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Adding a fiscal rule into a DSGE model: How much does it change the forecasts?

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Abstract

This article analyses an expansion of the dynamic stochastic general equilibrium model presented in Kreptsev, Seleznev (2017) and used by the Bank of Russia to forecast macroeconomic variables. The model was supplemented with an extended description of the fiscal sector, which formalises the fiscal rule in effect in Russia and which is similar to the one used in Medina, Soto (2007). The model was estimated on the basis of Russian data. Based on impulse response functions, we analyse the stabilising effect of the fiscal rule on macroeconomic variables. It was found that the fiscal rule leads to a decrease in output volatility, a slight decrease in exchange rate volatility and a stronger disinflationary effect in response to a positive oil price shock. The forecast errors were used to analyse whether it is possible to apply the formalisation of the fiscal rule in order to improve the forecast of macroeconomic variables within the DSGE model. We found that the fiscal rule does not improve the quality of the forecasts.

Key words: DSGE model, fiscal rule, reserve fund, credit cycle, commodity prices, financial frictions, monetary policy

JEL codes: D58, E47, E62, E63.

Content

1. Introduction

Before 2015, oil prices were one of the key factors that affected the Russian economy and considerably contributed to the volatility of macroeconomic indicators (*Polbin (2014) and Kreptsev, Seleznev (2018)*). After the 2014-2015 crisis, the Russian economy has demonstrated a weaker dependence of its macroeconomic indicators on oil prices (Figure 1, Appendix A) and a weaker correlation with oil prices and with each other. In view of this, the previous forecast models based on the oil price effect may turn out to be less relevant. It is important now to understand the reasons why, starting from 2015, the oil price has affected the Russian economy to a lesser degree. Knowledge of these reasons will enable us to model the economic dynamics more correctly and to make more accurate forecasts.

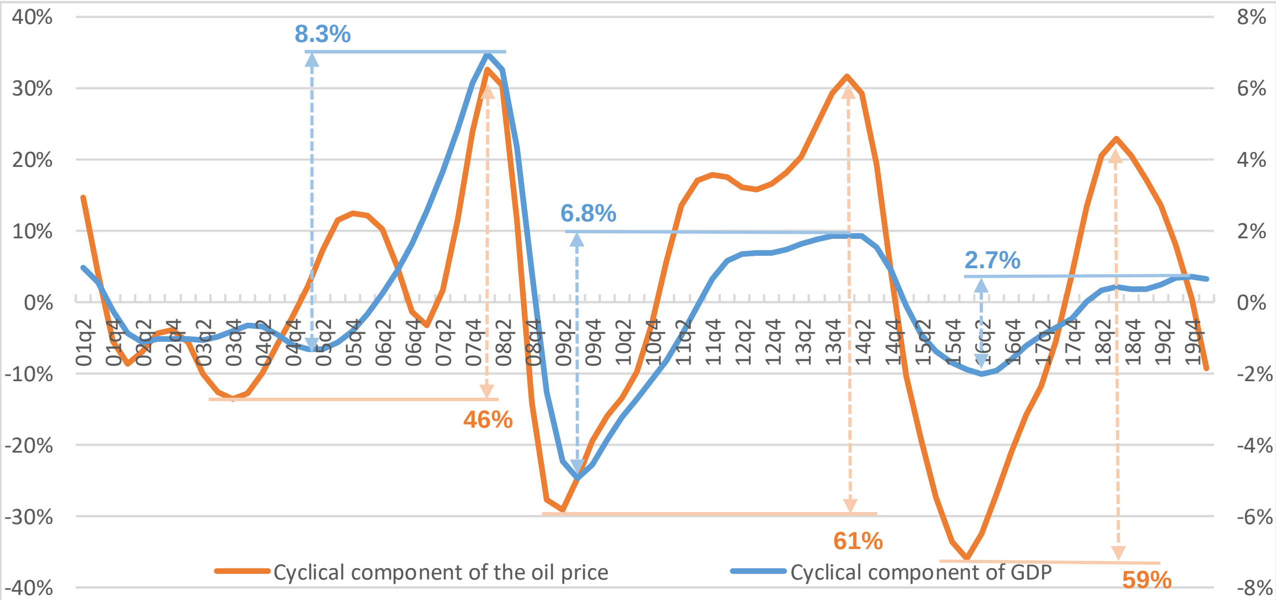

Figure 1 – Cyclical components of the oil price (the left scale) and of the Russian GDP (the right scale) cleaned from the high frequency component (HP filter with parameter value of 2). As a percentage of the trend (HP filter parameter value of 1600)

The reasons for the decreased effect of oil prices on the Russian economy may lie in various layers. First, the sanctions imposed by the developed countries on the Russian banks after 2015 caused the banking sector's liabilities to non-residents to consistently decrease owing to reduced flow of new liabilities. Direct investments in and out of Russia also decreased (Figure 2). The decreased activity of external flows may have reduced the degree of the external shock pass-through into the economy.

Figure 2 – Direct investments in and out of Russia in the manufacturing sector[1] (the left scale) and the banking sector's liabilities to non-residents (the right scale), USD bln

The second reason could be the change in the monetary policy. With a floating exchange rate, GDP is less volatile in response to both an external interest rate shock (*Gertler et al. (2007)*) and an oil price shock (*Cespedes et al. (2004) and Andreyev, Polbin (2019)*). The Bank of Russia has officially switched to inflation targeting since 3Q 2014. Starting from mid-2015, the Bank of Russia has not resorted to FX interventions but rather performed operations in the interests of the Russian Finance Ministry on the FX market.[2]

Another reason could be that, starting from 2017, the Russian Finance Ministry has used a fiscal rule[3] which provides for the smoothing of oil and gas revenues flowing into the state budget, as well as limitations on borrowings.

In this article, we examine the possibility of the fiscal rule having an impact on the (in)dependence of the Russian key indicators from commodity prices. As the main instrument, we use the dynamic stochastic general equilibrium model (*Kreptsev, Seleznev (2017)*) and expand it by describing the dynamics of the budget funds and the external currency fund.

Further, in Section 2, we briefly describe the model used from *Kreptsev, Seleznev (2017)*. Section 3 contains everything pertaining to the fiscal rule: a description of the current fiscal rule, an analysis of the fiscal rule from the perspective of the equations of mathematical economics, and a description of the specifications of the fiscal rule used. The technical details pertaining to the estimation of the model are provided in Section 4. In Section 5, based on impulse response

[1] “Other sectors” according to the BOP methodology

[2] https://www.cbr.ru/statistics/flikvid/

[3] Various rules for replenishing the budget from oil and gas revenues have existed since at least 2004. Here, by the fiscal rule, we mean the rule (Federal Law No. 262-FZ, dated 29 July 2017), introduced in 2017

functions, we analyse the impact of the oil price shock and the stabilising capabilities of the fiscal rule. In Section 6, we analyse whether it is possible to use the formalisation of the fiscal rule in order to improve forecast quality.

The application of fiscal rules for resource-rich countries is a moderately researched issue. *Jalali-Naini, Naderian (2020)* estimated a DSGE model for Iranian economy and explored the interaction of monetary and fiscal policy. A variety of fiscal policies is represented by an alternative to fully spent the cyclical component of oil revenue or to accumulate it in an oil stabilization fund. If there is no stabilization fund, then the real exchange rate targeting policy turns out to be optimal for the purposes of macroeconomic stability. Although this policy loses out in stabilizing inflation, it stabilizes the output gap and exchange rate in the best way, making the economy less vulnerable to oil price shock. If the oil stabilization fund is involved, then the volatility decreases. The CPI targeting policy becomes preferable because the fiscal rule creates an environment in which exchange rate fluctuations are less disruptive, and monetary authority can focus on inflation and output stabilization resulting in less aggregate volatility.

Medina, Soto (2007) considered several fiscal rules for the Chilean economy. Switching from expansive fiscal policy to the policy that saves most of the extra revenues from higher copper price results in significantly lower output and exchange rate volatility and mild deflation instead of inflation.

As in *Medina, Soto (2007)*, we found that a positive commodity shock leads to deflation. However, in contrast to *Medina, Soto (2007)* and *Jalali-Naini, Naderian (2020)*, we found that implementation of the fiscal rule only slightly reduces exchange rate volatility. This is due to the fact that oil revenues generate government demand only for domestic product, and the change in state's demand for foreign exchange is offset by changes in the owners' demand for foreign bonds.

According to *Pieschacon (2012)*, fiscal policy is an important transmission mechanism for oil price shocks, since the response of macrovariables varies greatly depending on fiscal policy in such countries such as Mexico and Norway. Fiscal policies that insulate the economy from oil price shocks (Norway), i.e. counter-cyclical ones, improve welfare over procyclical ones.

Cespedes, Velasco (2014) investigated the fiscal policies of resource-rich countries during the boom episodes of the 1970s and 2008 using indices. They conclude that pro-cyclical policies prevailed in 1970s, whereas in 2008 episode policies became more counter-cyclical due to improvements in institutional quality. They state that countries that use fiscal rules displayed a larger shift toward fiscal countercyclicality between the two episodes.

In contrast to *Pieschacon (2012)* and *Cespedes, Velasco (2014)*, *Bjornland, Thorsrud (2019)* argue that the Norwegian fiscal policy has been more procyclical since the adoption of the fiscal rule. The fiscal policy has not effectively protected the economy from oil price shocks. They separated the impact of oil price shock and global activity shock in the DFM model. They found fiscal policy to be partly counter-cyclical with global activity shock, which also affects oil prices.

Ivashchenko (2018) attempted to identify the combination of Russian monetary and fiscal policies within DSGE framework, but the results were contradictory.

2. Benchmark DSGE model

As the basic model for the analysis, we chose the dynamic stochastic general equilibrium (DSGE) model developed by the Bank of Russia Research and Forecasting Department (*Kreptsev, Seleznev (2017)*). First, this model is used by the Bank of Russia to forecast inflation and other indicators and, second, it has a rather broad structure that makes this model suitable for theoretic and practical experiments.

The dynamics of macroeconomic variables are described in the model as the result of interaction among agents of several types: households, producers, entrepreneurs, banks, the monetary authority, and external economy.

Households maximize the utility of consumption and minimise the disutility of labour, and they invest in foreign bonds and domestic deposits. There are no non-Ricardian households in the model.

Producers use labour and capital to produce the final domestic product. The domestic product is utilised for government consumption and, after packaging agents have combined the domestic product with imported products, it is used to produce the final consumption of households and investments. This model structure enables us to account for the impact of the exchange rate (and the shocks affecting the rate) on inflation and investment cost.

The description of the banks in the model partially overlaps with the one provided in *Gerali et al (2010)*. Banks borrow money from households and the monetary authority; they form equity and lend money to entrepreneurs. Each bank consists of a deposit branch and a credit one. Banks have monopoly power in the credit and deposit market. As a result, there is a non-zero spread between credit and deposit rates.

Entrepreneurs are described in accordance with *Bernanke, Gertler, Gilchrist (1999)*. They use equity and borrowed capital to form assets, i.e. production capital, which they lease out. Depending on the economic conditions, in each time period a certain portion of entrepreneurs go bankrupt.

The model implements the New-Keynesian approach, according to which markets operate imperfectly. The model has the following imperfections: the rigidity of domestic, import and export prices, wages rigidity, consumption habits, investment adjustment costs, financial frictions, as well as the imperfections associated with the banking sector: the bank capital adequacy requirements and the costs of setting loan and deposit rates.

