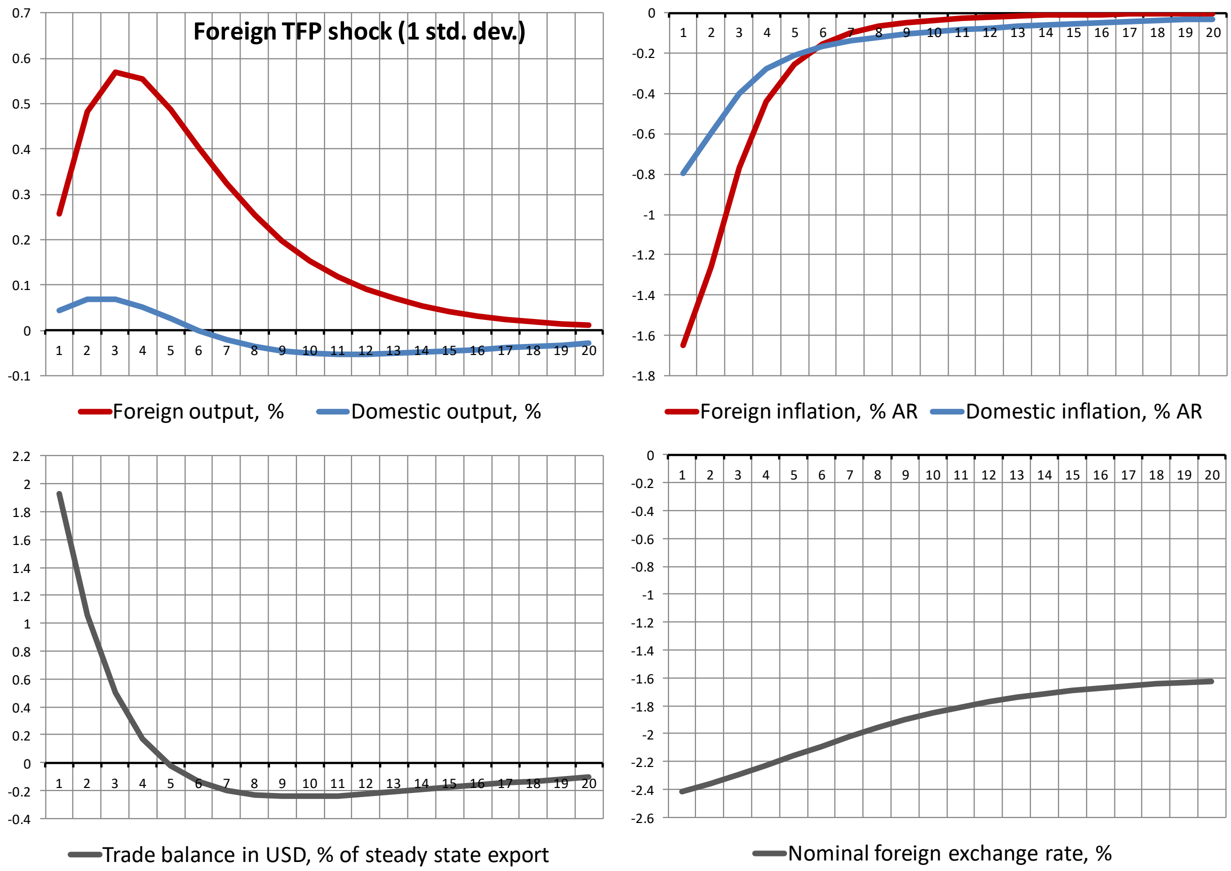

Figure A2. IRF on foreign productivity shock ε_{A^*t} (1 std. dev.)

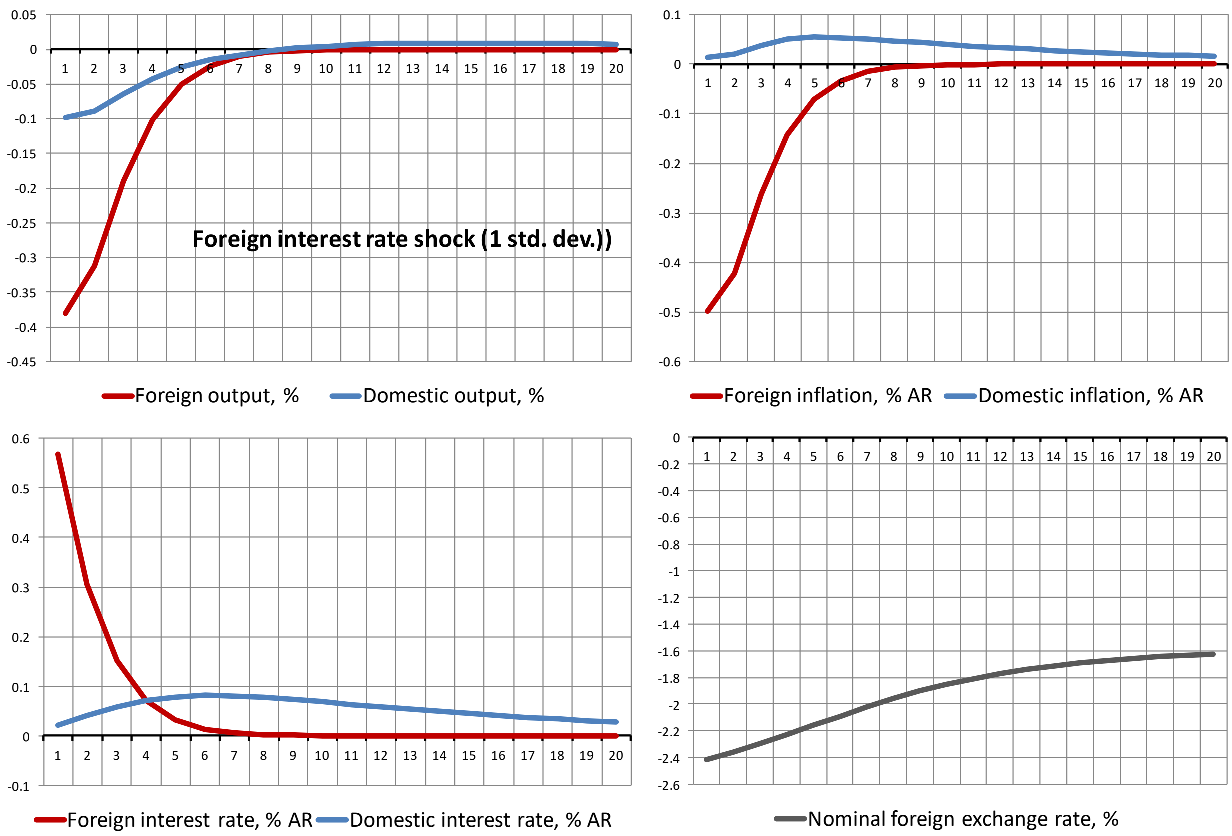

Figure A3. IRF on foreign interest rate shock ε_{i^*t} (1 std. dev.)

IRF on other regional shocks

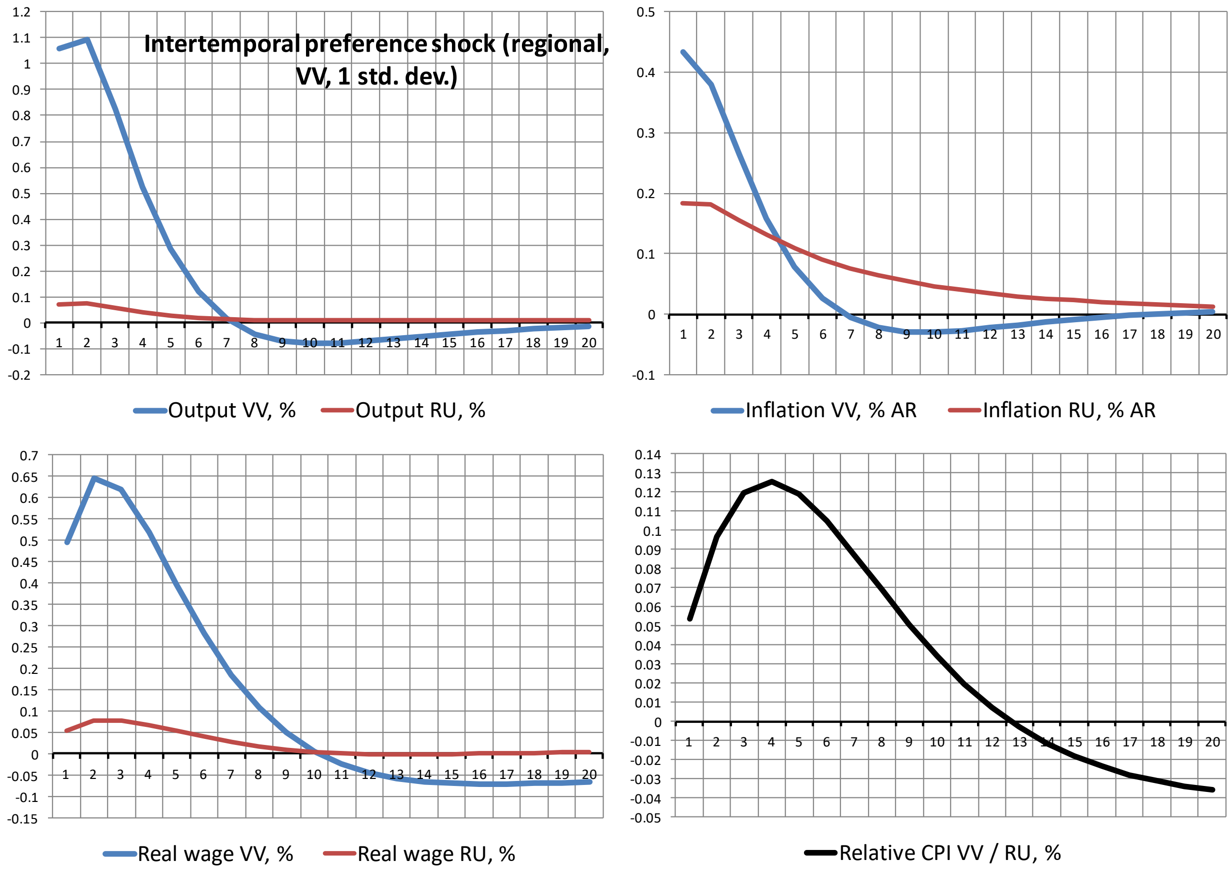

Figure A4. IRF on intertemporal preference shock $\varepsilon_{\beta VVt}$ (regional, VV, 1 std. dev.)

