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Measuring the Debt Service Ratio in Russia:

micro-level data approach

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Abstract

A new micro-level database was used to estimate the debt service ratio (DSR) for the private non-banking sector in Russia. This is the first work presenting a loan-based DSR estimate for Russia. The micro-level database contains information on the remaining maturities and lending rates for each loan issued in 2017–2019 by resident banks to the private non-banking sector in Russia. Estimated levels of the DSR were considerably higher than previous results obtained with the assumptions of constant maturity structure and prevailing lending rates. New results revealed that the aggregate assumptions are not sufficiently granular. Utilisation of actual remaining maturity at each estimation point improved the accuracy of DSR estimates by 10 p.p. (from 16% to 26% for 2019 Q4). The loan-level database provides new insight into the composition of the corporate debt servicing burden in Russia: prevalence of domestic currency loans, higher debt servicing cost for debt with shorter remaining maturity, and the sectoral heterogeneity of the DSRs.

JEL-classification: E44, F34, G21

Keywords: DSR, debt servicing burden, micro-level database, credit registry

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1. Introduction

The concept of the debt service ratio (DSR) was introduced in [Drehmann and Juselius \(2012\)](#) as a measure of financial constraints imposed by the indebtedness of the private sector – both households and private non-financial corporations. The direct link between the levels of the DSR and observable economic factors (such as prevailing lending rates, and the amount of borrower's disposable income), makes the DSR a valuable tool for measuring the debt servicing burden at different levels of data aggregation (at firm, sector, or economy-level). In [Drehmann and Juselius \(2012\)](#), the DSR was primarily used to proxy the level of debt servicing burden at an aggregate level for a set of countries¹.

Constructing the DSR at disaggregate level requires loan-level information for debt at each estimation point. A new micro-level database (credit registry) contains information on the amounts of debt outstanding, actual remaining maturities and lending rates for each loan in 2017–2019. The availability of loan-level data makes it possible to estimate the burden of servicing the debt with sufficient granularity and without making the aggregate assumption on constant maturity structure or prevailing lending rates. In line with the original methodology of [Drehmann and Juselius \(2012\)](#), we assume that the debt is repaid gradually over the time of remaining maturity. We depart from the assumption of constant maturity structure and use the actual remaining maturity for each loan at each estimation point. The effect of change in maturity parameter shifts the estimated level of DSR from 16%² to 26% (2019 Q4). New results are closer to the actual debt servicing payments as reported by banks (which includes the amount of interest payments and amortisation of debt).

We construct a quarterly time series (a total of 12 quarters for 2017–2019) of the DSR for Russia at economy level, separate DSRs for 15 aggregate industries, and for 61 disaggregate industries to observe the shifts in different sectors of the real economy. Indebtedness of different sectors can provide alternative outlook, when analysed in the context of debt servicing costs related to the level of current economic activities. Higher levels of existing debt servicing burden influence the availability of new funds and adversely affect the dynamics of capital expenditures (for extended discussion, refer to [Drehmann et al \(2017\)](#)). We decompose the changes in the DSR into the interest rate, maturity, and the stock of debt-related components, and conclude on the association of changes in the levels of DSR with the change in maturity structure.

In this paper, we estimate the DSR for debt issued by resident banks to the private non-banking sector in Russia. We do not account for the corporate bonds and debt issued by the non-resident banks. We leave this point to future research. The paper is structured as follows. Section 2 presents the background of the subject and the mainstream literature

¹ For 27 countries starting from the 1980s, for the detailed list of countries and the constructed time series, refer to [Drehmann and Juselius \(2012\)](#).

² [Bank of Russia Monetary Policy Report \(2019\)](#). The DSR was estimated in accordance with the methodology and assumptions described in [Donets and Ponomarenko \(2015\)](#).

review. Sections 3 and 4 outline the original methodology as presented in [Drehmann and Juselius \(2012\)](#), and introduce its application to a new micro-level database. Section 5 highlights alternative assumptions. In Section 6, main results are displayed and discussed. Section 7 concludes.

2. Literature Review

The comprehensive nature of the DSR is different from the measure of *credit-to-GDP*. The DSR explicitly accounts for the costs of servicing borrowed funds through the inclusion of interests accrued on the amount of debt outstanding, and the amortised amount of debt (assuming the debt is repaid gradually). The credit-to-GDP measures an exposure to borrowed funds, without taking into consideration costs of servicing the debt. For that reason, the DSR provides a more comprehensive assessment of the debt servicing burden³.