Monetary policy involves targeting inflation by following the Taylor rule in the following form:

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\phi_R} \left(\frac{\pi_t}{\pi_{ss}}\right)^{(1-\phi_R)\phi_\pi} e^{\varepsilon_t^R},$$ [4]

where R_t, π_t are the nominal interest rate and inflation, R_{ss}, π_{ss} are their steady states and ε_t^R is the monetary policy shock. In addition, foreign exchange interventions are also an element of the monetary policy. The volume of interventions is described as a separate shock.

Government consumption G_t is described by an AR(1) process:

$$\frac{G_t}{G_{ss}} = \left(\frac{G_{t-1}}{G_{ss}}\right)^{\rho_G} e^{\varepsilon_t^G} \tag{1}$$

The feature of the model in *Kreptsev, Seleznev (2017)* is the assumption that there exists a random walk in the level of technological progress. This assumption requires a special technique to express the model equations. This technique is described in *Seleznev (2016)*.

In order to solve the model, the authors used a log-linear approximation of the equations. The model was solved in Matlab using a code written by the authors based on the algorithms proposed in *Sims (2002)*. 63 model parameters were estimated on the basis of 22 data series over a period from 2006 until 2016.

The historic decomposition of the shock contribution revealed that, during the 2008-2009 and 2014-2015 crises, the oil price shock considerably contributed to the dynamics of consumption, investments and exchange rate, while the dynamics of GDP were explained to a greater extent by the exports shock and TFP shock. In addition to the oil price shock, the impact of the external risk premium shock turned out to be important too. Together with the oil price shock, the external risk premium shock has largely accounted for the volatility of inflation. Having compared the quality indicators for the forecasts based on several models, the authors concluded that the given model is at least as good as the other models.

[4] Hereinafter we have tried to be as close as possible to the notations used in *Kreptsev, Seleznev (2017)*.

3. Fiscal policy

3.1. The fiscal rule according to law

According to legislation,[5] the fiscal rule may be characterised by the following two main provisions.

First, smoothed oil and gas revenues (OGRs) flow into the federal budget. The smoothed value is calculated on the basis of the "base" Urals oil price. The base price is a fixed value which is annually adjusted according to the average inflation abroad. The difference between the unsmoothed and smoothed OGRs is used to form the National Wealth Fund (NWF).

Second, the federal budget expenses are limited by the amount of non-oil and gas revenues (NOGRs), the amount of smoothed OGRs, the debt interest payments and, from 2019 until 2024, also by an additional 0.5% of GDP as a surcharge to implement infrastructural projects.[6] In 2017-2019, in terms of annual data, this limitation was fulfilled (Figure 3).

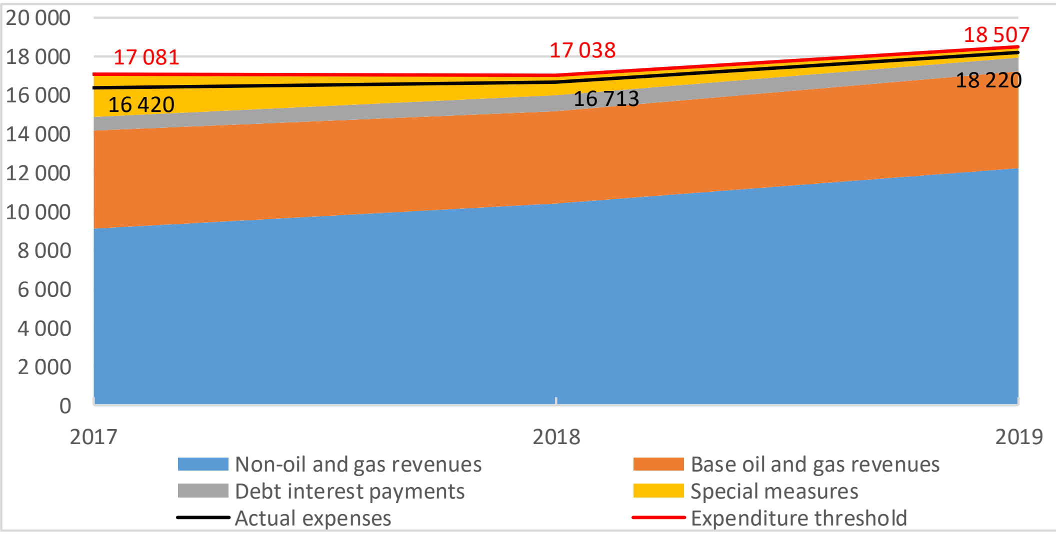

Figure 3 – Actual federal budget expenses and the components of the threshold federal budget expenditures in 2017-2019, RUB bln

The fiscal rule should help to reduce the dependence of the economy on changes in the phases of the commodity price cycle. Also the rule is deemed to limit borrowings, which would limit federal budget expenditures.

In addition to the aforementioned provisions, there is also a limitation on the use of NWF funds if the fund shrinks to a certain level. However, we will not take this provision into account. The fiscal rule is discussed in more detail in *Vlasov (2020)*.

[5] Federal Law No. 262-FZ "On Amending the Budgetary Code of the Russian Federation in respect of the Use of Oil and Gas Revenues of the Federal Budget", dated 29 July 2017.
[6] https://www.minfin.ru/common/upload/library/2019/10/main/2020-2022.pdf, Table 4.1.1

It is important to note forward that NOGRs also include income from the placement of NWF funds, while the federal budget expenditures include debt interest payments.

3.2. Model assumptions

As an idealised scheme of a part of reality, it is impossible to make any model without simplifying "model" assumptions. Below we will use a number of simplifications, including the following.

- The fiscal rule pertains to the federal budget, while the broadest type of budget is the consolidated budget. Therefore, the fiscal rule has a limited scope. We will further hold that, in the model, the fiscal rule applies to the entire fiscal system.
- We will hold that all loans are borrowed for one model time period, namely, a quarter. While it is possible to model the lending for a greater number of periods, this is not a subject of interest in this paper.
- Borrowing restrictions imposed by the fiscal rule must be met on a per calendar year basis. For simplicity, we will hold that the limitations are complied with for each period, i.e. a quarter.
- Government expenditures are set based on OGR value. OGR is valued at projected exchange rate, not the actual one. We will hold that the government is using the actual exchange rate. We will weaken this assumption in an additional experiment (chapter 5) by assuming that the government uses a lagged exchange rate.
- Apart from debt interest payments, state budget expenditures may be conventionally divided into the following large items: government consumption, transfers to households (including pensions), as well as transfers and investments to the national economy. Due to the limitations imposed by the DSGE model, we will hold that all state budget expenditures, apart from debt interest payments, are expenses on government consumption.
- We will hold that the Bank of Russia's liabilities to the federal government, except for the NWF accounts (the "Treasury's accounts with the BoR"), are a constant value. The Treasury's accounts with the BoR increase, for instance, when funds from the placement of federal bonds are received. Therefore, this assumption spares us the need to model, in particular, the impractical strategy where the fiscal authorities borrow funds on the domestic market and subsequently accumulate funds on the Treasury's accounts with the BoR without spending them, which formally does not contradict the fiscal rule. The assumption introduced is close to the actual state of affairs since, as may be seen from Figure 4, the rouble-denominated portion of the liabilities pertaining to the state budget settlements is relatively constant while the currency-denominated portion pertaining to the NWF varies considerably.

- We will hold that the fiscal authorities do not borrow funds abroad. In reality, this is not the case. However, the NWF funds placed abroad do not exceed external government borrowings. If necessary, the model NWF funds may be interpreted as the actual NWF funds minus the external government debt.
- All NWF funds are invested in foreign assets and are denominated in foreign currency.

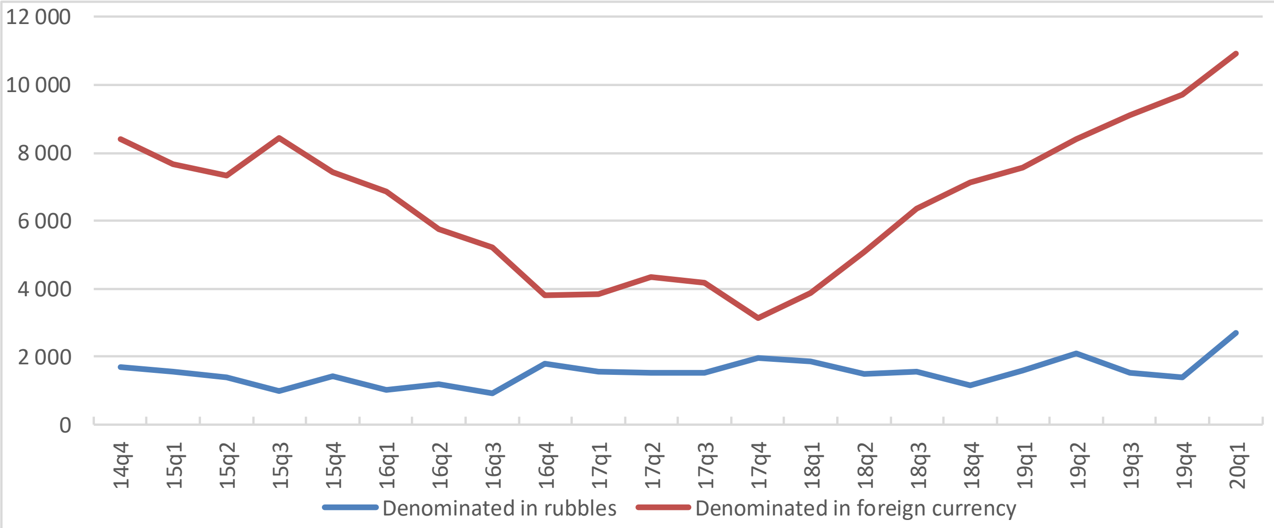

Figure 4 – The Bank of Russia's liabilities to the federal government, RUB bln

3.3. The fiscal sector in the model

We have expanded the model proposed by *Kreptsev, Seleznev (2017)* with a more detailed description of the fiscal sector. We will account for the smoothing of the fiscal oil and gas revenues by allocating a certain portion of funds to/from the NWF using the following equation[7]:

$$\tilde{B}^*_{G,t} = \tilde{B}^*_{G,t-1} + \tau^{oil} S_t \left(\tilde{P}^{oil}_t - P^{oil}_{ss} P^*_t\right) \quad , \qquad (2)$$

where $\tilde{B}^*_{G,t}$ stands for the NWF funds denominated in foreign currency, S_t – oil exports, $\tilde{P}^{oil}_t$ – oil price, P^*_t – price index in the external economy, τ^{oil} – the average tax burden on oil and gas exports. The P^{oil}_{ss} value is the cut-off price in the fiscal rule in a given base year. This value is adjusted annually to account for the external inflation and, therefore, $P^{oil}_{ss} P^*_t$ is the cut-off price for period t.

[7] The equation (2) has a unit root. To get rid of this, we used a practically equivalent equation in the programming code: $\tilde{B}^*_{G,t} = \rho_{\tilde{B}^*} \tilde{B}^*_{G,t-1} + \left(1-\rho_{\tilde{B}^*}\right) \tilde{B}^*_{G,ss} + \tau^{oil} S_t \left(\tilde{P}^{oil}_t - P^{oil}_{ss} P^*_t\right)$. Parameter $\rho_{\tilde{B}^*}$ was set close to 1.