VV = VVMB; RU = Other regions of Russia

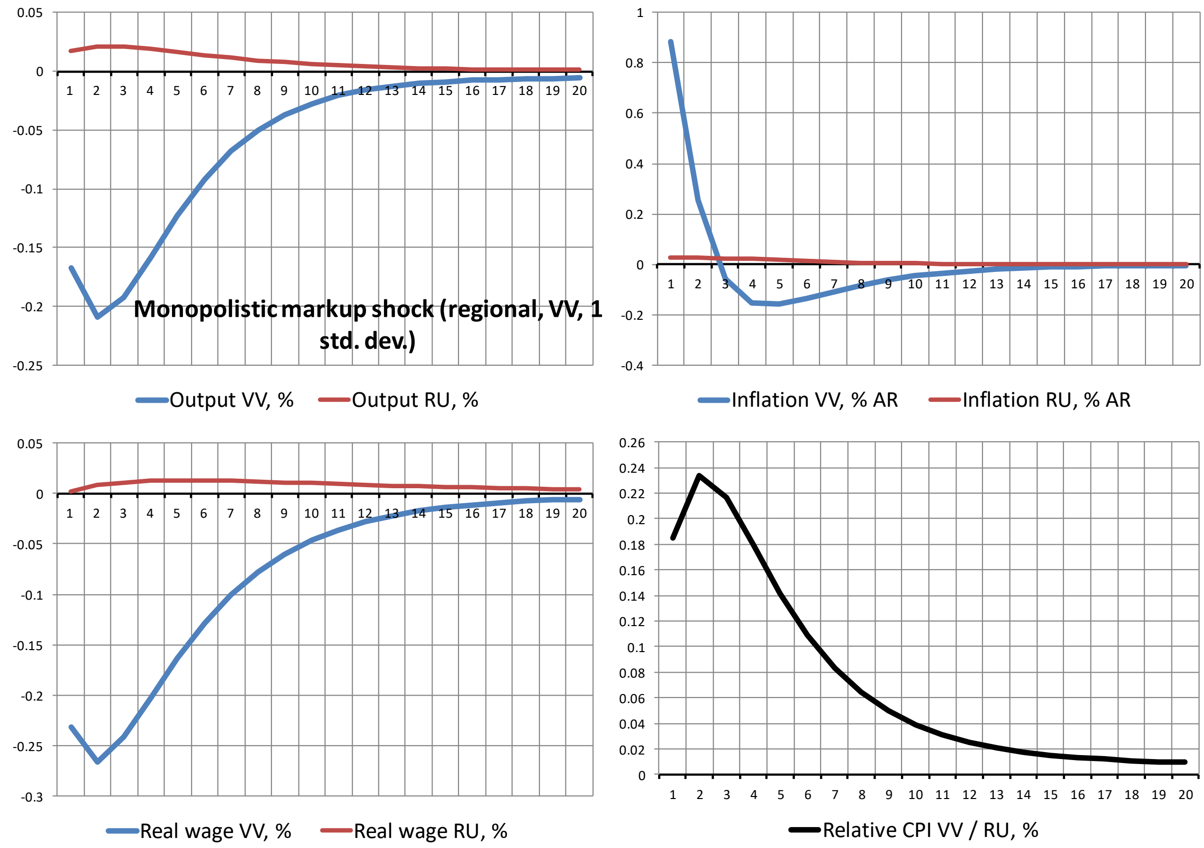

Figure A5. IRF on monopolistic mark-up shock $\varepsilon_{\mu VVt}$ (regional, VV, 1 std. dev.)

VV = VVMB; RU = Other regions of Russia

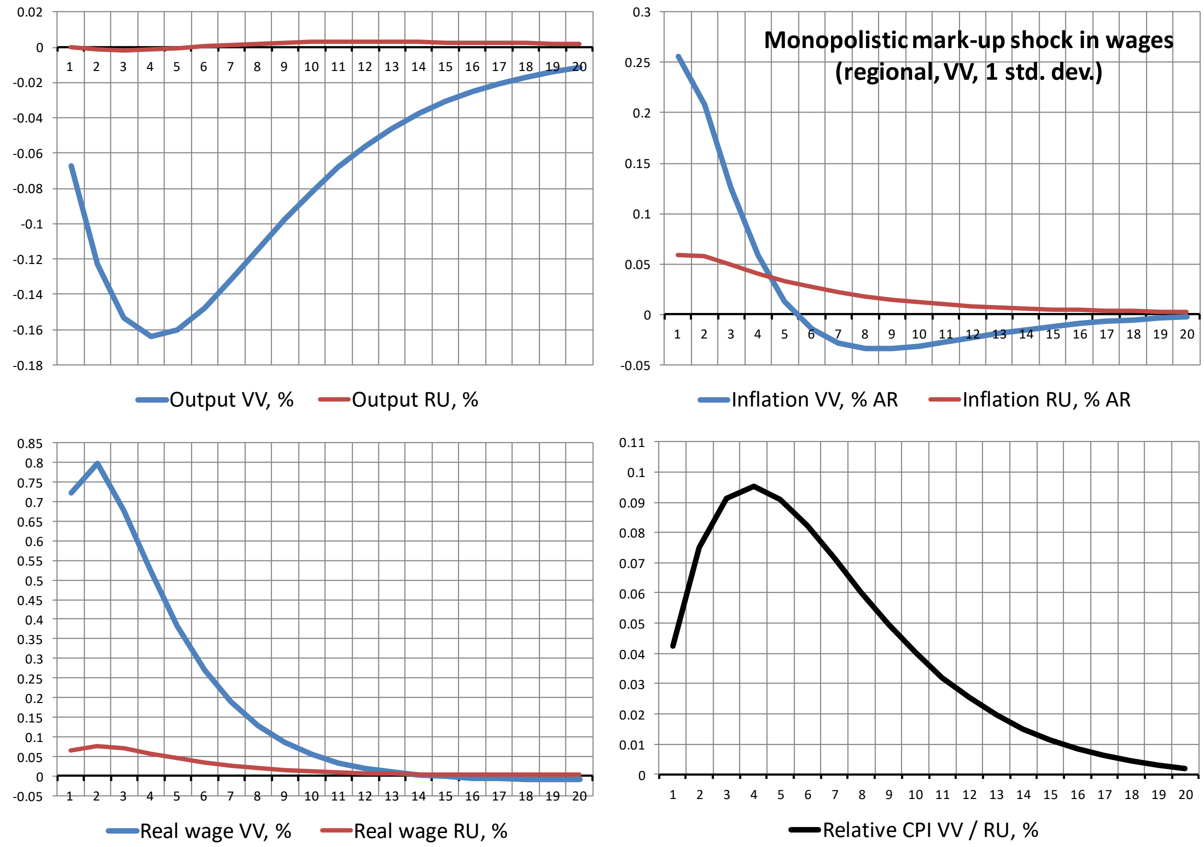

Figure A6. IRF on monopolistic mark-up shock in wages ε_{WVVt} (regional, VV, 1 std. dev.)