[Drehmann and Juselius \(2012\)](#) originally introduce and analyse the applicability of the DSR in the context of economic downturn, business cycles, and financial crises. The context consists from an excessive debt build-up, overextension of private sector, and the related economic constraints for over-indebted sectors. The authors propose the DSR as an accurate early warning signal of an upcoming banking crisis and as a supplementary indicator for accumulating financial vulnerabilities. The DSR was recommended as a useful measure to capture the cost of servicing the debt more comprehensively than the alternative measures (credit-to-GDP). Rising DSR is interpreted as a sign of increasing financial vulnerabilities. In a subsequent work, [Drehmann and Juselius \(2013\)](#) evaluate the plausibility of the DSR as an early warning signal of vulnerability build-up based on the criteria of stability and interpretability. [Juselius and Drehmann \(2015\)](#) analyse the path of debt and investments development and find the negative effects of debt servicing cost on capital expenditure growth in the short and long-run.

[Donets and Ponomarenko \(2015\)](#) construct a time series of the DSR for Russia based on the total aggregate data on corporate and household debt for the period 2001–2017 with quarterly frequency. The levels of DSR were estimated at around 10% for households and the private non-financial corporations for the 2016 Q4 (which is substantially lower compared to the current results estimated with loan-level data). The assumption on the maturity parameter in DSR estimation was made based on the weighted average remaining maturities of corporate loans in different maturity buckets (less than 1 year, from 1 to 3 years, more than 3 years). [Drehmann et al \(2015\)](#) discuss the impact of aggregate assumption (such as using aggregates for the interest rates, and fixed maturity assumption), and the assumption about repayment schemes (instalment loans vs bullet loans) on the accuracy of DSR estimates. In the subsequent publication of [BIS \(2017\)](#), main proxies for inputs in the DSR estimation were reviewed on the aggregate level, based

³ For a discussion of credit-to-GDP measure and its impact on economic development, refer to [Arcand et al \(2012\)](#), [Cecchetti and Kharroubi \(2012\)](#), [Juselius and Drehmann \(2015\)](#), [Donets and Ponomarenko \(2015\)](#), [Alessi and Detken \(2018\)](#), [Bank of Russia \(2019\)](#).

on the unification of methodologies for compiling the internationally comparable datasets. Subsequently, [BIS Statistical Bulletin \(2019\)](#) estimate DSRs for the three sectors: private non-financial sector as a whole, households, and the private non-financial corporations separately (depends on the data availability in different countries). It was reemphasised that the higher level of DSR could have a strong negative effect on consumption and investments; the institutional and behavioural borrowing patterns are among the additional sources of volatility of the DSR in different countries.

This paper contributes to existing literature as the first outline of the loan-based estimate of the DSR for Russia. Results include the details on debt servicing costs attributable to a short, medium, and long-term debt, currency structure of the DSRs, and the sector-specific DSR estimates. The credit registry contains extensive details on the amounts of loans and credits issued by the resident banks to the private non-banking sector in Russia. It contains updated information on actual remaining maturity and interest rates for each amount of debt outstanding. The DSR estimated with loan-level data produced meaningful insight into the processes of debt build-up and the stretching debt servicing capacity for different sectors of economy in Russia.

3. Methodology

The DSR as a measure of debt servicing burden at aggregate level takes the following form⁴:

$$DSR_{j,t} = \frac{i_{j,t}}{(1-(1+i_{j,t})^{-s_{j,t}})} * \frac{D_{j,t}}{Y_{j,t}} \quad (1)$$

This functional form captures the non-linearity of changes in remaining maturity (S), lending rates (i), and the amount of debt outstanding (D) for sector j at time point t , normalised by the amount of income (Y). Applying formula (1) to a micro-level database, we can re-write as follows:

$$DSR_t = \frac{\sum_{n=1}^N \frac{i_{j,t} * D_{j,t}}{(1-(1+i_{j,t})^{-s_{j,t}})}}{Y_t} \quad (2)$$

where n is individual loan and N is total number of entries (loans) in the database.

The concept of the DSR was methodologically introduced in [Drehmann and Juselius \(2012\)](#). It assumes that the principal amount of debt is amortised over the period of its *remaining* maturity (instalment loans). This assumption could cause an upward bias in case the “interest only” payment scheme is realised, i.e. where the principal amount of debt is due at maturity (bullet loans). However, this assumption is plausible for evaluating implicit debt servicing burden at each estimation point and its evolution over time.

⁴ For a formal derivation, refer to [Drehmann and Juselius \(2012\)](#).

Approximation error and plausibility of [formula \(1\)](#) for estimating the DSRs at different levels of data aggregation are discussed in [Drehmann and Juselius \(2012\)](#).

To check the robustness of micro-level DSR estimate, we recalculate the amount of debt repaid in each quarter using the accounting roll-forward exercise:

$$Debt\ redeemed = Stock\ of\ Debt_t - Stock\ of\ debt_{t+1} + Debt\ issued \quad (3)$$

where the amount of debt redeemed in each quarter is defined as the difference in amount of debt outstanding between two quarters, amplified by the amount of debt issued during that quarter. Accounting roll-forward exercise could be criticised based on several points. First, when calculated at an individual loan level, the amount of debt repaid will be misleading if some loans are consolidated (i.e. added up) during that period. Second, when calculated at sectoral level, the same issue will appear if some loans are reclassified from one sector of economy to another. Those issues could be resolved if [formula \(3\)](#) is applied at economy level.