In accordance with equation (2), during period t, the NWF will change by $\tau^{oil} S_t \left(\tilde{P}_t^{oil} - P_{ss}^{oil} P_t^*\right)$. Therefore, each period t, the amount

$$S_t \tilde{P}_t^{oil} - \tau^{oil} S_t \left(\tilde{P}_t^{oil} - P_{ss}^{oil} P_t^*\right) = \left(1-\tau^{oil}\right) S_t \tilde{P}_t^{oil} + \tau^{oil} S_t P_{ss}^{oil} P_t^*$$

will pass through the FX market: households will receive the unsmoothed portion of the funds $\left(1-\tau^{oil}\right) S_t \tilde{P}_t^{oil}$, while the state budget will receive the smoothed portion $\tau^{oil} S_t P_{ss}^{oil} P_t^*$.

The dynamics of the funds kept in the Treasury's accounts with the BoR A_t is described by the following equation:

$$A_t - A_{t-1} = -\varepsilon_t \tilde{B}_{G,t}^* + \varepsilon_t \left(1+i_{t-1}^*\right) \tilde{B}_{G,t-1}^* + \tilde{B}_{G,t} - \left(1+i_{t-1}\right) \tilde{B}_{G,t-1} + \tau_h \tilde{P}_t^H Y_t^H + \varepsilon_t \tau^{oil} \tilde{P}_t^{oil} S_t - \tilde{P}_t^Y G_t \;, \quad (3)$$

where ε_t stands for the nominal exchange rate, i_t^* – the yield of the NWF funds, i_t – the domestic loan rate, $\tilde{B}_{G,t}$ – government loans on the domestic market, $\tilde{P}_t^H Y_t^H$ – nominal output, τ_h – tax burden on the non-oil and gas sector, and $\tilde{P}_t^Y G_t$ – nominal expenses on government consumption.

In accordance with equation (3), the funds on the accounts change due to operations with the NWF, operations with domestic debt, the proceeds from unsmoothed tax NOGRs $\tau_h \tilde{P}_t^H Y_t^H$, the proceeds from unsmoothed OGRs $\varepsilon_t \tau^{oil} \tilde{P}_t^{oil} S_t$, and the expenses on government consumption.

Given the assumption made that the Treasury's accounts with the BoR are constant, the right part of (3) is zeroed out:

$$0 = -\varepsilon_t \tilde{B}_{G,t}^* + \varepsilon_t \left(1+i_{t-1}^*\right) \tilde{B}_{G,t-1}^* + \tilde{B}_{G,t} - \left(1+i_{t-1}\right) \tilde{B}_{G,t-1} + \tau_h \tilde{P}_t^H Y_t^H + \varepsilon_t \tau^{oil} \tilde{P}_t^{oil} S_t - \tilde{P}_t^Y G_t \quad . \quad (4)$$

The main limitation of the fiscal rule, according to which the state budget expenditures may not exceed the sum of the unsmoothed NOGRs, the smoothed OGRs, debt interest payments and 0.5% of GDP as a surcharge to implement infrastructural projects, may be notated with the variables introduced, as follows:

$$\tilde{P}_t^Y G_t \le \varepsilon_t i_{t-1}^* \tilde{B}_{G,t-1}^* + \tau_h \tilde{P}_t^H Y_t^H + \varepsilon_t \tau^{oil} P_{ss}^{oil} P_t^* S_t + M_t \quad . \quad (5)$$

The left part stands for nominal government consumption. The debt interest payments are zeroed out in both sides of the equation. The right side includes income from investing the NWF funds $\varepsilon_t\, i^*_{t-1}\tilde{B}^*_{G,t-1}$ as a component of NOGRs (in accordance with the legislation), unsmoothed tax NOGRs $\tau_h \tilde{P}^H_t Y^H_t$, smoothed OGRs $\varepsilon_t\, \tau^{oil} P^{oil}_{ss} P^*_t S_t$, and M_t – the surcharge of 0.5% of GDP as a surcharge to implement infrastructural projects, when it comes to the 2019-2020 period, or other hypothetical surcharges when it comes to future periods.

Let us analyse the equations obtained. One can express the NWF value $\tilde{B}_{G,t}$ from equation (2), substitute it in (4) and express the government consumption as:

$$\tilde{P}^Y_t G_t = \varepsilon_t\, i^*_{t-1}\tilde{B}^*_{G,t-1} + \tilde{B}_{G,t} - \left(1+i_{t-1}\right)\tilde{B}_{G,t-1} + \tau_h \tilde{P}^H_t Y^H_t + \varepsilon_t\, \tau^{oil} P^{oil}_{ss} P^*_t S_t \quad . \tag{6}$$

Then one can subtract (6) from (5):

$$\tilde{B}_{G,t} \le \left(1+i_{t-1}\right)\tilde{B}_{G,t-1} + M_t \ .$$

The obtained inequality says that the domestic debt during period t may increase by no more than the expenses for maintaining domestic debt $i_{t-1}\tilde{B}_{G,t-1}$ and the value of special measures M_t .

Let us take Δ_t as the discrepancy of equation (5):

$$\Delta_t = \varepsilon_t\, i^*_{t-1}\tilde{B}^*_{G,t-1} + \tau_h \tilde{P}^H_t Y^H_t + \varepsilon_t\, \tau^{oil} P^{oil}_{ss} P^*_t S_t + M_t - \tilde{P}^Y_t G_t \, .$$

Then $\Delta_t \ge 0$ is the value of the "underperformed" limitation (5) of the fiscal rule; and

$$\tilde{P}^Y_t G_t = \varepsilon_t\, i^*_{t-1}\tilde{B}^*_{G,t-1} + \tau_h \tilde{P}^H_t Y^H_t + \varepsilon_t\, \tau^{oil} P^{oil}_{ss} P^*_t S_t + M_t - \Delta_t \tag{7}$$

Subtracting expression (6) from (7), we will get:

$$\tilde{B}_{G,t} = \left(1+i_{t-1}\right)\tilde{B}_{G,t-1} + M_t - \Delta_t \quad . \tag{8}$$

It turns out that domestic government debt increases due to debt interest payments $i_{t-1}\tilde{B}_{G,t-1}$, changes (the direction of the change is undefined) due to special measures M_t and decreases due to the funds "saved" (in the sense of unperformed limitation (5) as an equation) on government consumption Δ_t.

Therefore, the values M_t and Δ_t are the parameters of the fiscal policy determining the level of domestic government debt. The difference between M_t and Δ_t lies only in the fact that M_t was legally introduced as a value set for 2019-2024, while Δ_t is a parameter that is not fixed in law. In view of this, it is practical to notate $\tilde{M}_t = M_t - \Delta_t$ and to subsequently understand $\tilde{M}_t$ not only as measures specifically prescribed but also as a result of performance of the fiscal rule inequality (5).

Then, the main limitation of the fiscal rule (5) can be written as an equation:

$$\tilde{P}_t^Y G_t = \varepsilon_t i_{t-1}^* \tilde{B}_{G,t-1}^* + \tau_h \tilde{P}_t^H Y_t^H + \varepsilon_t \tau^{oil} P_{ss}^{oil} P_t^* S_t + \tilde{M}_t \quad , \tag{9}$$

and the expression (8) – in the form of

$$\tilde{B}_{G,t} = \left(1 + i_{t-1}\right) \tilde{B}_{G,t-1} + \tilde{M}_t \quad . \tag{10}$$

The last expression may determine domestic debt (including as a ratio to the nominal GDP) as both a stable value over time and an unstable value depending on $\tilde{M}_t$. The models of the given type, i.e. DSGE, require that domestic debt be stable[8]. If, for example, $\tilde{M}_t = 0$ and $\tilde{B}_{G,0} = 0$ are zero initial borrowings and special measures, then

$$\tilde{B}_{G,t} = 0$$

and, obviously, domestic debt is stable.

In 2017-2019, Russian domestic debt was positive; limitation (5) of the fiscal rule was complied with as an equality ($\Delta_t = 0$); from 2019, the special measures caused an increase in

[8] More precisely, Blanchard-Kahn's condition must be satisfied

domestic debt ($\tilde{M}_t > 0$), while the domestic debt interest rate i_t exceeded the nominal GDP growth rates. This means that, if the current trends continue, not only the domestic debt value, but also the debt to GDP ratio will remain unstable. Two conclusions may be drawn here.

First, the fiscal rule inequality (5) should indeed be understood as *in*equality: as a general rule, it should be performed, with the right side exceeding the left side. It is incorrect to refer to ratio (5) as a consistently true equation, either from the perspective of long-term stability of government finances or from the perspective of mathematical models. It is advisable to understand the "fiscal rule", in the meaning of an equation, as ratio (9), taking into account the uncertainty accounted for within $\tilde{M}_t$.

Second, due to the uncertainty of the policy regarding the extent and direction of special measures $\tilde{M}_t$, we need to make a model assumption as to the dynamics of domestic debt to ensure domestic debt stability. Further, we will use two ways to define the dynamics of of government debt.

The first and simplest way is to set the dynamics of real government debt $B_{G,t} = \tilde{B}_{G,t} / P_t^C$ using the AR(1) process:

$$B_{G,t} = \rho_G \, B_{G,t-1} + (1-\rho_G) B_{G,ss} + B_{G,ss} e^{\varepsilon_t^G} \tag{11}$$

The second way assumes that domestic borrowings are aimed at smoothing the receipts of real NOGRs:

$$B_{G,t} = B_{G,t-1} + \tau_h \left(\tilde{P}_{ss}^H Y_{ss}^H / P_{ss}^C - \tilde{P}_t^H Y_t^H / P_t^C \right) + B_{G,ss} e^{\varepsilon_t^G} \tag{12}$$

In both expressions above, the shock ε_t^G is the budget borrowing shock. Given that, if the loans $\tilde{B}_{G,t}$ are excluded, the fiscal rule (9) begins to depend on the shock ε_t^G, and this shock may also be interpreted as a budget expense shock.

As a result, we have four variables that characterise fiscal policy and four equations that determine these variables: the volume of the NWF funds $\tilde{B}_{G,t}^*$ is described by expression (2), government consumption G_t – by (6), domestic government debt $\tilde{B}_{G,t}$ – by (11) or (12), the volume of special measures $\tilde{M}_t$ – by (10). Expression (10) may be excluded from the model as an expression that determines the isolated variable $\tilde{M}_t$.

In addition to the fiscal sector, the fiscal rule also affects the foreign exchange market. Foreign exchange market equilibrium condition in the notation of *Kreptsev, Seleznev (2017)* is as follows:

$$D_t^* - D_{t-1}^* R_{t-1}^* + P_t^{*H} Y_t^{*H} - P_t^{F*} IM_t^* + \tilde{P}_t^{oil} S_t + \left(1 + i_{t-1}^*\right) \tilde{B}_{G,t-1}^* - \tilde{B}_{G,t}^* = 0 \tag{13}$$

The first two terms represent household transactions with foreign bonds, the third term is non oil and gas exports, the fourth term is imports, the fifth term is oil and gas export earnings, and the last two terms are transactions with the NWF. NWF transactions smooth out the cyclical component of oil and gas export earnings and thus affect the exchange rate and the variables presented in the equation.

3.4. Comparison between the description of the fiscal rule with a similar one in *Medina, Soto (2007)*

Among DSGE models for commodity-based economies, the closest description of the fiscal sector is presented in *Medina, Soto (2007)*. The model describes the Chilean economy, which exports copper. The authors analyse several rules for organising the state budget, among which "Rule C" is the most interesting. This rule is based on the premise that there are no domestic loans and only external loans are used. The fiscal rule has the following form in the notations similar to the model in *Kreptsev, Seleznev (2017)*:

$$\tilde{P}_t^G G_t e^{-\varepsilon_t^G} = \left(1 - \frac{1}{\left(1 + i_{t-1}^*\right)\Theta_{t-1}}\right) \varepsilon_t \tilde{B}_{G,t-1}^* + \tau \tilde{P}_t^Y \bar{Y}_t + \varepsilon_t \chi P_{ss}^{oil} P_t^* S_t - B_{S,t} \tag{14}$$ [9]

Here $B_{S,t}$ is the "structural" budget surplus, $\bar{Y}_t$ – the output Y_t smoothed to account for the economic cycle, Θ – some external debt function which reflects a non-linear relationship of the external debt cost.