VV = VVMB; RU = Other regions of Russia

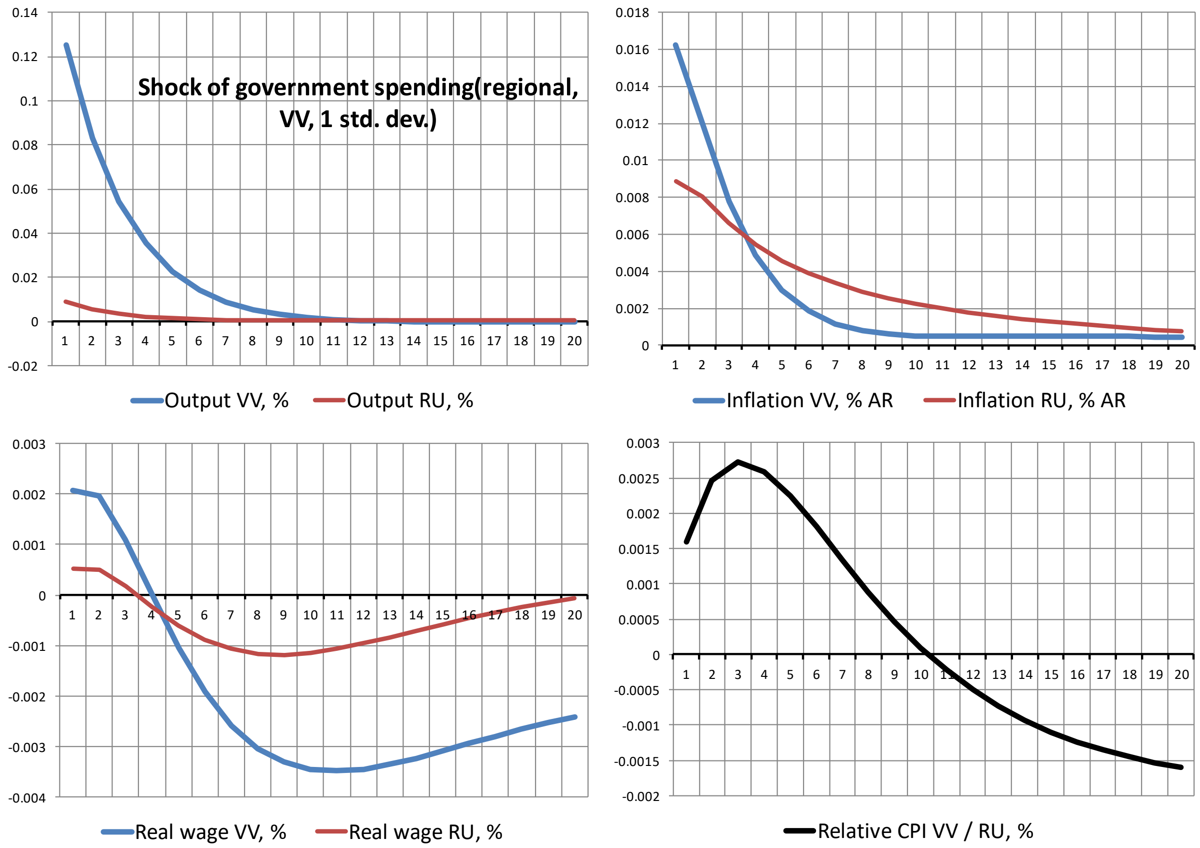

Figure A7. IRF on government spending shock ε_{GVVt} (regional, VV, 1 std. dev.)

VV = VVMB; RU = Other regions of Russia

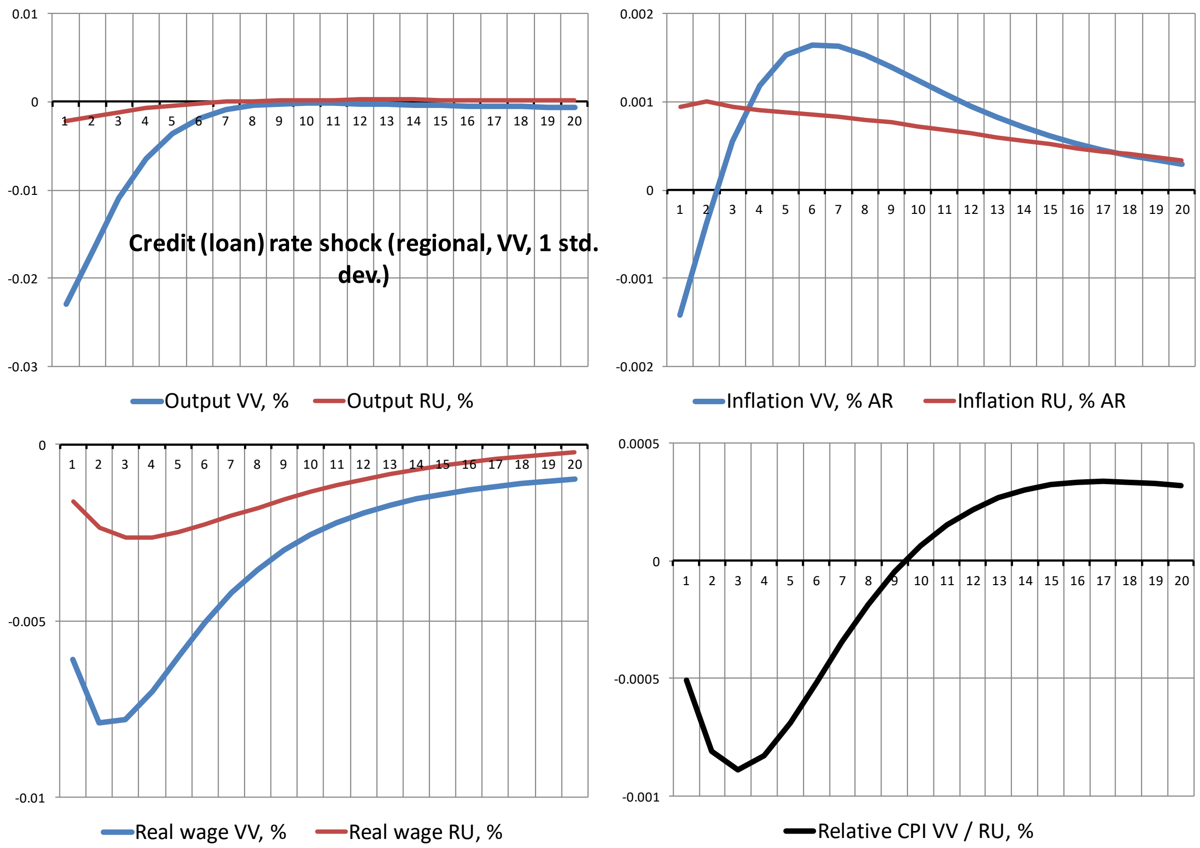

Figure A8. IRF on credit rate shock ε_{iVVt} (regional, VV, 1 std. dev.)

VV = VVMB; RU = Other regions of Russia

IRF on other common shocks

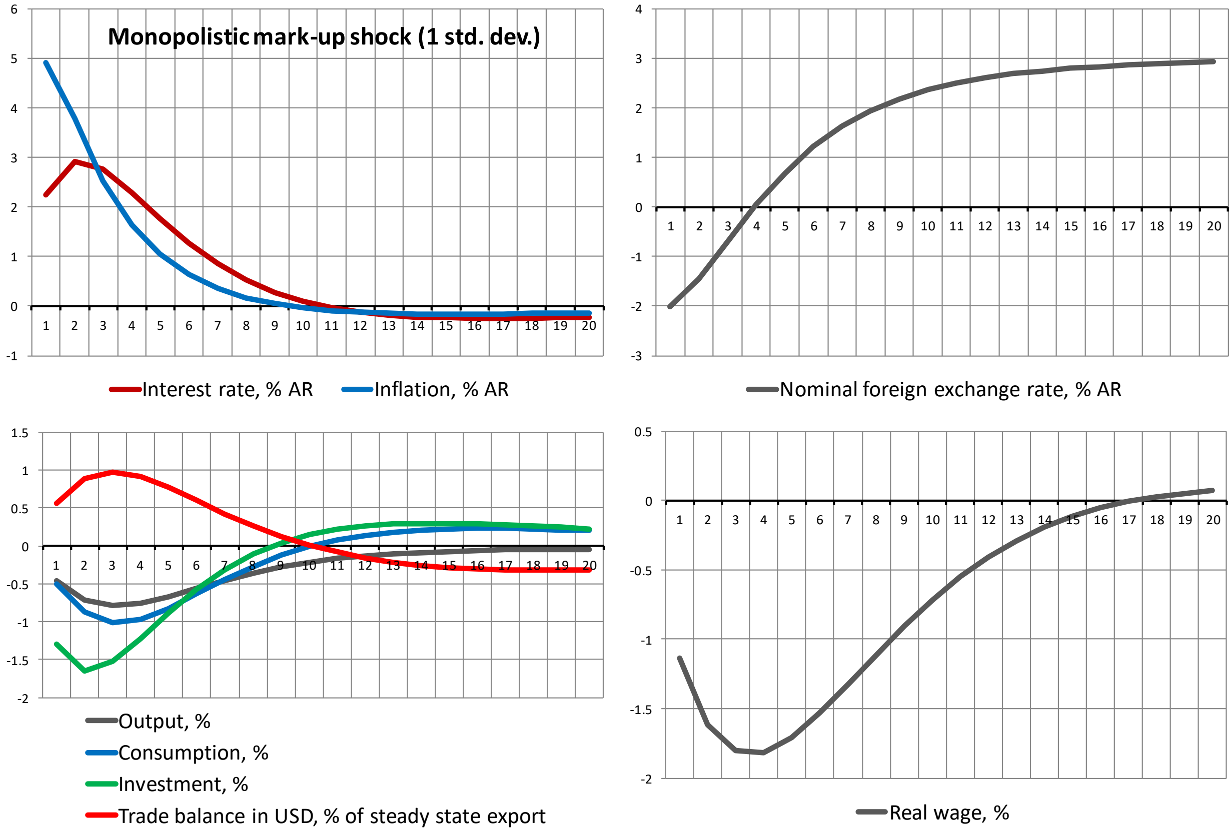

Figure A9. IRF on monopolistic mark-up shock $\varepsilon_{\mu t}$ (1 std. dev.)