4. Micro-level database

To the best of our knowledge, this is the first piece of research implementing a micro-level database (loan-level credit registry⁵) to estimate the DSR for the private non-banking sector in Russia. The database contains detailed information on the amount of debt outstanding, new loans issued, actual lending rates for each loan, initial maturity, actual remaining maturity, and the amounts of actual debt repayments as reported by banks monthly (including interest payments and amortisation of the principal amount of debt) in 2017–2019.

The stock of debt (D) is the loans issued by resident banks to private firms in Russia and outstanding at each estimation point. The lending rate (i) is the corresponding interest rate for a particular amount of loan issued. The remaining maturity (S) is the actual time in quarters remained until the redemption date of each loan. The amount of income is proxied by the nominal GDP at economy-level, by the GDP allocated pro rata with shares of EVA produced for sectoral DSR estimates.

5. Impact of changing aggregate assumptions

In order to understand how the levels of DSR are changed when different assumptions on maturities and lending rates are used, we perform the following procedure. First, we estimate the DSR with [formula \(1\)](#) using the aggregate assumption of *constant* maturity structure ([Fig. 1, blue solid line](#)). The stock of debt outstanding is aggregated by

⁵ We have access to banking reporting form No. 303 (Russian). Banks submit it to the Bank of Russia on a quarterly basis. We refer to it as a ‘credit registry’, although it is not the data from the credit registry bureaus.

initial maturity buckets, and prevailing lending rates are applied⁶. The assumption of prevailing lending rates reflects composite nature of the stock of debt at each estimation point.

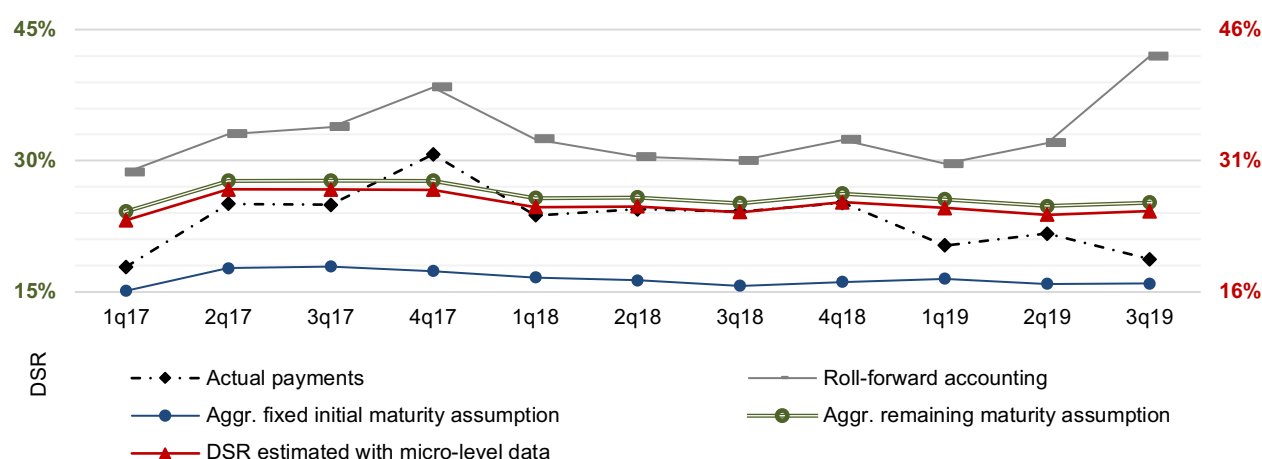
Second, we estimate the DSR using aggregate assumption of *changing* remaining maturity (Fig. 1, green line). The stock of debt outstanding is aggregated by remaining maturity buckets. We have obtained considerably higher levels of the DSR than those estimated with constant maturity assumption (e.g. 25.23% vs 15.97% for the 2019 Q3).

Next, the DSRs estimated with aggregate assumptions are compared to the DSR estimate without aggregate assumption, i.e. using formula (2) and granular data on actual remaining maturity and lending rates for each loan from the credit registry. The loan-level DSR estimates (Fig. 1, red line) are not significantly different from the DSR estimated with the aggregate remaining maturity assumption (Fig. 1, green line). Nevertheless, it is considerably higher than the results obtained with aggregate constant maturity assumption (Fig. 1, blue solid line).

Subsequently, we compare estimated levels of the DSR to the *actual* amount of debt repayments reported by banks (Fig. 1, black dotted line) consisting of the interest payments and amortisation of the principal amount of debt (this data is also available from the credit registry). We find that the levels of DSR estimated with loan-level data were closer to the actual amount of debt servicing costs reported by banks.