The fiscal rules (9) and (14) differ meaningfully in that, first, rule (14) proceeds from some smoothed production $\bar{Y}_t$, while rule (9) proceeds from unsmoothed NOGRs. Second, rule (9) has an additional instrument of domestic loans $\tilde{B}_{G,t}$, which is indirectly presented by the variable $\tilde{M}_t$.

In which case, will the forms of fiscal rules (9) and (14) be the same? For instance, if, in (10) we first take $i_{t-1}\tilde{B}_{G,t-1} = B_{S,t}$, i.e. the structural surplus, as equal to domestic debt cost. Second, if we take the change in domestic debt in accordance with the expression:

[9] Expression (27) in *Medina, Soto (2007)*

$$\tilde{B}_{G,t} = \tilde{B}_{G,t-1} + \tau_h \tilde{P}_t^H \left(\bar{Y}_t - Y_t^H\right)$$

– domestic debt is intended to smooth NOGRs. Then, having substituted $\tilde{M}_t$ from (10) in (9), we will obtain the rule in the form of (14) from *Medina, Soto (2007)*, which meaningfully differs, however, in terms of the fact that there are loans on the domestic market.

The way to determine domestic debt (12), as intended to smooth NOGRs, meaningfully repeats the idea in *Medina, Soto (2007)*.

3.5. Model specifications compared

Hereinafter we will compare several specifications of the model.

The first one is the initial model from *Kreptsev, Seleznev (2017)*, in which government consumption G_t is set by the AR(1) process (1). This specification is of the most pronounced procyclical nature of fiscal policy among all the specifications under examination, since it requires increasing taxes during an economic crisis and decreasing taxes during an economic recovery.

The second specification is the same model; however, instead of equation (1), we will use the AR(1) process to determine the ratio of government consumption to GDP $\frac{G_t}{Y_t}$:

$$\frac{G_t}{Y_t} = \left(\frac{G_{t-1}}{Y_{t-1}}\right)^{\rho_{GY}} e^{\varepsilon_t^G} \quad . \tag{15}$$

This method to determine government consumption has several advantages over ratio (1). First, the ratio of government consumption to GDP is one of the most stable ratios in the GDP structure and may, therefore, be interpreted as part of the fiscal policy. Second, with specification (15), the dynamics of government spending may be explained by the shocks that affect GDP and that are different from the shock of the AR(1) process ε_t^G.

The third specification are equations (2), (6) and (11), which substitute equation (1) in the initial model and which determine the following three variables: government consumption, the NWF funds and the domestic government debt. The fiscal rule (9) is a consequence of the balance of the government (6) and the domestic debt equation (11). Hereinafter we will refer to this specification as "the fiscal rule with AR(1) process for domestic debt".

The fourth specification is similar to the third one with the only difference that domestic debt is determined by ratio (12). Hereinafter, we will refer to this specification as "the fiscal rule with

NOGRs smoothing". Unlike the first two specifications, the specifications with the fiscal rule have a counter-cyclical nature and should have a stabilising effect on the economy.

Finally, specifications 5 and 6 repeat the first and second ones in terms of the set of equations but differ from them in the parameter estimation method. In particular, for specifications 1 through 4, first, an estimation based on data from 2006 until 2016 inclusively is made and, afterwards, posterior distribution of the parameters is used as a prior distribution to make the estimation for the period from 2017 (when the fiscal rule was introduced). As for specifications 5 and 6, we used all the data for the period from 2006 to 2019 at once to estimate the parameters. In this case, the model is less based on the data from 2017 to 2019 than in specifications 1 and 2.

4. Normalisation of variables, calibration and estimation

Before the model is solved, the variables of the model are expressed in real values and relative prices. Then, the equations are modified in accordance with the requirement for models that takes into account the random walk trend (*Seleznev (2016)*).

The new parameters and values are calibrated as follows. Although the tax burden on oil and gas exports τ^{oil} is not a tax set directly, it is, however, relatively stable in time and is estimated by the value 0.55. The coefficient τ_h was selected with a view to match the calibration of the current model with the calibration of the model in *Kreptsev, Seleznev (2017)*. It turned out that $\tau_h = 0.193$.

The autocorrelations parameters and the shock values for equations (11), (12) and (15) were estimated using the OLS method. As the statistic of domestic borrowings by the Russian Finance Ministry, we used the amount of the GKO-OFZ federal bonds.

The long-term value of domestic borrowings $\tilde{B}_{G,ss}$ was taken at 32% of the quarterly GDP, which is in line with the average ratio of the GKO-OFZ bond volume to GDP for the period 2018-2019. The long-term value of the NWF $\tilde{B}^*_{G,ss}$ was assessed on the basis of its current position. The upper bound of this value is the entire volume of the NWF (as at 1 May 2020), being 10.9% of the annualised GDP. However, given that over 9% of the NWF funds has been invested in Russia and Russia's external government debt in the form of bonds accounts for 22% of the NWF funds (as at 1 May 2020), the lower bound of the parameter $\tilde{B}^*_{G,ss}$ may be 6.7% of the annualised GDP. In the end, the value $\tilde{B}^*_{G,ss}$ was taken as 27% of the quarterly GDP.

To start with, we estimated the initial model based on the quarterly data for the 2006-2016 period. We characterise this period as a period with no fiscal rule. The observable variables set matched those used in *Kreptsev, Seleznev (2017)*, 22 series in total. We used the posterior average values of the estimated parameters as prior average values to estimate the model specifications for the periods beginning from 2017. Such an approach allows us to take into account information based on data up to 2016. The need to divide the observation period 2006-2019 into 2 periods is justified by the emergence of a budget rule in 2017.

Further in Section 5, for specifications 1 through 4, as well as for some other cases where the fiscal rule smooths oil and gas revenues only partially, we present impulse response functions for the oil price shock.

In Section 6, for each of the six model specifications, we present forecast errors for 1 period ahead forecasts.

5. Impulse response functions

Figure 5 shows the impulse response functions[10] for a positive 10% oil price shock for all the model variables observed. The impulse response functions are provided for model specifications 1 through 4, i.e. for the specifications for which the estimation was performed separately for the 2006-2016 period and 2017-2019 period.

For the specifications without the fiscal rule (specifications 1 and 2), the GDP response to the 10% positive oil price shock turned out to be positive – 0.09% and 0.06% during the first time period and 0.24% and 0.19% at the maximum.[11] However, for the specifications with the fiscal rule, the GDP response turns out to be negative during the first two quarters (-0.2% and -0.24% during the first quarter). The reason for the negative GDP response lies in the negative response of real government consumption (1.2% and 1.5%) during the first quarter. The negative government consumption response is explained by the strengthening of the exchange rate and, as a consequence, by the reduction of the smoothed OGRs $\varepsilon_t \tau^{oil} P_{ss}^{oil} P_t^* S_t$ in the fiscal rule (9). The income effect becomes visible several periods after the shock: household consumption grows in response to the increased income due to export revenues. When the income effect prevails over the effect of reduction in the real government consumption, GDP exceeds the initial level.

[10] Given that the model used in *Kreptsev, Seleznev (2017)* is classified as a model with stochastic trends, the values of the variables do not have to return to their initial values. Due to economic shocks, the system may find a new equilibrium point. The system in issue has an infinite number of equilibria. As a rule, the equilibrium point is determined as the position in which the system remains for an indefinitely long period unless affected by a shock.

[11] The value of the response turns to be smaller than in *Kreptsev, Seleznev. (2017)*. The order of response is the same. The reduction in the response value is explained by the adjustment of the model parameters to the statistic that reflects a smaller influence of the oil price shock on GDP after 2017.

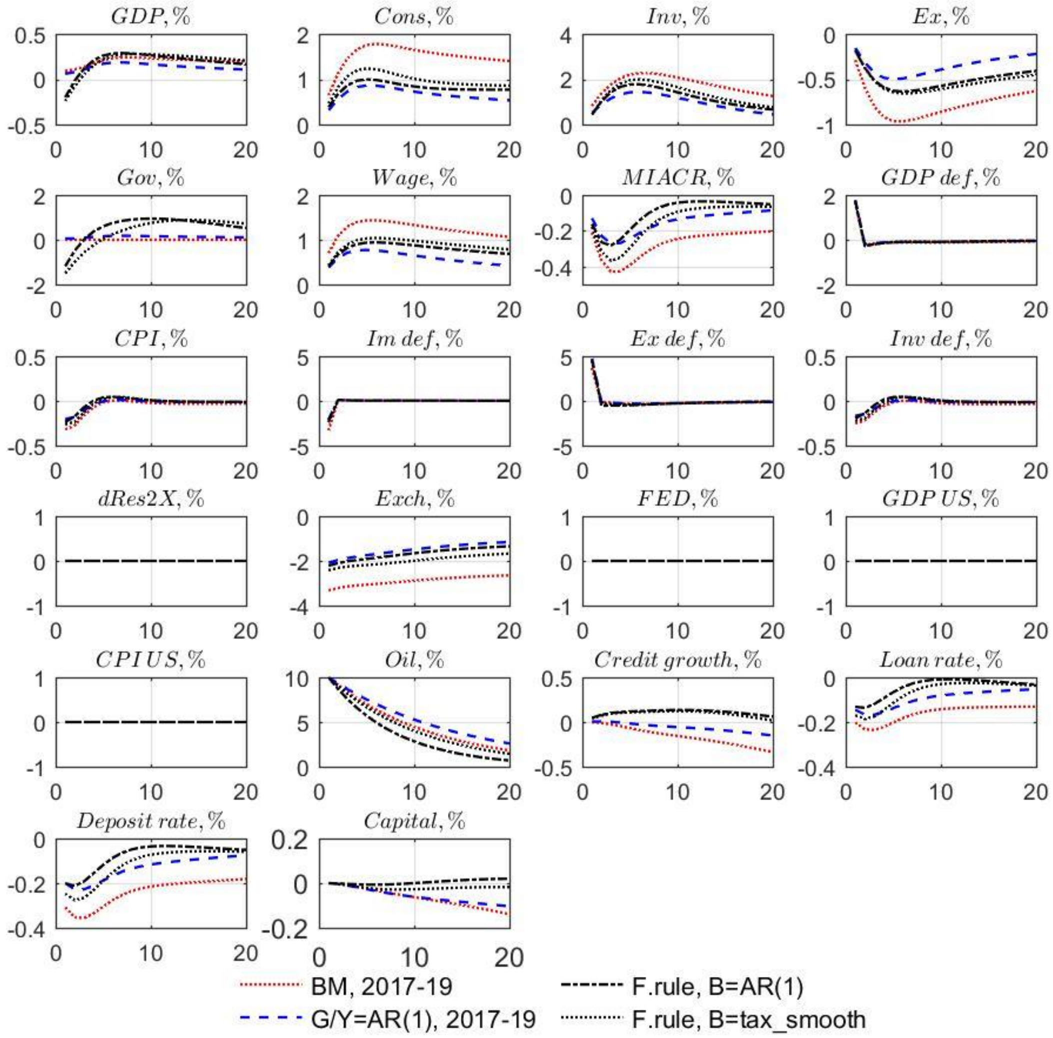

Figure 5 – Observable variables response to a 10% positive oil price shock. For specifications 1 through 4[12]

A similar decrease in GDP during the first time periods, in response to the growth of commodity prices, was received in *Kumhof, Laxton (2009)* for a model calibrated for the Chilean economy. In this case, the fiscal rule came down to smoothing NOGRs and smoothing copper export revenues flowing into the state budget. Specification 4 with the fiscal rule with NOGRs smoothing used in this paper approximates this case most closely. *Kumhof, Laxton (2009)* differs in that the surplus commodity revenues are invested on the domestic market rather than on the external market. The same authors in *Kumhof, Laxton (2009)* have demonstrated that the effect of the GDP decrease

[12] Hereinafter, in the legend: "BM, 2017-19" – specification 1, "G/Y=AR(1), 2017-19" – specification 2, "F.rule, B=AR(1)" – specification 3, "F.rule, B=tax_smooth" – specification 4, "BM, 2006-19" – specification 5 and "G/Y=AR(1), 2006-19" – specification 6. The names and dimensions of variables are presented in the table 1.

repeats also for the case when the government expenditures are fixed and when the active instrument of the fiscal policy is the NOGRs tax rate.