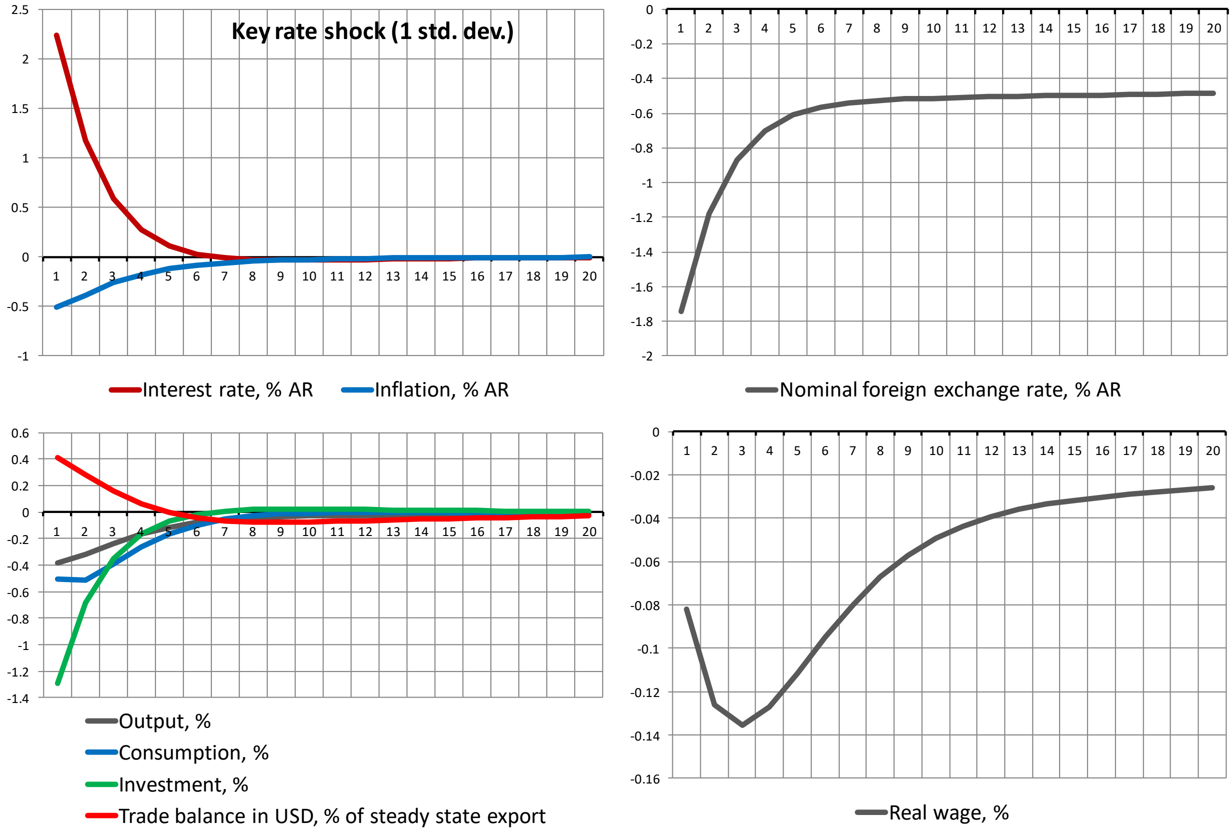

Figure A10. IRF on monetary policy (key rate) shock ε_{ikt} (1 std. dev.)

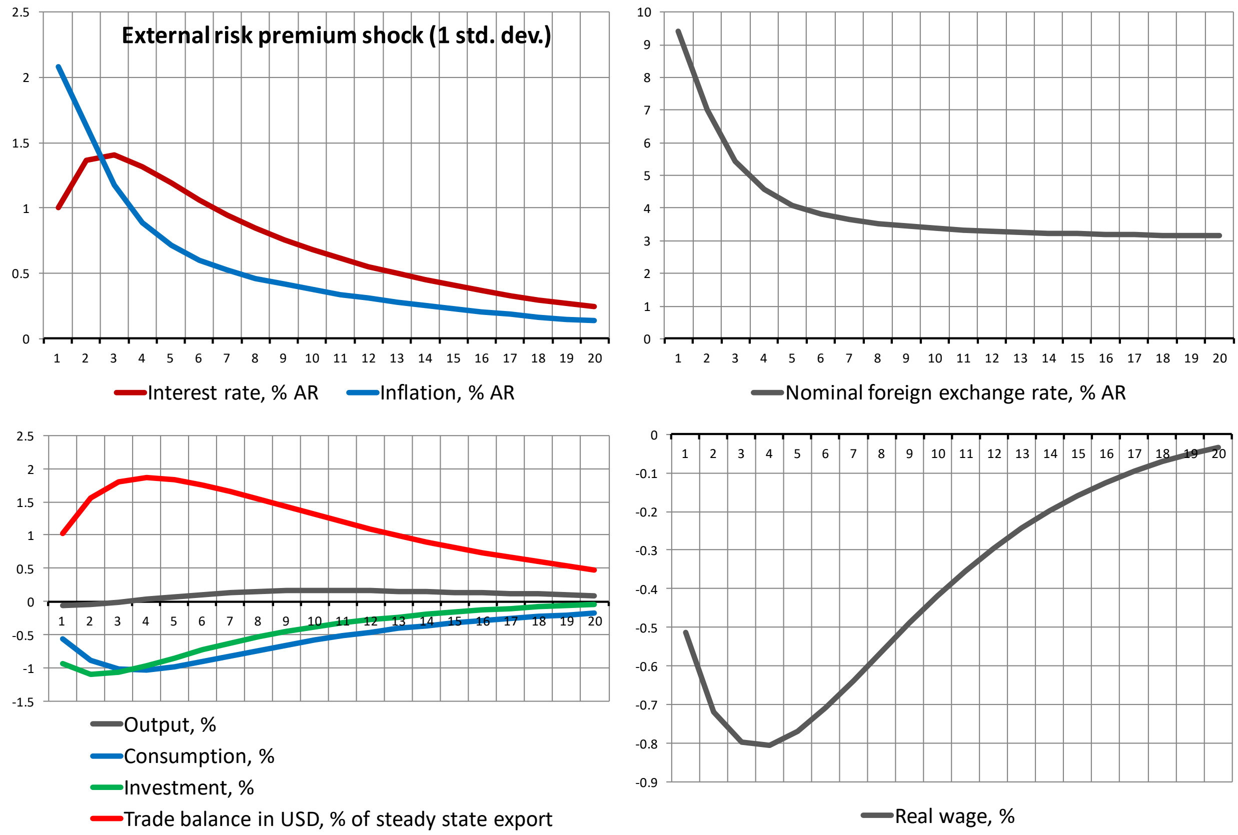

Figure A11. IRF on external risk premium shock ε_{rpt} (1 std. dev.)

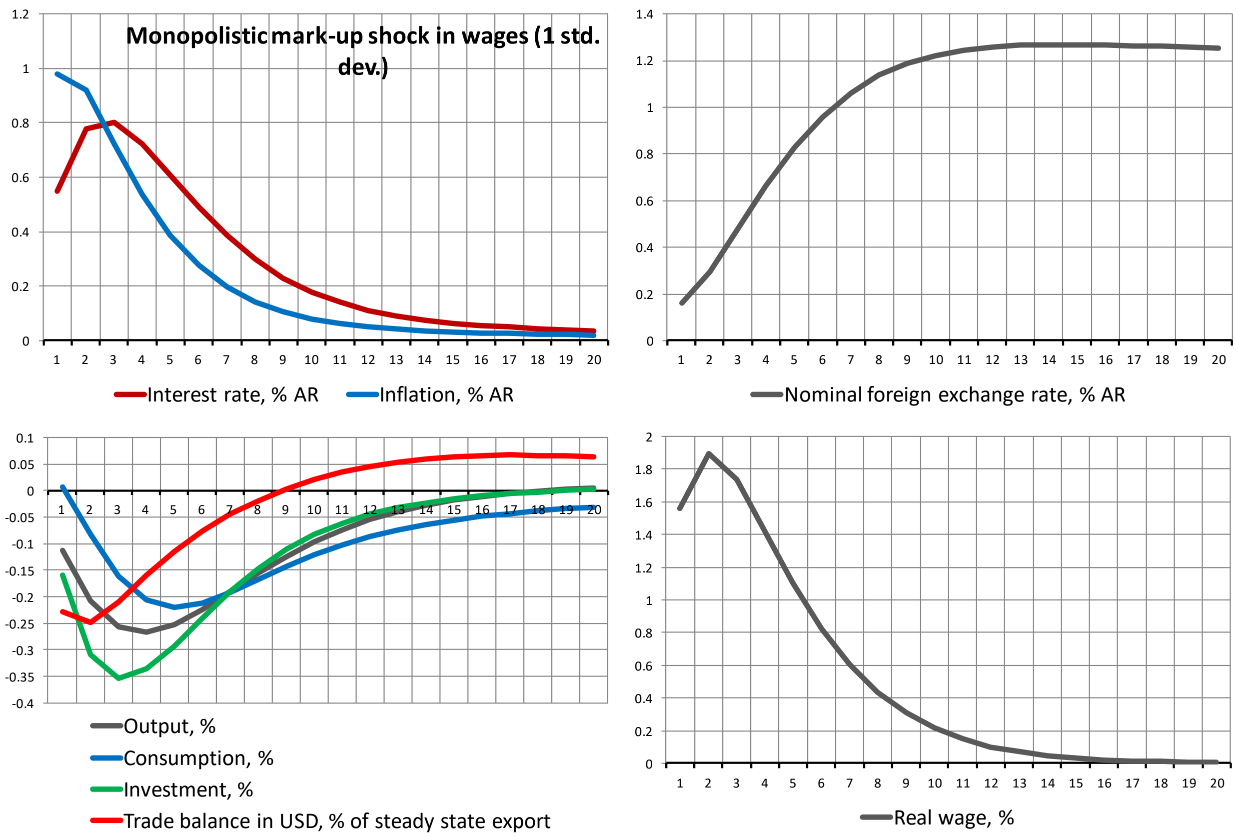

Figure A12. IRF on monopolistic mark-up shock in wages ε_{Wt} (1 std. dev.)