Results obtained with the accounting roll-forward exercise (Fig. 1, grey line) were even higher than the micro-level DSR estimates and the *actual* amount of debt repayments. Thus, we find the DSR⁷ to be a valid approximation to measure of debt servicing burden for the economy.

Figure 1. DSR estimates with the alternative assumptions



Source: Bank of Russia, author's calculations

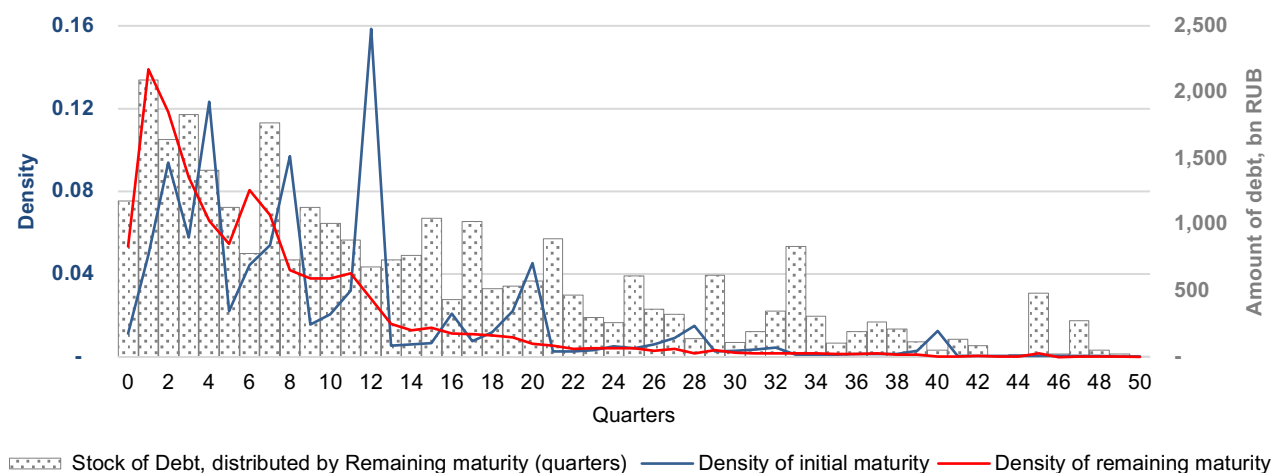
⁶ Data on Loans in Rubles, in US dollars, in Euros to Non-Financial Organisations. Total for the Russian Federation.

The Central Bank of the Russian Federation. [Available online] https://www.cbr.ru/eng/statistics/pdco/int_rat/

⁷ If estimated with the loan-level information on lending rates and actual remaining maturity for each amount of debt outstanding and applying the methodology introduced by Drehmann and Juselius (2012).

The difference in DSR estimates is rooted in the fact that loans with longer initial maturity have shorter actual remaining maturity, i.e. the disaggregate amount of debt outstanding at each point are not symmetrically distributed around the mean value of remaining maturity (Fig. 2). For more formal definition of this point, refer to Appendix 1.

Figure 2. Distribution of the stock of debt, 2019 Q4



Source: Bank of Russia, author's calculations

6. Results

The level of DSR estimates with micro-level data (26% for 2019 Q4) is considerably higher than the results obtained with constant maturity assumptions (BIS (2019) the latest available estimate for Russia of 8%, fixed maturity buckets assumed by Donets and Ponomarenko (2015) with the estimated DSR of 16% for 2019 Q4).

The difference in the DSR levels between *non-banking* sector of economy (includes financial corporations engaged in providing financial and insurance services⁸), and *non-financial* sector is 3% on average. For details, please refer to Table 1.

Table 1. DSR (%) estimates

	1q17	2q17	3q17	4q17	1q18	2q18	3q18	4q18	1q19	2q19	3q19	4q19
Private non-financial sector	21.26	24.61	24.25	24.67	22.69	22.73	21.56	22.48	21.52	21.75	21.95	22.66
Private non-banking sector	24.20	27.71	27.72	27.70	25.72	25.79	25.14	26.25	25.59	24.83	25.23	25.83

Source: Bank of Russia, author's calculations

⁸ For a full definition, refer to BIS Glossary <https://www.bis.org/statistics/glossary.htm>

The breakdown of the DSR by the remaining maturity of debt outstanding (Table 2) shows the major part of it attributable to servicing the debt with a shorter remaining maturity of less than 1 year. This is consistent with the stance of corporate operating activities – financing working capital needs and carry-trade. Currency breakdown of the DSR shows 85% of the debt servicing costs attributable to servicing domestic currency debt.