Does the obtained effect of the GDP decrease mean that similar fiscal rules also give a negative GDP response during the first time periods after the shock? No, it does not mean this. There may be changes in the fiscal rule that lead to the GDP increase in response to the oil price shock.

The first possibility is pretty obvious: only a portion of the cyclical component of oil revenues can be transferred to the NWF. For clarity, we will consider the specification "the fiscal rule with AR(1) process for domestic debt"[13] under the assumption that the OGRs are not smoothed – the entire OGRs flow into the state budget and the NWF is not, accordingly, replenished. In this case, equation (2) for the NWF and fiscal rule (9) will be as follows:

$$\tilde{B}^*_{G,t} = \tilde{B}^*_{G,t-1},$$

$$\tilde{P}^Y_t G_t = \varepsilon_t\, i^*_{t-1} \tilde{B}^*_{G,t-1} + \tau_h \tilde{P}^H_t Y^H_t + \varepsilon_t\, \tau^{oil} \tilde{P}^{oil}_t S_t + \tilde{M}_t \quad . \tag{16}$$

Second, as noted earlier, when planning government expenditures, OGR is valued at projected exchange rate, not the actual one. Thus, if the projected exchange rate is used, the OGRs could depreciate with delay as a result of the national currency appreciation. This can delay or even eliminate the decline in government consumption and GDP.

To accommodate this, we will use the following rule instead of the fiscal rule (6):

$$\tilde{P}^Y_t G_t = \varepsilon_t\, i^*_{t-1} \tilde{B}^*_{G,t-1} + \tilde{B}_{G,t} - \left(1 + i_{t-1}\right) \tilde{B}_{G,t-1} + \tau_h \tilde{P}^H_t Y^H_t + \varepsilon^{gov}_t\, \tau^{oil} P^{oil}_{ss} P^*_t S_t ,$$

where the projected exchange rate ε^{gov}_t is used instead of the actual exchange rate ε_t. The projected exchange rate is assumed to be equal to the actual one with a lag of 1 quarter:

$$\varepsilon^{gov}_t = \varepsilon_{t-1}.$$

Figure 6 presents IRFs for: 1) the base case with OGRs smoothing and the actual exchange rate in the fiscal rule, 2) the case with non-smoothed OGRs, 3) the case with lagged value exchange rate in the fiscal rule.

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[13] The results for the specification with the fiscal rule with NOGRs smoothing are similar

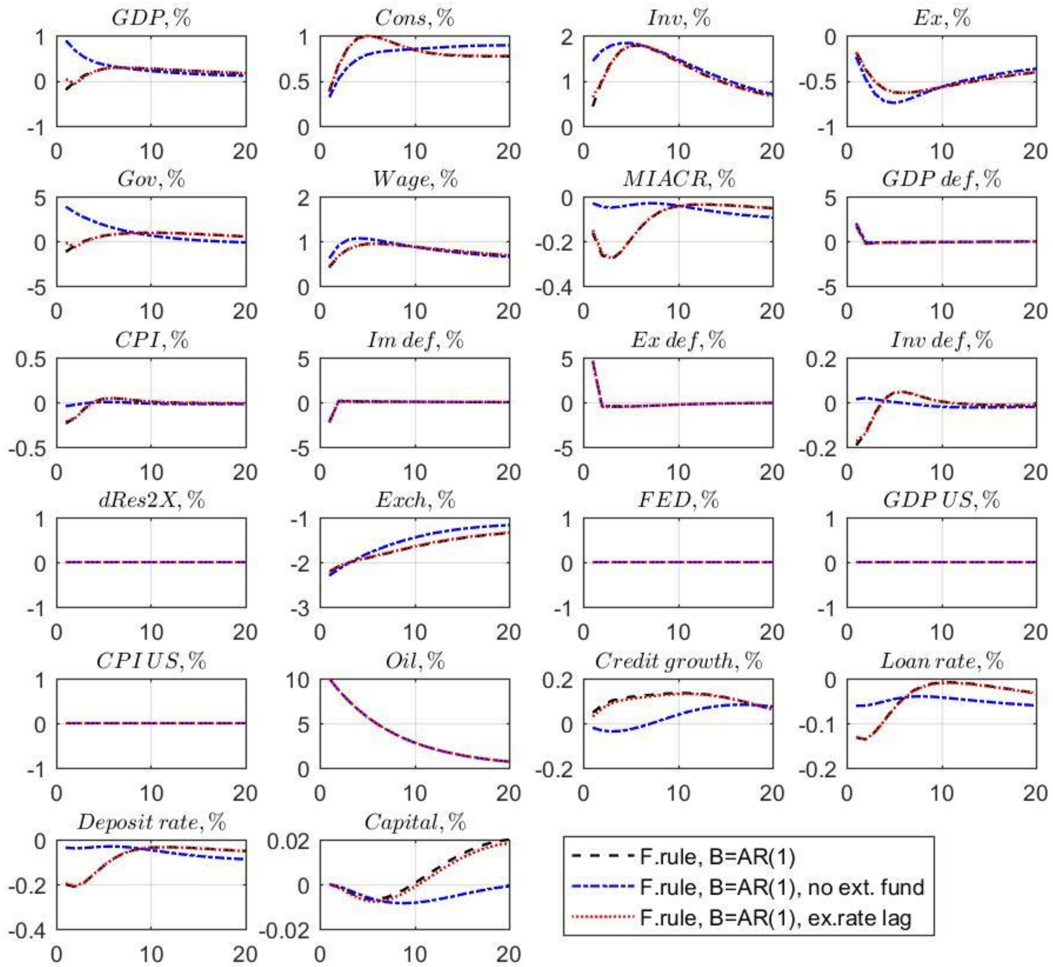


Figure 6 – Observable variables response to a 10% positive oil price shock for the specification with the fiscal rule with AR(1) process (specification 3). In the legend, "no ext. fund" is relevant for the case where OGRs are not smoothed, "ex. rate lag" is relevant for the case with lagged exchange rate in fiscal rule.

If OGRs are not smoothed, then GDP response amounts to 0.9% in response to a 10% oil price shock. In this case, the devaluation of OGRs flowing into the state budget is even stronger since the exchange rate strengthens more. However, the effect from the increased foreign currency funds flowing into the state budget exceeds the effect of their devaluation in the rouble equivalent and, as a result, in accordance with modified fiscal rule (16), real government consumption increases by 3.9% during the first time period after the shock.

Households' consumption includes import and domestic components and, therefore, inflation depends on both the growth rate of the import cost, i.e. the growth rate of the exchange rate, and the growth rate of prices for domestic products. The exchange rate response does not differ significantly between the fully smoothed and unsmoothed OGR cases, and hence the import inflation is -0.79% and -0.80% respectively. In the case of smoothed OGRs, the government demand for domestic products is low and, therefore, domestic inflation is lower than in the case of unsmoothed OGRs: 0.06% and 0.36% respectively. The contribution of import and domestic inflation to the CPI is 1:2 which corresponds to a negative CPI -0.23% (deflation) in the case of completely smoothed OGRs and -0.04% in the case of unsmoothed OGRs.

This experiment eventually demonstrates that the fiscal rule used by the Russian Finance Ministry does smooth the impact of the oil price shock on the output. In addition to the output, the stabilising effect also impacts investments, exports and government consumption, while exchange rate volatility reduces slightly. The magnitude of the stabilizing effect is presented in the Appendix B based on model simulations.

The experiment with the lagged exchange rate in the fiscal rule shows that the lagged exchange rate leads to smaller reductions in government spending in the first two quarters after the shock. As a result, the influence of the income effect on GDP compensates for the decrease in government spending and so the response of the GDP turns out to be non-negative.

Thus, both additional experiments with IRF show that some shifts in the fiscal rule can lead to an entirely positive GDP response in the presence of the positive oil price shock, as opposed to purely theoretical modeling of the fiscal rule[14].

It should also be noted that, according to Figure 6 and model simulations (Appendix B), the fiscal rule results in only a small decrease in exchange rate volatility. To understand the reasons for this, Figure 7 shows the responses of groups of variables involved in the foreign exchange market equilibrium condition (13): 1) households transactions with foreign bonds, 2) non oil and gas exports, 2) imports, and 4) the combined effect of oil revenues and transactions with NWF.

[14] In addition to the model specifications described in section 3.5 and 6, we took into account these ideas in forecasting experiment presented in chapter 6. We estimated the share of the cyclical component of oil revenues transmitted to the NWF, and also we set the exchange rate delay in the fiscal rule at 1-4 quarters and at steady state level. The qualitative results remained the same as in section 6.

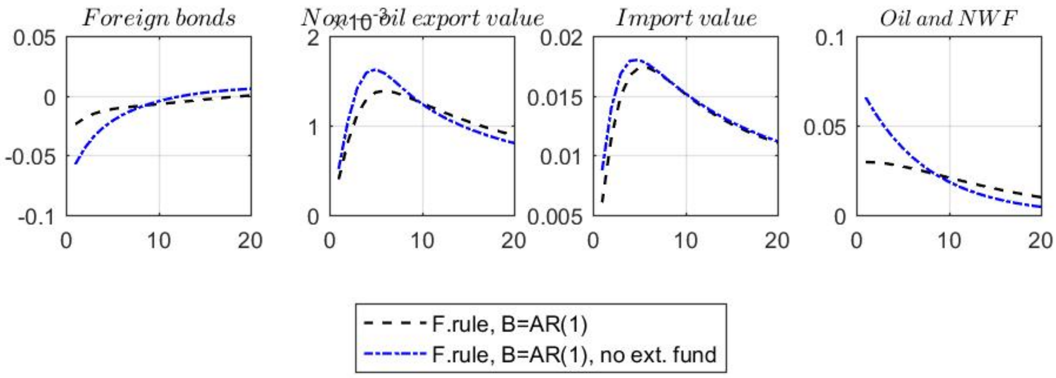


Figure 7 – The absolute deviation of the balance of payments components in response to a positive oil price shock

First, the change in import value response is quite small. The reason is that the government has a demand for a domestic product, the production of which does not use imported components. This seems plausible, since the input-output statistics indicate the presence of only 3-4% of the import component in government consumption. A small change in import value reaction cannot lead to a significant change in the exchange rate reaction.