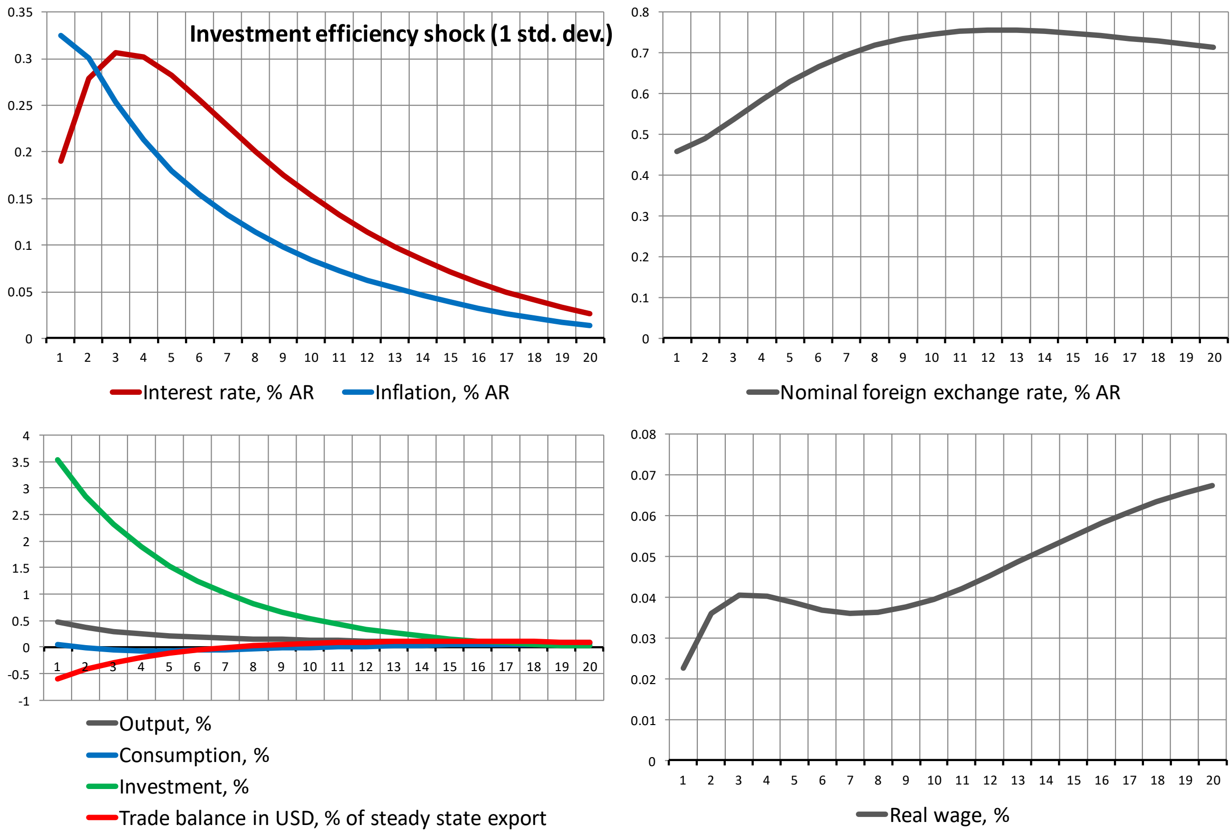

Figure A13. IRF on investment efficiency shock ε_{It} (1 std. dev.)

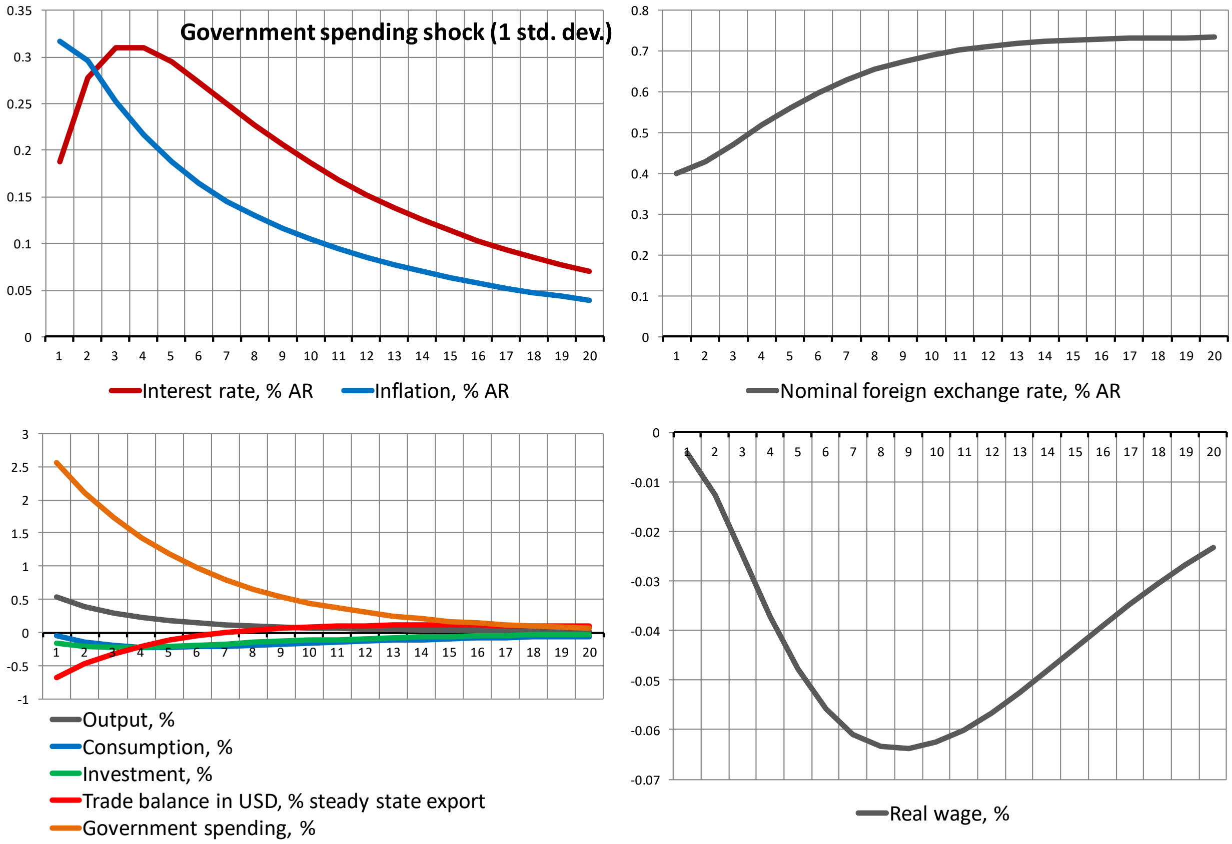

Figure A14. IRF on government spending shock ε_{Gt} (1 std. dev.)

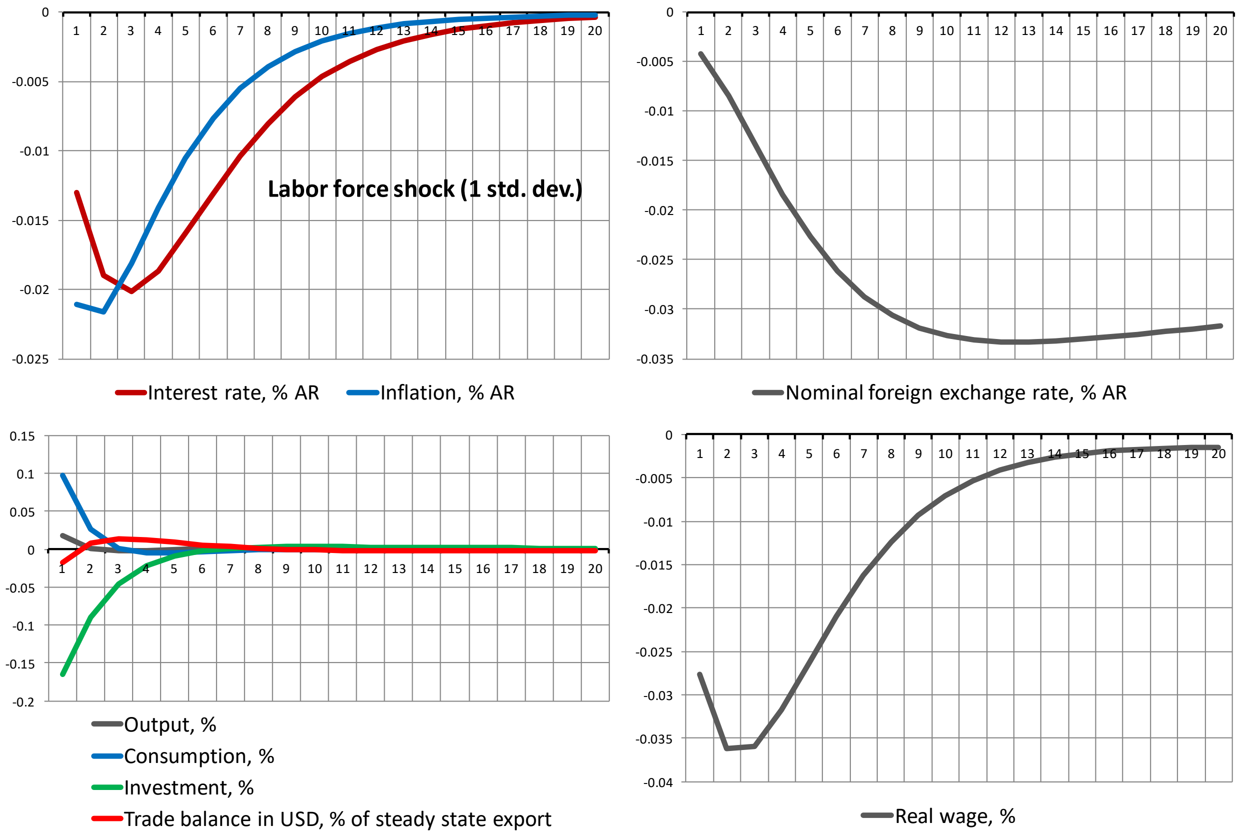

Figure A15. IRF on labour force shock ε_{Nt} (1 std. dev.)