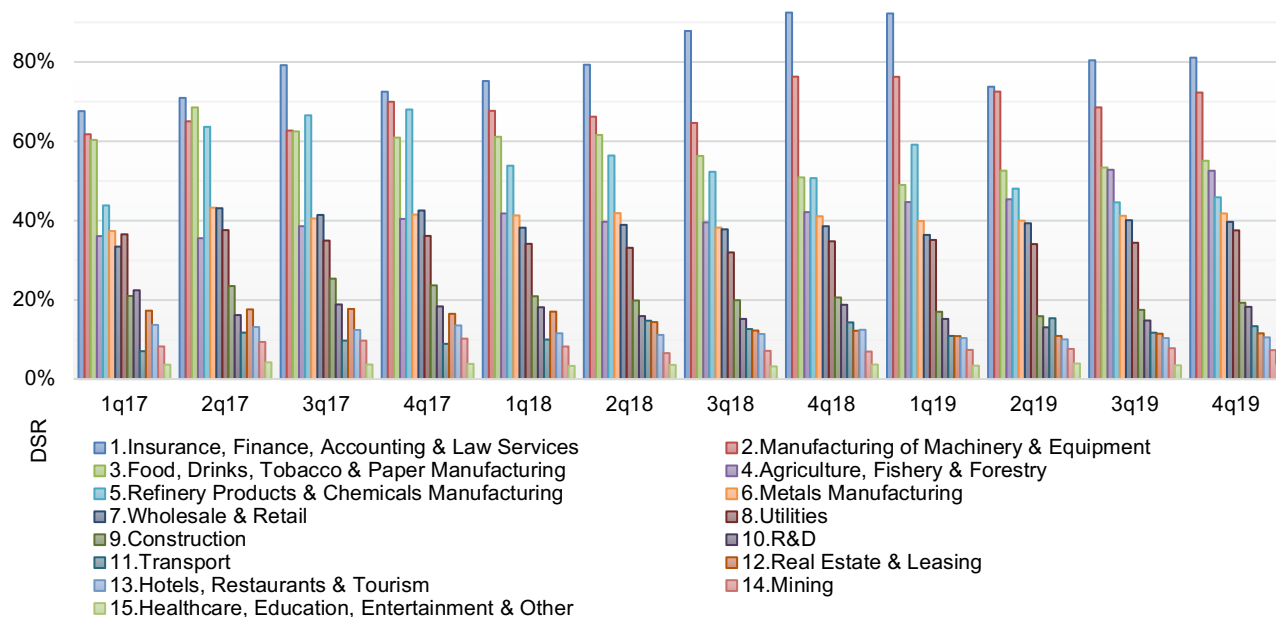
Table 2. DSRs (%) breakdown by remaining maturity of loans

	1q17	2q17	3q17	4q17	1q18	2q18	3q18	4q18	1q19	2q19	3q19	4q19
More than 3 yrs	3.97	3.96	3.99	4.02	3.93	3.83	3.83	3.84	3.61	3.60	3.61	3.60
From 1 to 3 yrs	4.40	4.48	4.47	4.65	4.22	3.77	3.72	3.83	3.94	4.16	4.33	4.23
Less than 1 yr	15.84	19.28	19.26	19.03	17.57	18.18	17.58	18.57	18.04	17.07	17.29	18.00

Source: Bank of Russia, author's calculations

Higher levels of the DSR are associated with the higher exposure to liquidity shocks (Drehmann and Juselius (2012)). A possibility of debt roll-over can change this exposure, but the interest rate risk and liquidity risk should be accounted for, i.e. lending conditions could change adversely and debt roll-overs could shrink. This will increase the exposure of entities with higher DSR levels. DSRs attributable to debt roll-overs (as reported by banks) are 3% on average for the private non-banking sector in Russia. We construct separate DSRs for 15 aggregated sectors of the real economy and find different shifts on the horizon of 12 quarters (Fig. 3). For the detailed list of 61 disaggregate sectoral DSRs, refer to Appendix 2.