Second, the combined effect of oil revenues and transactions with NWF is largely offset by households' foreign bond transactions. Note, that model households receive all profits from all producers. The change in government demand is approximately equal to the change in households' profits caused by a change in government demand. As a result, households withdraw funds from the country and compensate an influence of the fiscal rule smoothing mechanism. In reality, it is difficult to associate such behavior with Russian wage workers, but it is quite possible to associate it with business owners and state corporations.

We have concluded that the OGRs smoothing mechanism affects the BOP components reaction, but cannot significantly alter the exchange rate reaction.

We believe that the way how agents alter import demand in response to OGRs smoothing has a key impact on the extent to which exchange rate volatility changes. In *Jalali-Naini, Naderian* (2020), the state spends a significant share of its expenditure (40%) on imported goods. Therefore, the exchange rate adjusts to the change in the volume of imports when the smoothing mechanism changes. As a result, the fiscal rule significantly affects the volatility of the exchange rate. Similarly, in models with Ricardian and non-Ricardian households (as in *Medina, Soto* (2007)), in which a share of OGRs is transferred by the state to both types of households, non-Ricardian households spend a share of the additional income on import goods. This can also lead to a high impact of the fiscal rule on exchange rate volatility. There is no division into Ricardian and non-Ricardian households in the model considered. It also seems doubtful that Russian non-Ricardian households receive a part of the OGRs' cyclical component. Moreover, in the case of Russia, the statistics do not confirm the high share of government spending on imported goods.

The division into Ricardian and non-Ricardian households together with OGRs transfers to households leads, in our opinion, to a predefined significant effect of the fiscal rule on exchange rate volatility. Instead of a predefined result, we use a flexible approach in the form of model parameters estimation. The estimated parameters, among other things, determine the reaction of the risk premium and, consequently, the willingness of the owners to invest in foreign bonds. The results of estimation show that owners largely adjust to changes in the fiscal rule by investing in foreign bonds, which leads to a weak fiscal rule effect on the import and the exchange rate.

6. Quality of the forecasts

6.1. Quality of the oil price conditional forecast

The quality of the model specifications may be characterised by the quality of the forecasts. The forecasts of macroeconomic variables in themselves are of practical interest for the Bank of Russia.

Below, Figures 8-10 show one period ahead forecast errors for three variables: inflation, GDP and government consumption. In this case and hereafter, the forecasts are conditional on the oil price: the future oil price path is known to the agents. Table 1 shows the average forecasts errors for the majority of the observed model variables.[15]

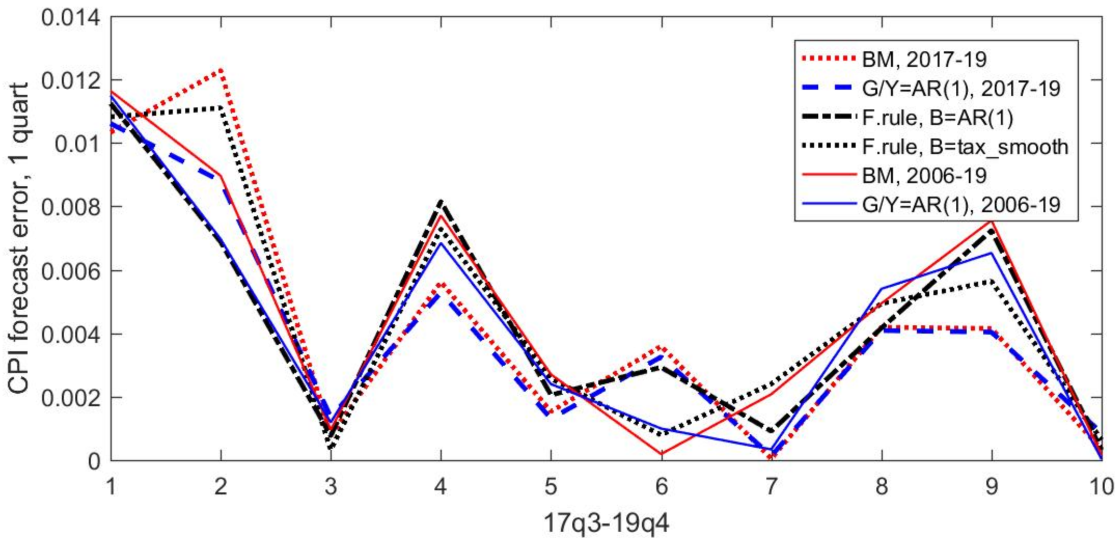


Figure 8 – The error of the inflation forecast conditional on the oil price for the six model specifications

[15] Except for the oil price and the variables characterising the external economy

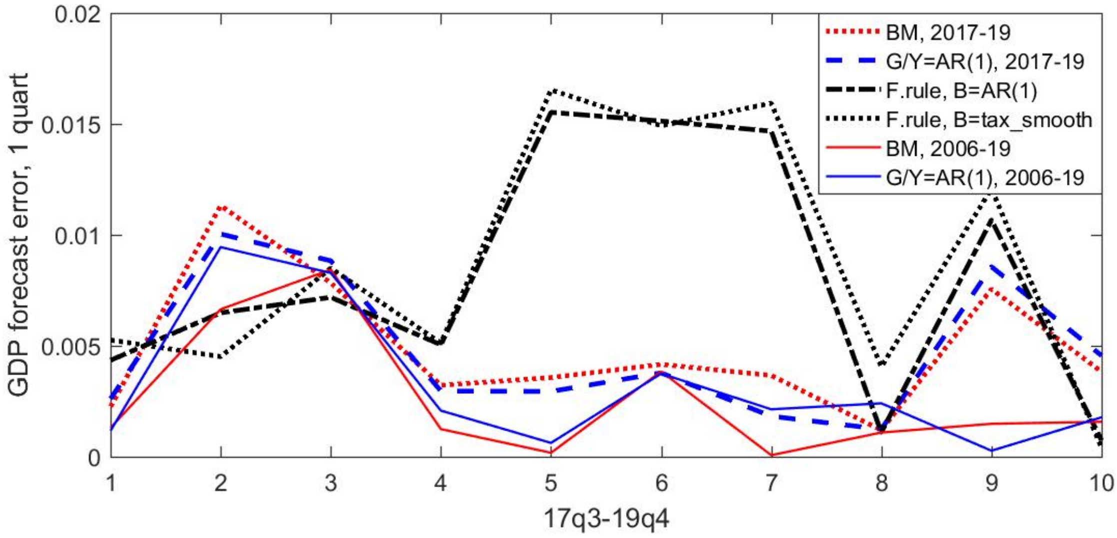


Figure 9 – The error of the real GDP forecast conditional on the oil price for the six model specifications

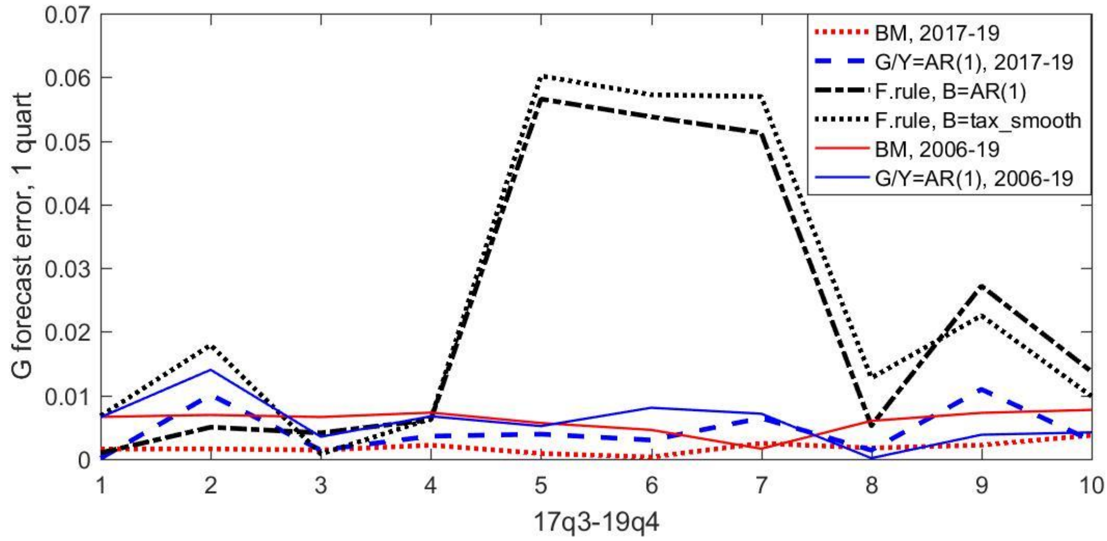


Figure 10 – The error of real government consumption forecast conditional on the oil price for the six model specifications

As one can see from figure 8, the quality of the inflation forecasts changes slightly depending on the model specification.

As for forecasting GDP and government consumption, the quality of the forecasts decreases considerably in the case of the fiscal rule (specifications 3 and 4). The quality of the GDP forecast decreases due to the decreased quality of the government consumption forecast.

Specifications 5 and 6, which use the data before 2017 more extensively, provide the best results in forecasting GDP. This could be interpreted as follows: the model cannot find anything crucially new in the GDP behaviour from the time when the fiscal rule was introduced in 2017.

Otherwise, if new mechanisms had begun to affect the GDP behaviour from 2017, then specifications 1 and 2, which allow for a greater use of the information from 2017, would adapt to the changes better and would produce a better GDP forecast. At the same time, specifications 5 and 6 are best at predicting only 2 variables.

Table 1 – Average (conditional on the oil prices) forecast errors of the observable variables during the period from 3Q 2017 until 4Q 2019

		Specifications					
Indicator	Unit	1	2	3	4	5	6
Inflation	Quarterly	0,429%	0,397%	0,447%	0,465%	0,469%	0,422%
Real GDP	Quarterly growth rate	0,486%	0,474%	0,809%	0,873%	0,259%	0,320%
Real household consumption	Quarterly growth rate	0,625%	0,491%	0,483%	0,598%	0,827%	0,832%
Real investment	Quarterly growth rate	3,862%	3,651%	3,389%	3,308%	4,154%	3,949%
Real export	Quarterly growth rate	2,225%	2,135%	2,156%	2,193%	2,214%	2,183%
Real government consumption	Quarterly growth rate	0,182%	0,439%	2,241%	2,514%	0,604%	0,594%
Real salaries	Quarterly growth rate	0,791%	0,686%	0,857%	0,689%	0,916%	0,953%
Interbank interest rate	Quarterly	0,068%	0,064%	0,067%	0,067%	0,072%	0,071%
GDP deflator	Quarterly	1,467%	1,477%	1,515%	1,448%	1,496%	1,589%
Import deflator	Quarterly	4,081%	3,551%	3,704%	4,042%	3,995%	3,696%
Export deflator	Quarterly	3,561%	3,550%	4,496%	3,975%	3,818%	3,857%
Investment deflator	Quarterly	2,086%	2,050%	2,087%	2,097%	2,182%	2,204%
Interventions to exports	In proportions	0,141	0,141	0,141	0,141	0,141	0,141
Exchange rate	Quarterly growth rate	4,526%	4,171%	4,258%	4,349%	4,831%	4,492%
Bank system loans	Quarterly growth rate	0,993%	1,024%	1,051%	1,020%	1,050%	1,107%
Loan rate	Quarterly	0,084%	0,087%	0,103%	0,097%	0,355%	0,346%

Deposit rate	Quarterly	0,083%	0,074%	0,069%	0,072%	0,081%	0,083%
Bank system capital	Quarterly growth rate	2,605%	2,625%	2,595%	2,648%	2,543%	2,486%

According to the table, the fiscal rule with AR(1) process for domestic debt (specification 3) turns out to be the best in forecasting household consumption, and the deposit rates, while the fiscal rule with NOGRs smoothing (specification 4) turns out to be the best in forecasting investments and GDP deflator. So specifications 3 and 4 are best at predicting 4 variables, specification 2 is best at predicting 8 variables, and specification 1 is best at predicting 3 variables.