IRF on other permanent shocks

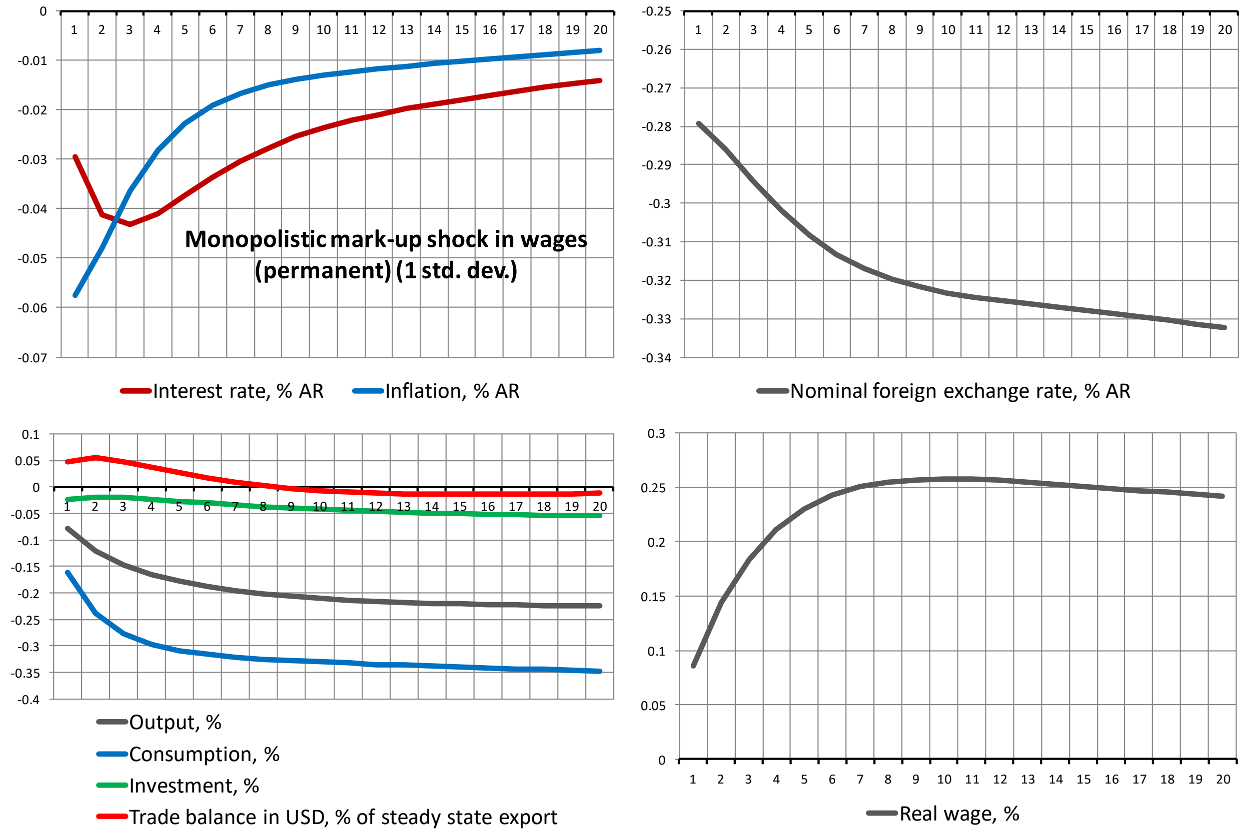

Figure A16. IRF on permanent wages shock υ_{Wt} (1 std. dev.)

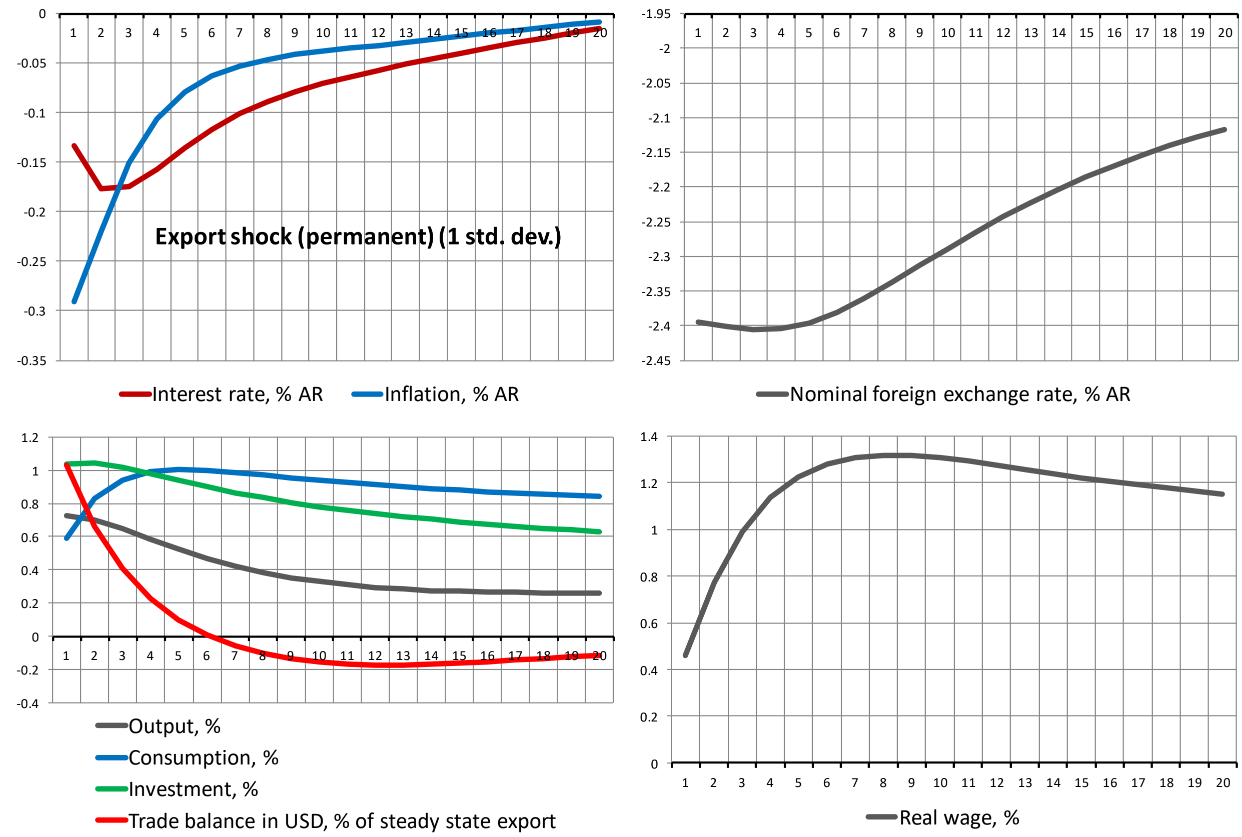

Figure A17. IRF on permanent export shock υ_{Ext} (1 std. dev.)