Figure 3. Sectoral DSR estimates



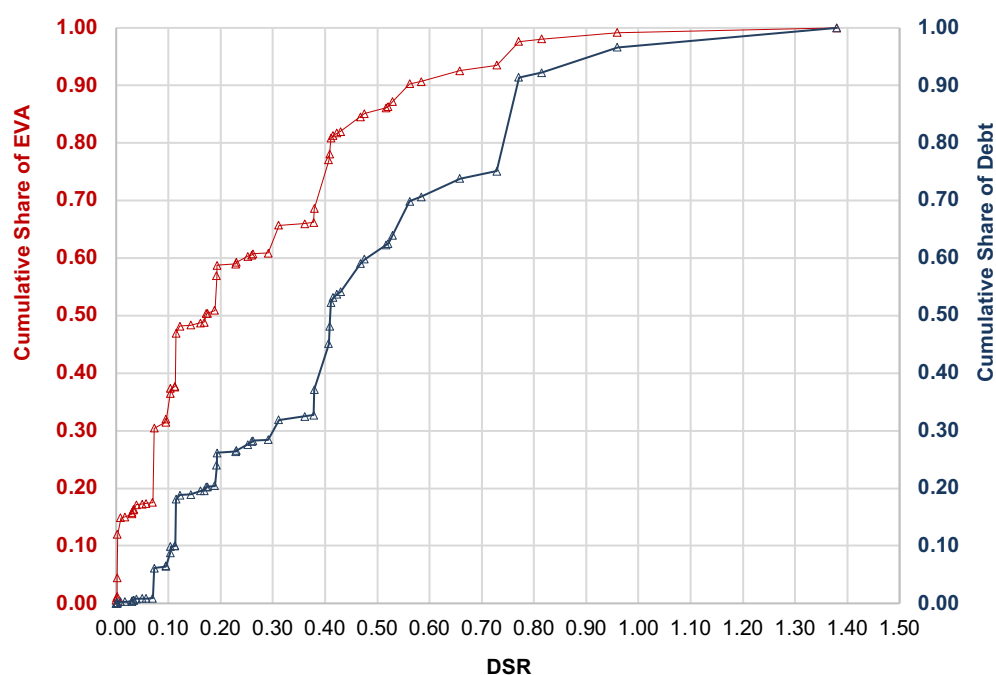
Source: Bank of Russia, author's calculations

We have identified sectoral heterogeneity of the DSRs, which is attributable to the different patterns of operating activities in industries, different degree of resilience and exposure to liquidity shocks. Sector-specific DSR estimates result in the following levels:

- the DSR above 75% was estimated for insurance, finance, law and accounting services and for manufacturing of machinery, equipment and other vehicles;
- the DSR above 50% was estimated for food, drinks and tobacco manufacturing, production of plants and cattle, manufacturing of metal goods, electric appliances, and chemicals;
- the DSR below 25% was estimated for healthcare, education, utilities, hotels and restaurants, forestry, mining, transportation, fishery, R&D sectors, maintenance of machinery and equipment.

We display the path of the DSR increase by sectors with the corresponding shares in the total amount of debt and the economic value added for 15 aggregated sectors of economy (Fig. 4). We have found that about 50% of the amount debt outstanding was placed in sectors with a DSR of less than 50%, and with the corresponding share in EVA of 80%.

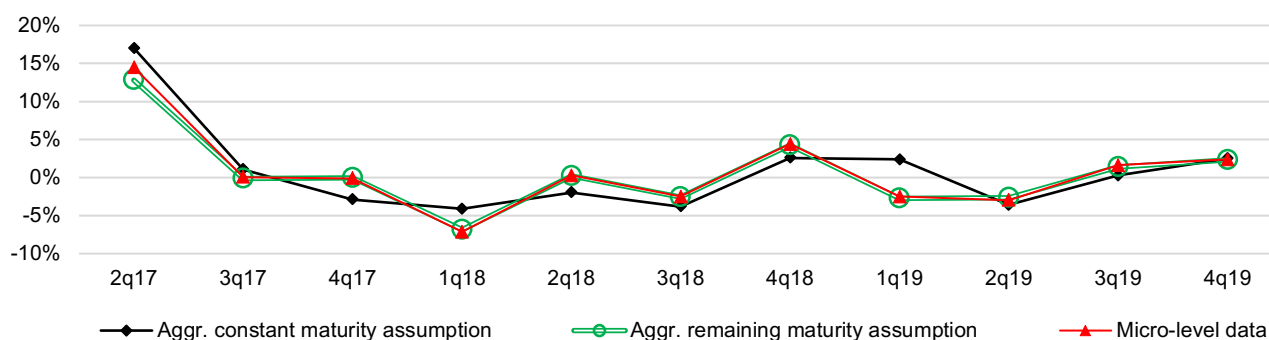
Figure 4. Sectoral DSRs, total Debt (blue line) and EVA, (red line) 2019 Q4



Source: Bank of Russia, author's calculations

The dynamics of DSRs estimated with alternative assumptions display comparable moves (for visual representation please refer to Fig. 5, for discussion of alternative assumptions please refer to section 5).