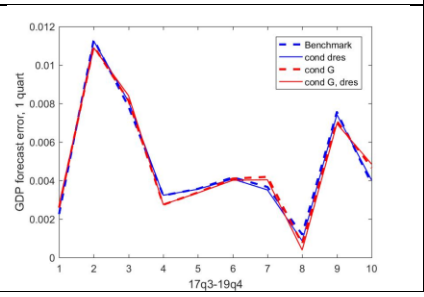
6.2. A general experiment to improve the forecast quality

None of the proposed specifications significantly improves the inflation forecast. To what extent is this result independent from the introduction of other model specifications which, theoretically, describe the budget sector better than the existing specifications?

This question may be answered as follows. Let's assume that we have an ideal specification of the fiscal sector, i.e. one that exactly predicts the behaviour of fiscal variables. If the ideal specification provides a better forecast of inflation and other variables than the existing specifications, then it makes sense to look for better specifications. However, if the ideal specification does not give a better forecast, this means that any change in the description of the fiscal sector is meaningless from the perspective of improving the quality of the forecast.

To imitate the existence of an ideal specification, we will build forecasts conditional on fiscal variables. The fiscal variables include, in the first place, government consumption. Second, the fiscal variables may also include the Bank of Russia's operations in buying and selling currency on the open market since, beginning from mid-2015, these operations are performed only in the interests of the Russian Finance Ministry.

Figure 11 below shows the errors of the forecasts conditional on the variables of government consumption, FX interventions and both these variables, for certain model specifications.

<table>
<tr><th></th><th>Real GDP</th><th>Real government consumption</th><th>Inflation</th></tr>
<tr><td>Specification 1</td>
<td>
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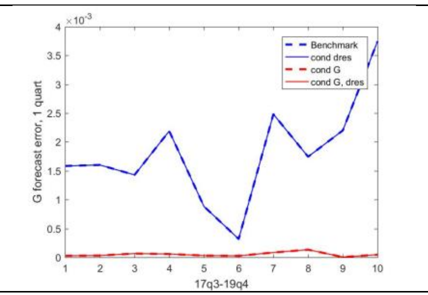
<td>
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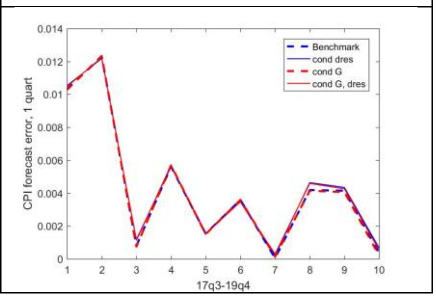
<td>
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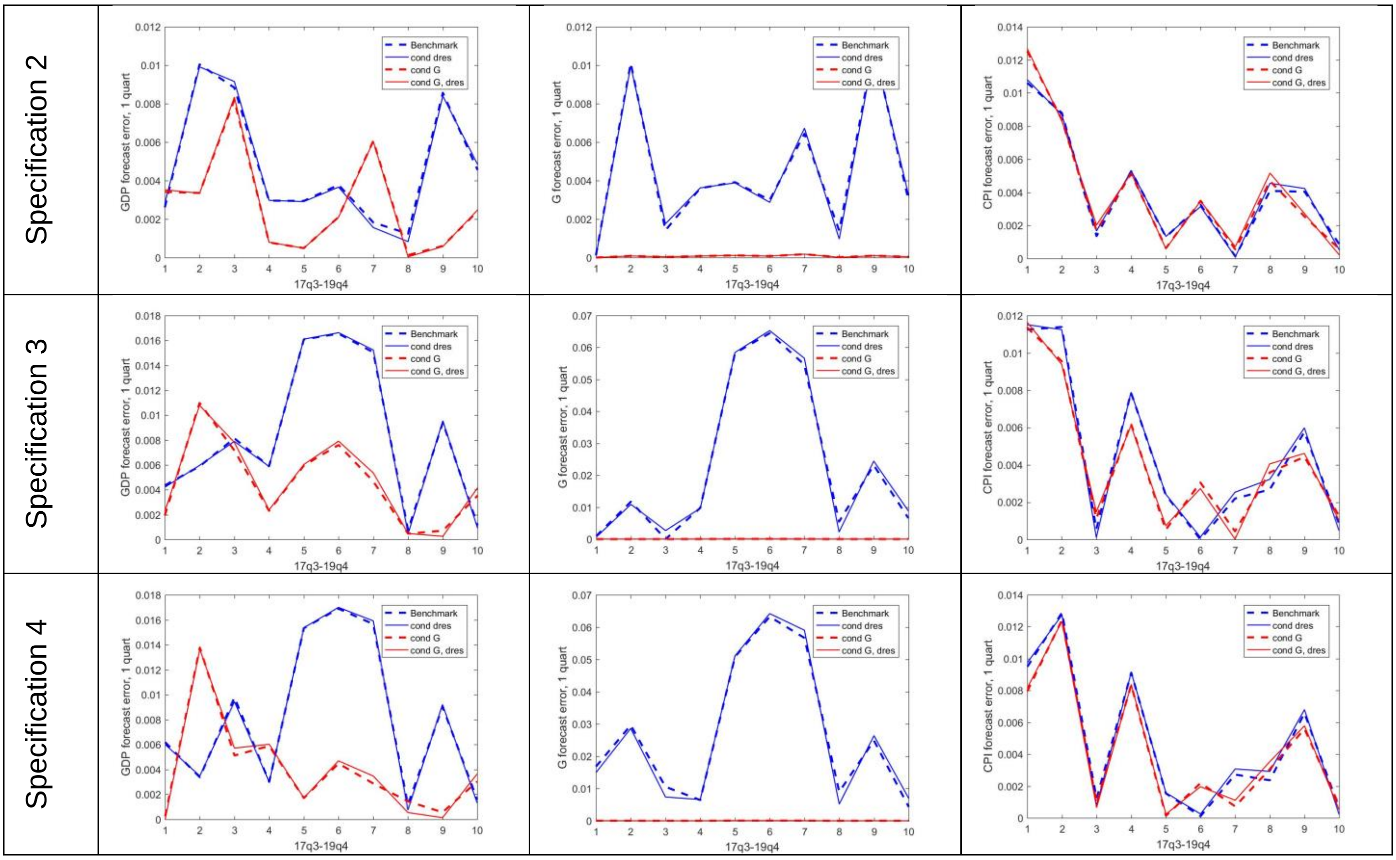

Figure 11 – The errors of the forecast conditional on government consumption and interventions for the periods from 3Q 2017 until 4Q 2019. Blue dash line – unconditional forecast (conditional on the oil price only), blue solid line – conditional on FX interventions, red dash line – conditional on government consumption, red dash line – conditional on both government consumption and FX interventions

The quality of the inflation forecast changes slightly depending on the agents' knowledge of fiscal variables. It follows that, in order to improve the quality of forecasting inflation, it does not make sense to modify the descriptions of the fiscal sector in this model.

The quality of the GDP forecast virtually does not depend on the agents' knowledge of the FX interventions: either the interventions have a slight effect on GDP or the volume of the interventions actually made in 2017-2019 is insignificant for changing GDP.

The quality of the GDP forecast may be considerably improved if government consumption is known and the fiscal rule is used. Therefore, in order to improve the GDP forecast, it is advisable to change the description of the fiscal sector.

How does the quality of a forecast conditional on government consumption depend on model specifications? Figures 12-13 below show the errors of the GDP and inflation forecast conditional on oil prices and government consumption, and Table 2 shows the average forecast errors for all observable variables.

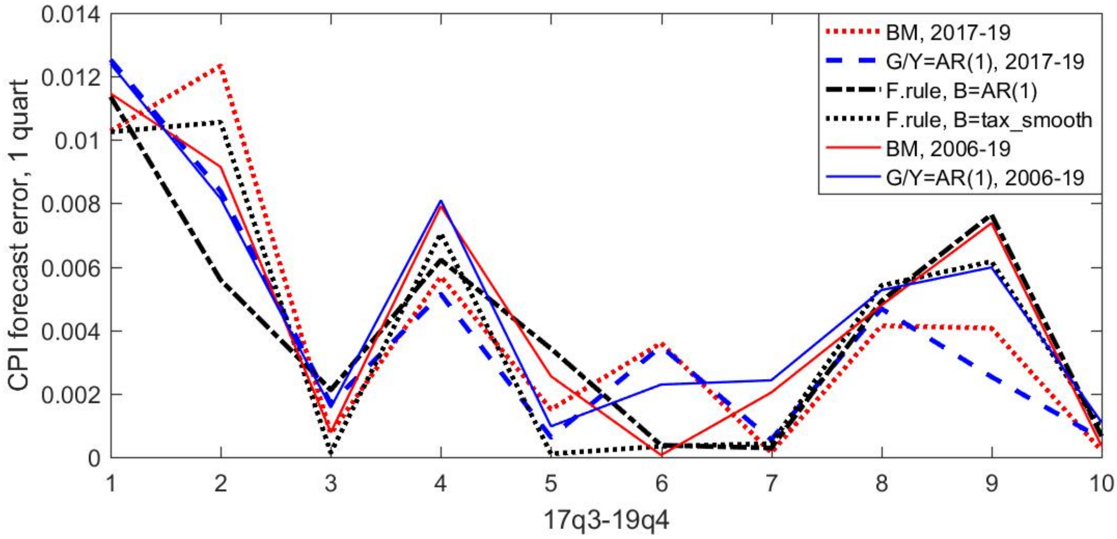

Figure 12 – The error of the inflation forecast conditional on the oil price and government consumption for the six model specifications

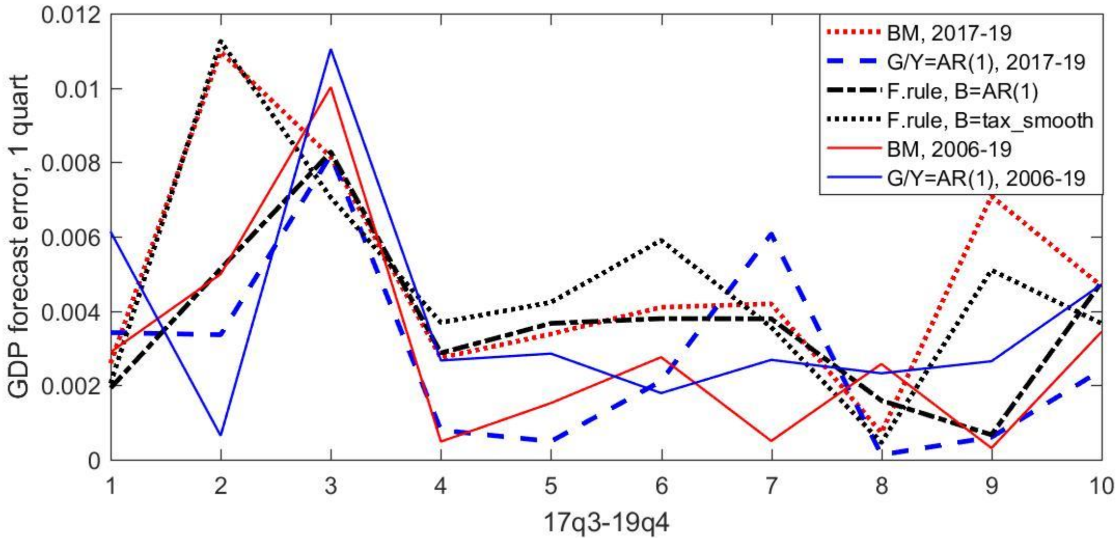

Figure 13 – The error of the real GDP forecast conditional on the oil price and government consumption for the six model specifications

As it has been shown earlier, the quality of the inflation forecast does not depend on the model specification (Figure 12). In the case of a fiscal rule, the quality of the GDP forecast improves and becomes comparable with the quality of other forecasts (Figure 13).