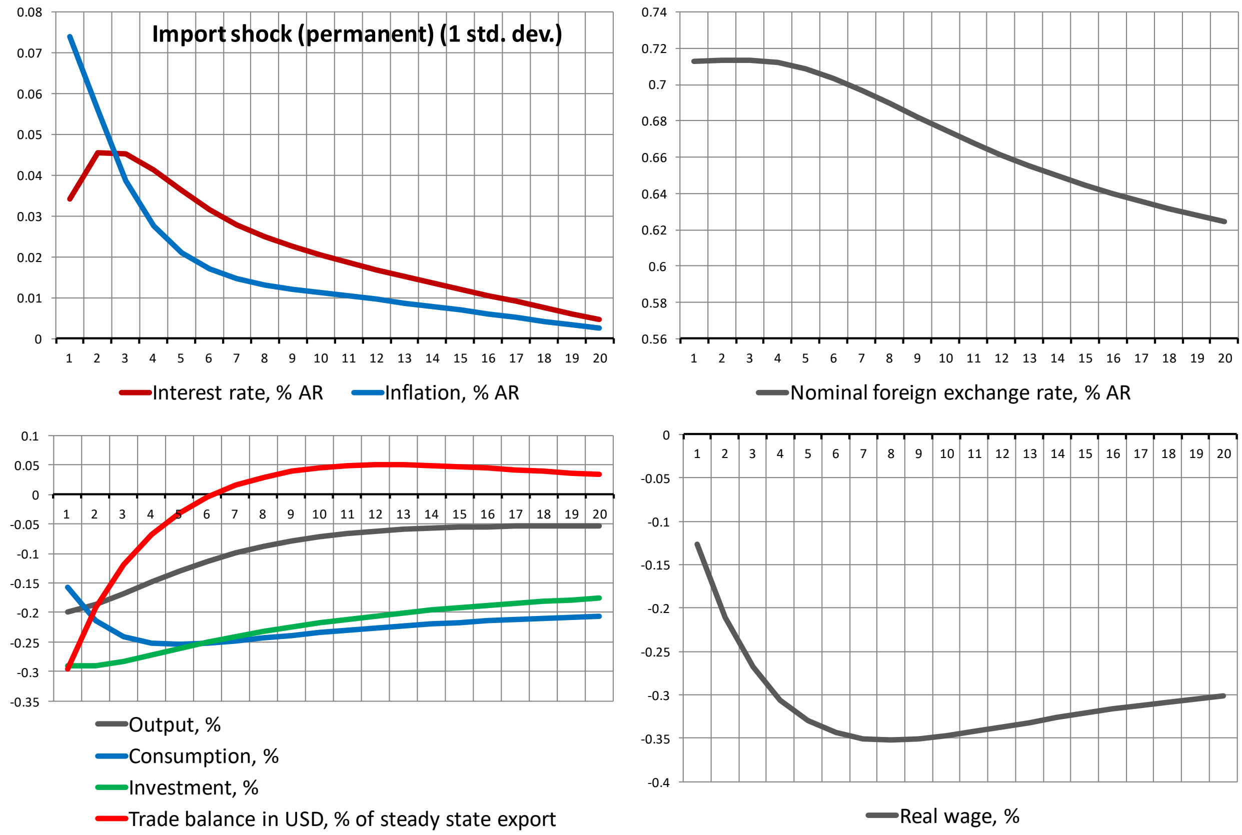

Figure A18. IRF on permanent import shock υ_{IMt} (1 std. dev.)

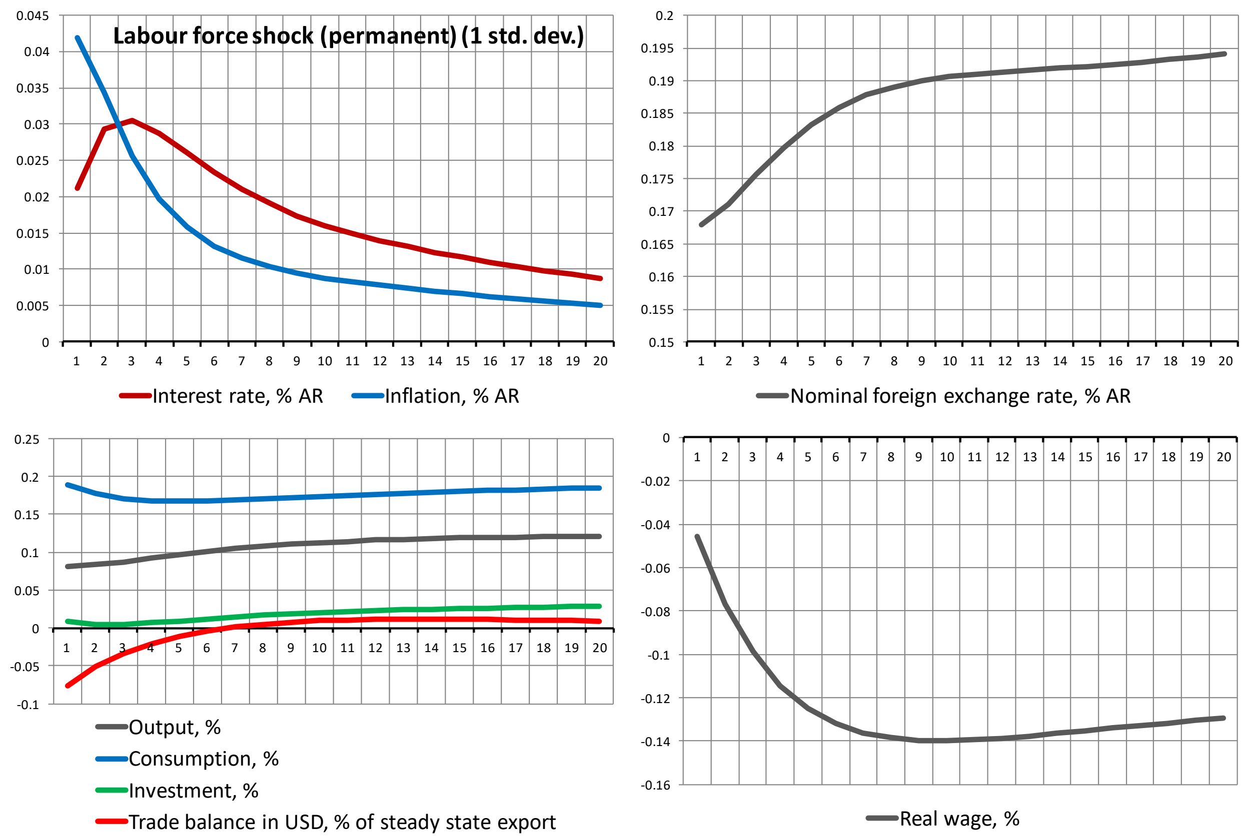

Figure A19. IRF on permanent labour force shock υ_{Nt} (1 std. dev.)

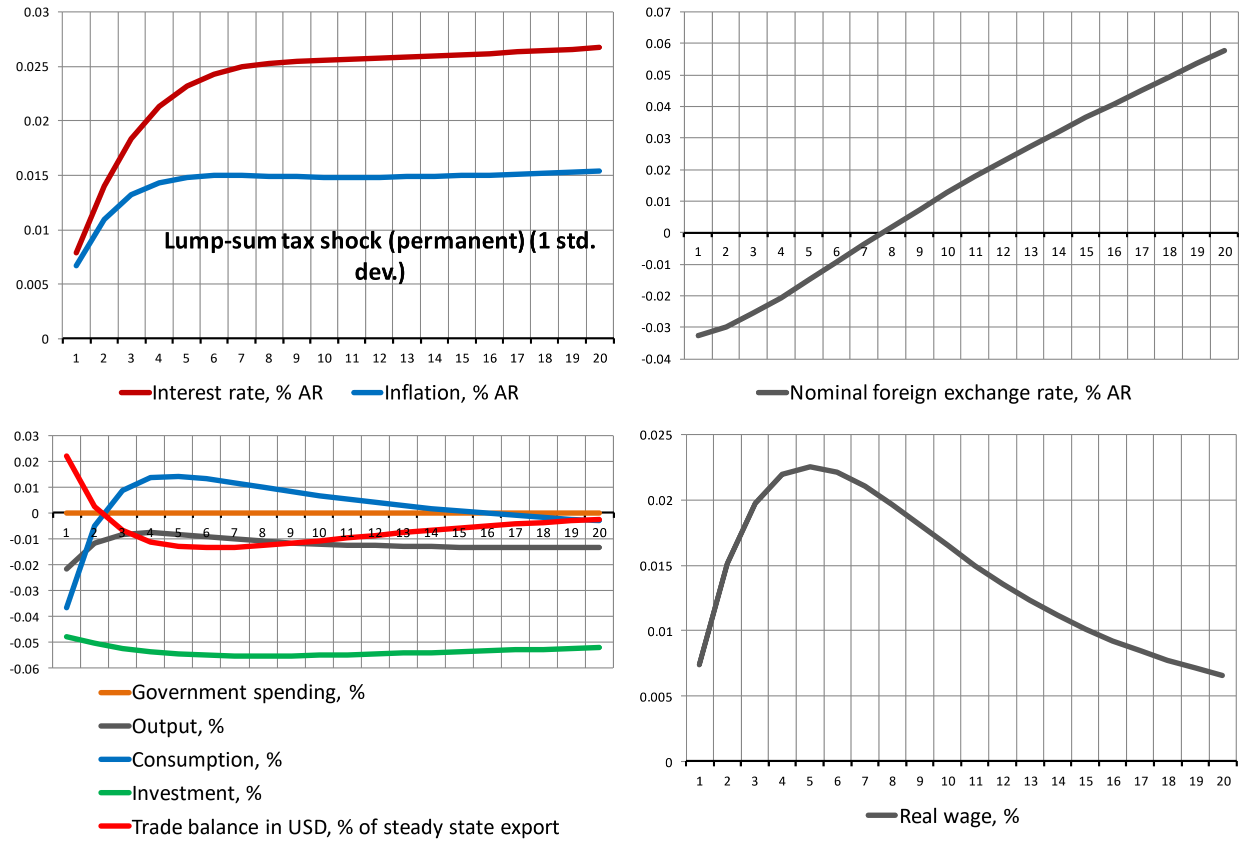

Figure A20. IRF on permanent lump-sum tax shock v_{Tt} (1 std. dev.)