Figure 5. Growth rates* of DSRs estimates with alternative assumptions

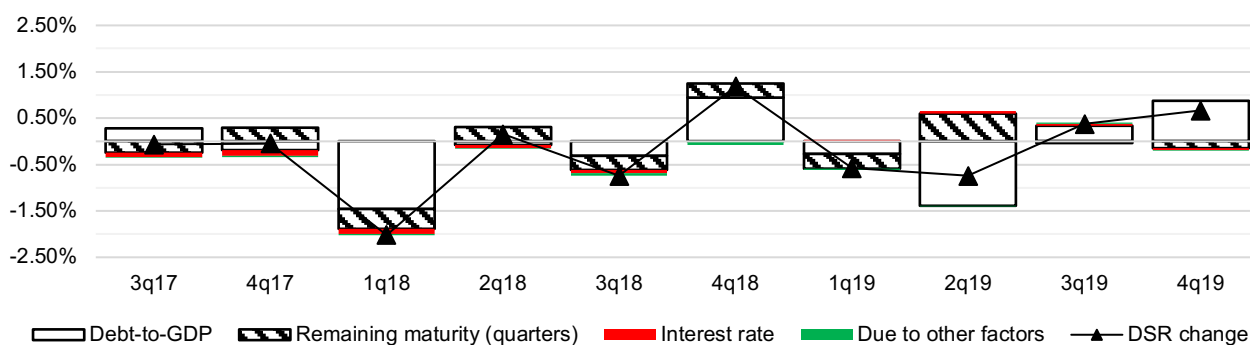


*Calculated as a percentage change to previous quarter.

Source: Bank of Russia, author's calculations

We decompose the changes in the DSR levels (Fig. 6) into the components related to the interest rate, the length of remaining maturity, and the level of debt in relation to GDP.

Figure 6. Factor analysis of changes in the DSR



Source: Bank of Russia, author's calculations

Changes in the DSR are mainly attributable to the changes in the relative level of indebtedness of the economy, i.e. to the change in debt-to-GDP component (from min. of -1.47 to max. of +0.95 p.p.). Changes in the remaining maturity of debt influenced the dynamics of DSR from min. of -0.42 to max. of +0.59 p.p. Changes in the interest rates influenced the dynamics of DSR less significantly (from min. of -0.13 to max. of +.06 p.p.). It could be argued that prudent maturity management can influence the level of debt servicing burden in the economy. From policy perspective, this means that for the private corporations in Russia, changes in lending rates were effectively translated into the changes in maturity structure and the amounts of debt borrowed.

In this research, we estimate the DSR for the stock of debt issued by resident banks to the private non-banking sector in Russia. Incorporating external debt and market debt (corporate bonds) would complement the research and create the basis for estimating the *total* debt servicing burden for the sectors of economy and corporations. We leave this point to future research.

7. Conclusion

We have estimated the DSR for the private non-banking sector in Russia for 12 quarters in 2017–2019. In line with the original methodology of [Drehmann and Juselius \(2012\)](#), we have assumed gradual repayment of debt over the time of its remaining maturity but have departed from the assumption of constant maturity structure of the stock of debt. Instead, we have used the actual remaining maturity and the interest rate for each loan available from the new micro-level database (credit registry). The levels of the DSR estimated with loan-level data were higher than the previous results obtained with the aggregate constant maturity assumption and were closer to the actual amounts of debt servicing costs as reported by banks (including interest payments and amortisation of debt). We conclude that the DSR estimated with granular data is a valid approximation of the debt servicing burden for the economy. We construct separate DSRs for the sectors of economy in Russia and find different shifts on the horizon of 12 quarters.

A positive effect of re-calibrating the levels of DSR would be a better approximation of exposure of different sectors of economy to liquidity shocks and the earlier identification of building constraints (e.g. on capital expenditures). A non-existence of analogues loan-level database in public domain of other countries complicates calibration and the comparison of DSRs globally.

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Appendix 1. Mathematical formulation of the phenomena of asymmetric distribution of the stock of debt around the mean value of remaining maturity

Let us introduce notations:

D – stock of debt

S – remaining maturity

\bar{S} – weighted average remaining maturity

N – number of entries in credit registry

i, j, k, r – loan index

ε_i – deviation of actual remaining maturity from weighted average

$I_{1,2}$ – two synthetic indexes

$$\text{Let } I_1 = \sum_{i=1}^N \frac{D_i}{S_i}, \quad I_2 = \frac{\sum_{i=1}^N D_i}{\frac{\sum_{j=1}^N D_j S_j}{\sum_{k=1}^N D_k}}$$

$$\text{Let } \bar{S} = \sum_{i=1}^N S_i w_j, \quad \text{where } w_j = \frac{D_j}{\sum_{k=1}^N D_k}$$

$$\text{Then } I_2 = \sum_{i=1}^N \left[\frac{D_i}{S_i} \right] \frac{S_i}{\bar{S}}$$

$$\text{Let } S_i = \bar{S} + \varepsilon_i$$

$$\text{If } (\varepsilon_i = 0, \forall i) \text{ or } \left(\frac{D_r}{S_r} = \frac{D_i}{S_i} \forall i \neq r \right) \Rightarrow I_2 = \sum_{i=1}^N \left[\frac{D_i}{S_i} \right] = I_1$$