Appendix 3. In- and Out-of-sample forecasting

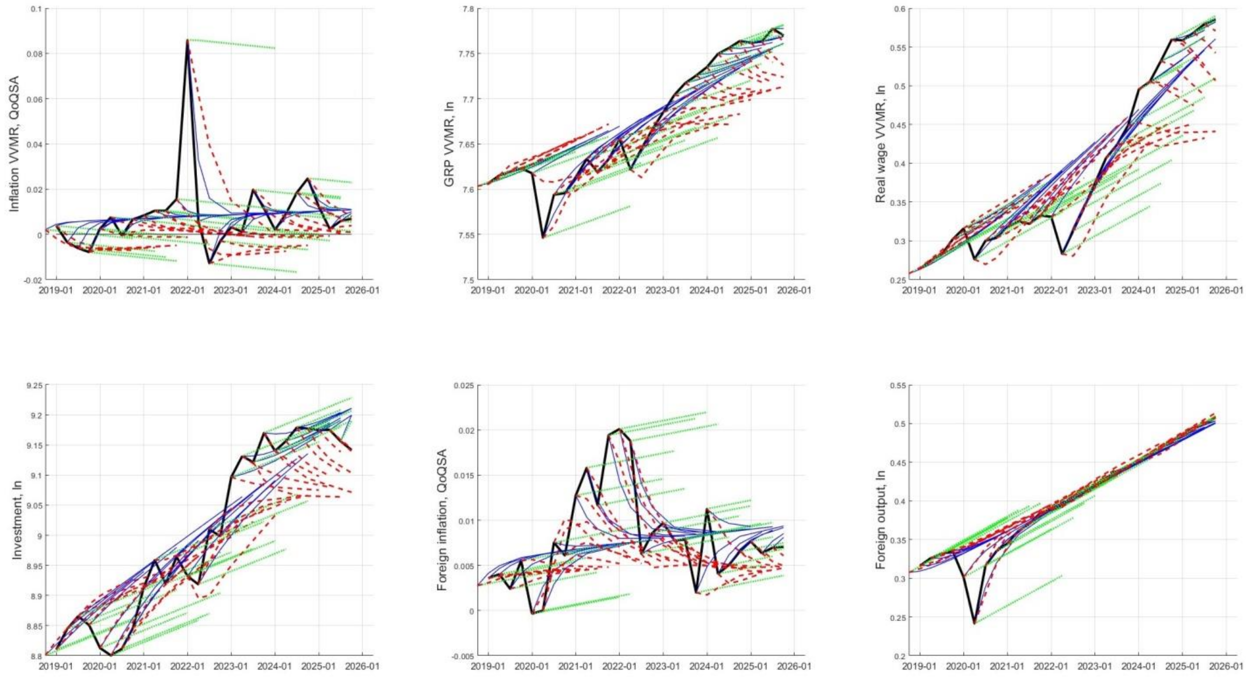

Figure P21. In-sample forecasts based on three models: MARS (red); AR1 (blue); RW (green).
Note: Actual (black)

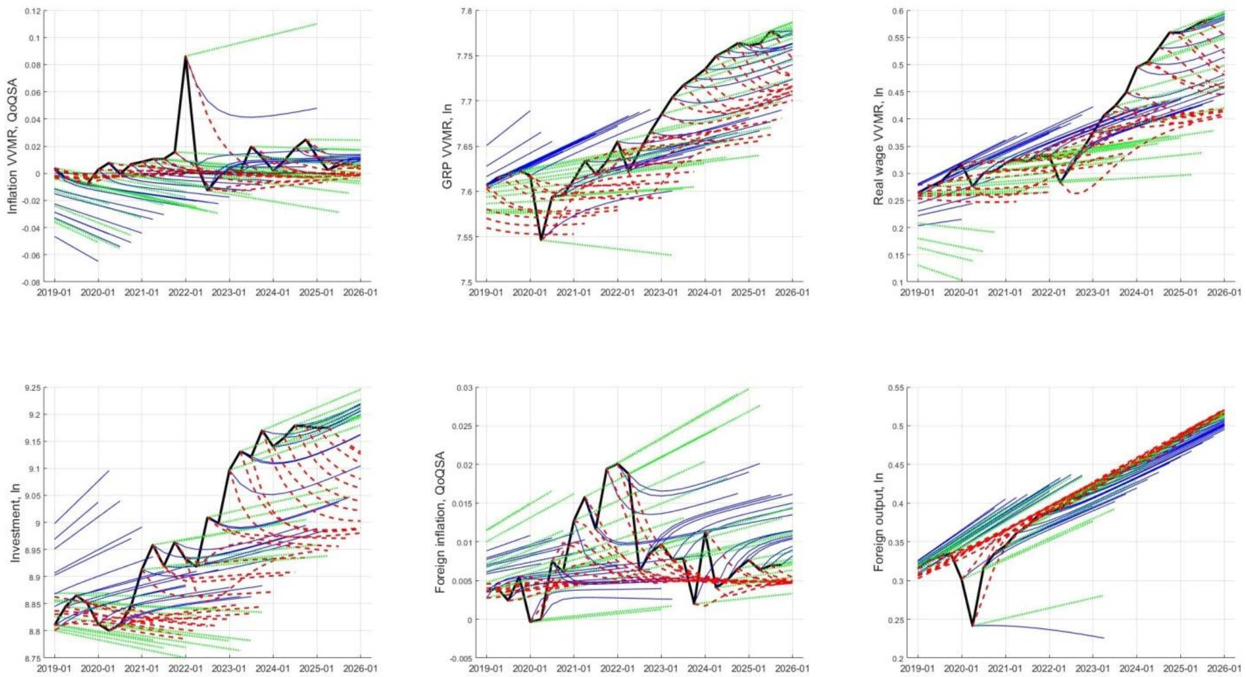

Figure P22. Out-of-sample forecasts based on three models: MARS (red); AR1 (blue); RW (green).
Note: Actual (black)

Appendix 4. Variance decomposition

Table P1. Contribution of shock groups to the variance of real wages in Russia, %

Quarters	1	4	8	12	16	20
Internal general	75.9	65.7	54.6	47.5	42.9	39.5
Permanent	3.9	9.3	17.6	25.1	31.4	36.6
Foreign exchange market	11.5	16.6	19.4	19.0	17.8	16.6
External	2.0	3.3	4.4	4.7	4.5	4.2
Monetary policy	0.1	0.1	0.1	0.1	0.1	0.1
Regional	5.6	3.9	2.7	2.2	2.0	1.8
Sectoral	0.9	1.1	1.2	1.3	1.3	1.2
Agent (household)	0.0	0.0	0.0	0.0	0.0	0.0
Sum total	100.0	100.0	100.0	100.0	100.0	100.0
Total heterogeneity shocks	6.6	4.9	3.9	3.5	3.3	3.1

Source: authors' calculations.

Table P2. Contribution of shock groups to the variance of real wages in the Volga-Vyatka MR, %

Quarters	1	4	8	12	16	20
Internal general	69.7	62.4	53.0	46.5	42.1	38.8
Permanent	3.7	8.9	17.0	24.5	30.8	35.9
Foreign exchange market	10.8	15.6	18.3	18.0	16.9	15.7
External	1.8	3.0	4.1	4.3	4.1	3.9
Monetary policy	0.1	0.1	0.1	0.1	0.1	0.1
Regional	13.1	9.0	6.3	5.3	4.8	4.4
Sectoral	0.9	1.0	1.2	1.2	1.2	1.2
Agent (household)	0.0	0.0	0.0	0.0	0.0	0.0
Sum total	100.0	100.0	100.0	100.0	100.0	100.0
Total heterogeneity shocks	13.9	10.0	7.5	6.5	6.0	5.6

Source: authors' calculations.

Table P3. Contribution of shock groups to the variance of the real foreign exchange rate, %

Quarters	1	4	8	12	16	20
Internal general	11.0	19.0	21.4	21.6	21.2	20.7
Permanent	5.4	8.7	12.5	15.8	18.7	21.3
Foreign exchange market	75.1	62.7	55.9	52.4	50.1	48.3
External	6.0	7.8	8.6	8.8	8.6	8.4
Monetary policy	2.2	1.4	1.1	1.0	0.9	0.9
Regional	0.2	0.2	0.3	0.3	0.3	0.3
Sectoral	0.1	0.1	0.1	0.1	0.1	0.1
Agent (household)	0.0	0.0	0.0	0.0	0.0	0.0
Sum total	100.0	100.0	100.0	100.0	100.0	100.0
Total heterogeneity shocks	0.3	0.4	0.4	0.5	0.5	0.5

Source: authors' calculations.

Table P4. Contribution of shock groups to the variance of the nominal foreign exchange rate, %

Quarters	1	4	8	12	16	20
Internal general	6.7	9.5	15.5	21.2	25.2	28.1
Permanent	5.4	8.3	10.2	10.8	11.0	11.0
Foreign exchange market	78.5	69.8	59.9	52.9	48.4	45.4
External	6.6	9.8	11.7	12.2	12.4	12.5
Monetary policy	2.4	1.7	1.3	1.1	1.0	0.9
Regional	0.3	0.7	1.2	1.5	1.6	1.7
Sectoral	0.1	0.1	0.2	0.3	0.3	0.4
Agent (household)	0.0	0.0	0.0	0.0	0.0	0.0
Sum total	100.0	100.0	100.0	100.0	100.0	100.0
Total heterogeneity shocks	0.4	0.8	1.4	1.8	2.0	2.1

Source: authors' calculations.