Otherwise \Rightarrow

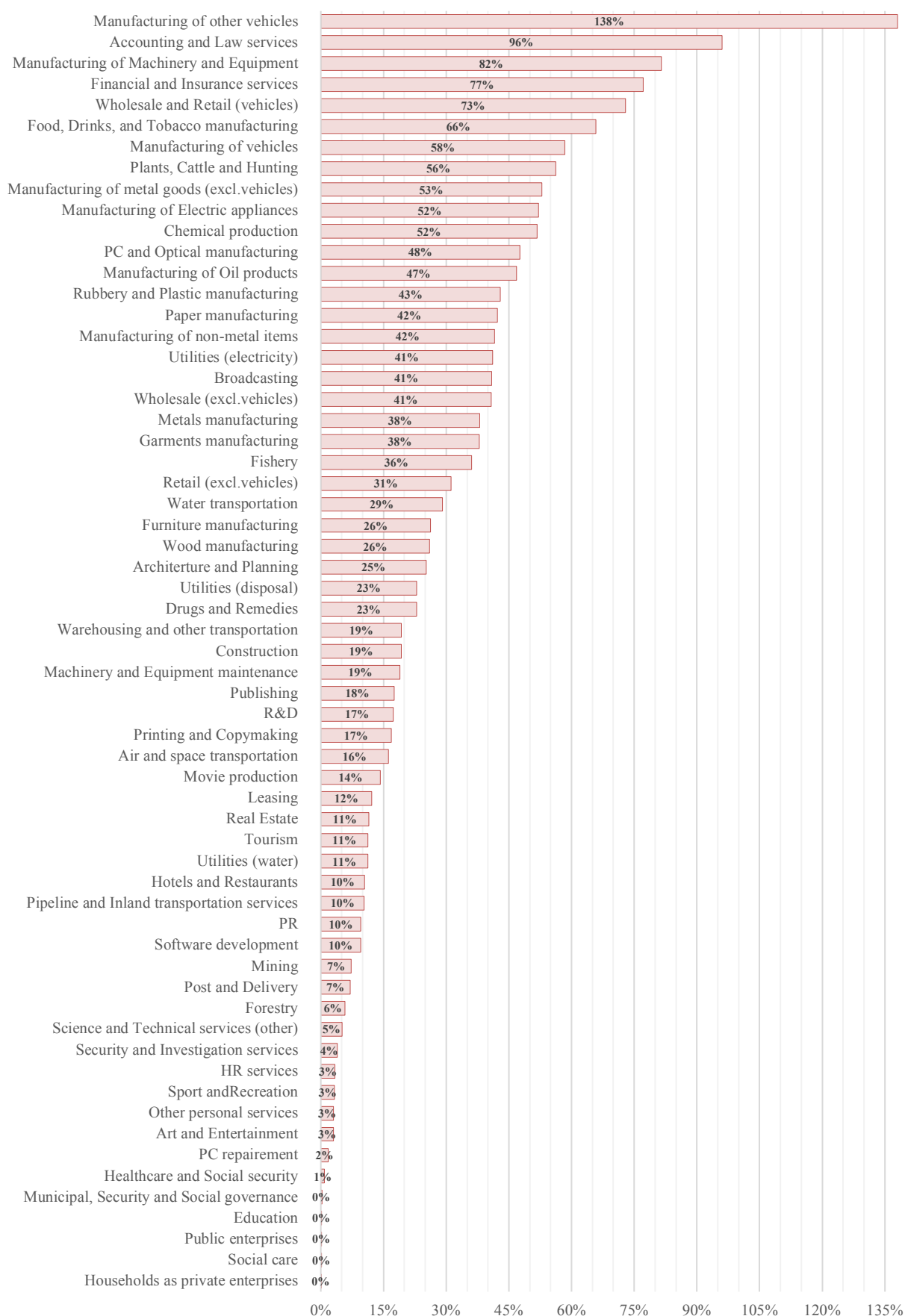
$$I_2 = \sum_{i=1}^N \left[\frac{D_i}{S_i} \right] \left[\frac{\varepsilon_i}{\bar{S}} + 1 \right] = I_1 + \sum_{i=1}^N \left[\frac{D_i}{S_i} \right] \frac{\varepsilon_i}{\bar{S}} = I_1 + \sum_{i=1}^N \left[\frac{D_i}{\varepsilon_i + \bar{S}} \right] \frac{\varepsilon_i}{\bar{S}} = I_1 + \sum_{i=1}^N \left[\frac{D_i}{\left(1 + \frac{\bar{S}}{\varepsilon_i}\right) \bar{S}} \right]$$

Thus, the following effects are observed:

Effect 1: Asymmetric distribution of the amounts of debt around the mean value of average remaining maturity

Effect 2: Asymmetric distribution of ε_i around 0

Appendix 2. Sector-specific DSR estimates (2019 Q4)



Source: Bank of Russia, author's calculations