Table 2 – Average (conditional on the oil prices and government consumption) forecast errors of the observable variables during the period from 3Q 2017 until 4Q 2019

		Specifications

Indicator	Unit	1	2	3	4	5	6
Inflation	Quarterly	0,427%	0,402%	0,426%	0,413%	0,465%	0,482%
Real GDP	Quarterly growth rate	0,486%	0,276%	0,365%	0,470%	0,295%	0,375%
Real household consumption	Quarterly growth rate	0,626%	0,441%	0,485%	0,638%	0,834%	0,767%
Real investment	Quarterly growth rate	3,852%	3,197%	3,467%	3,435%	4,315%	4,060%
Real export	Quarterly growth rate	2,224%	2,170%	2,037%	1,984%	2,214%	2,206%
Real government consumption	Quarterly growth rate	0,005%	0,008%	0,003%	0,003%	0,020%	0,010%
Real salaries	Quarterly growth rate	0,794%	0,706%	0,856%	0,639%	0,962%	1,086%
Interbank interest rate	Quarterly	0,067%	0,076%	0,077%	0,079%	0,072%	0,084%
GDP deflator	Quarterly	1,465%	1,527%	1,524%	1,414%	1,499%	1,597%
Import deflator	Quarterly	4,086%	3,463%	3,582%	4,063%	4,004%	3,661%
Export deflator	Quarterly	3,562%	3,726%	4,663%	4,307%	3,831%	3,929%
Investment deflator	Quarterly	2,088%	2,041%	2,015%	2,061%	2,188%	2,210%
Interventions to exports	In proportions	0,141	0,140	0,146	0,147	0,141	0,140
Exchange rate	Quarterly growth rate	4,522%	4,350%	4,091%	4,054%	4,843%	4,589%
Bank system loans	Quarterly growth rate	0,993%	1,033%	1,069%	1,043%	1,049%	1,090%
Loan rate	Quarterly	0,084%	0,106%	0,115%	0,095%	0,353%	0,388%
Deposit rate	Quarterly	0,082%	0,077%	0,068%	0,071%	0,082%	0,082%
Bank system capital	Quarterly growth rate	2,605%	2,668%	2,593%	2,635%	2,543%	2,428%

Specifications 1 through 6 are best when predicting 4, 5, 2, 4, 0 and 1 variables, respectively. Therefore, in the case of forecast conditional on the government consumption, specifications with a fiscal rule start to perform better. Specification 2 continues to outperform other specifications by predicting 5 variables including CPI in the best way.

7. Conclusion

In order to analyse how the fiscal rule affects the relationship between the volatility of GDP and oil prices and to analyse whether it is possible to use the model description of the fiscal rule in order to improve the forecast quality, we supplemented the DSGE model used by the Bank of Russia with a more detailed description of the fiscal sector.

The analysis of the fiscal rule used in Russia has revealed that the legislature does not determine precisely the behaviour of the three fiscal variables we have identified: government spending, domestic borrowings by the fiscal authority, and external investment (the dynamics of the Russian NWF funds). The uncertainly stems from the lack of a precise prescription as to the volume of borrowings by the Russian Finance Ministry, while there is only an upper limit on the borrowings. Extrapolating the current trends in the fiscal sector would be an unstable way to develop government finances. This is why, having assumed the behaviour of domestic debt, we determined for our analysis two specifications with the fiscal rule, which demonstrate stable dynamics of government finances. In the model, the specifications with the fiscal rule, as prescribed by legislature, have fully smoothed oil and gas revenues flowing into the budget.

First, in terms of impulse response functions, the fiscal rule leads to a decrease in output volatility and a slight decrease in exchange rate volatility. The decline in GDP volatility is due to the smoothing of the government's demand for final products. The fiscal rule only slightly reduces exchange rate volatility due to the absence of the government demand for imported goods and the offsetting effect of changes in owner's demand for foreign bonds in response to changes in government demand for foreign exchange. It turned out that under the fiscal rule, the disinflationary effect is more pronounced in response to a positive shock in the oil price. Stronger disinflationary effect is explained by the almost fixed response of import inflation (exchange rate) and the fall in domestic inflation in response to a drop in government demand.

Second, where there is a fiscal rule in place that fully smooths oil and gas revenues, the GDP reacts negatively during the first quarters after a positive oil price shock. This effect is explained by the strengthening of the exchange rate and devaluation of oil and gas revenues, which decreases government consumption and is similar to the results obtained in scientific literature. We have demonstrated that some shifts in the fiscal rule like lagged exchange rate or partial government consumption of the cyclical component of the oil and gas revenues lead to positive GDP response.

Making forecasts on a short time interval from 2017 until 2019 shows that the use of the fiscal rule in the model does not improve the forecast quality: the quality of the inflation forecast remains the same and the quality of the GDP forecast worsens. Therefore, simpler descriptions of the dynamics of government consumption, as compared to the fiscal rule, provide forecasts of at least the same quality. This means that, at present in the existing circumstances, when making forecasts, we can use simple descriptions of the fiscal variables. We can use the fiscal rule when modelling qualitative shifts in the fiscal policy.

We have also identified that specifications with the fiscal rule start to perform better (comparative with other specifications) when making forecasts conditional on government consumption.

In the end, as for the possible causes that might decrease the dependence of GDP on oil prices over the last years, the results obtained turn out to be contradictory: in theory, at the level of impulse response functions, the stabilising effect of the fiscal rule on the output is confirmed; however, forecast errors do not testify in favour of the assumption that the fiscal rule significantly influences the observed dynamics of economic indicators.

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9. Appendix A: business cycle moments

Table3 presents standard deviations of the main macroeconomic variables. Table 4 presents correlation matrix for the main variables.

Table 3 – Standard deviations of the main macroeconomic variables for Russia. The data is presented in percent deviation from the long-term trend (HP filter, 1600)

	Standard deviation				Ratio: variable st.d./ oil price st.d.			
Data series	99q1-13q4	14q1-19q3	14q1-16q4	17q1-19q3	99q1-13q4	14q1-19q3	14q1-16q4	17q1-19q3
Real GDP	2,7%	1,3%	1,6%	0,8%	0,131	0,052	0,051	0,056
Real household consumption	3,3%	3,8%	5,0%	1,7%	0,157	0,150	0,155	0,118
Real government consumption	1,3%	1,3%	1,7%	0,5%	0,063	0,050	0,052	0,037
Real investment	17,3%	5,8%	7,6%	2,7%	0,838	0,229	0,236	0,187
Oil price	20,6%	25,2%	32,0%	14,7%	1,000	1,000	1,000	1,000
Exchange rate	7,5%	14,1%	18,7%	5,9%	0,364	0,560	0,586	0,402
Interest rate (MIACR)	2,5%	1,9%	2,5%	0,9%	0,121	0,076	0,078	0,064
CPI	1,3%	1,2%	1,6%	0,6%	0,063	0,049	0,051	0,039

Table 4 – Correlation matrix. The data is taken as a percentage deviation from the long-term trend (HP filter, 1600). The periods are 99q1-13q4 / 14q1-19q3

Data series		1	2	3	4	5	6	7	8
Real GDP	1	1/1	0,94/0,83	0,65/0,47	0,75/0,8	0,81/0,7	-0,61/-0,66	-0,44/-0,19	0,16/0,02
Real household consumption	2	0,94/0,83	1/1	0,63/0,54	0,64/0,49	0,82/0,44	-0,61/-0,65	-0,38/0,14	0,17/0,2
Real government consumption	3	0,65/0,47	0,63/0,54	1/1	0,82/0,14	0,74/0,31	-0,83/-0,5	-0,8/-0,07	-0,28/-0,05
Real investment	4	0,75/0,8	0,64/0,49	0,82/0,14	1/1	0,67/0,69	-0,71/-0,47	-0,75/-0,36	-0,23/-0,14
Oil price	5	0,81/0,7	0,82/0,44	0,74/0,31	0,67/0,69	1/1	-0,8/-0,62	-0,61/-0,59	-0,08/-0,24
Exchange rate	6	-0,61/-0,66	-0,61/-0,65	-0,83/-0,5	-0,71/-0,47	-0,8/-0,62	1/1	0,74/0,24	0,39/-0,13

Interest rate (MIACR)	7	-0,44/-0,19	-0,38/0,14	-0,8/-0,07	-0,75/-0,36	-0,61/-0,59	0,74/0,24	1/1	0,62/0,46
CPI	8	0,16/0,02	0,17/0,2	-0,28/-0,05	-0,23/-0,14	-0,08/-0,24	0,39/-0,13	0,62/0,46	1/1

Note that estimates for the periods 14q1-16q4 and 17q1-19q3 should be treated with caution. First, the number of observations for these periods is small, and the confidence interval is wide. Secondly, these periods are shorter than the length of one business cycle.

The estimates show that the volatility of Russian macroeconomic variables has decreased since 2014. The decline in volatility is more pronounced for GDP, investment and interest rate. For the exchange rate, household consumption, government consumption and inflation, this conclusion turns out to be true only for the 17q1-19q3 period. If we assume that all the volatility of the variables is due to the change in the oil price, then the ratio of the standard deviation of the variable to the standard deviation of the oil price will demonstrate the degree of the oil price impact. Then, in accordance with Table 3, the impact of oil prices on household consumption, exchange rate and inflation was generally the same for the 17q1-19q3 period as for the 99q1-13q4 period. The impact of oil prices on GDP decreased by 2 times.

According to Table 4, correlations between macroeconomic variables have generally declined since 2014.

10. Appendix B: standard deviations of simulated variables

Here we present the standard deviations of the simulated variables. One case concerns the fiscal rule with AR(1) process for domestic debt (specification 3). This case corresponds to the complete smoothing of OGRs. Another case is the same case but with unsmoothed OGRs.

Volatility is the result of a simulated oil price shock with the estimated standard deviation of 17.9%.

Table 5 – Standard deviations of simulated variables. Specification 3 for completely smoothed and unsmoothed cyclical component of oil and gas revenues cases.

Variable	Unit	Model specifications	
		Unsmoothed OGRs	completely smoothed OGRs (specification 3)
Inflation	Quarterly	0,23%	0,59%

Real GDP	Quarterly growth rate	1,63%	0,57%
Real household consumption	Quarterly growth rate	0,75%	0,97%
Real investment	Quarterly growth rate	2,61%	1,68%
Real export	Quarterly growth rate	0,73%	0,58%
Real government consumption	Quarterly growth rate	7,02%	2,71%
Real salaries	Quarterly growth rate	1,25%	0,94%
Interbank interest rate	Quarterly	0,33%	0,37%
GDP deflator	Quarterly	3,65%	3,16%
Import deflator	Quarterly	4,06%	3,88%
Export deflator	Quarterly	8,45%	8,52%
Investment deflator	Quarterly	0,24%	0,50%
Oil price	Quarterly growth rate	17,89%	17,89%
Exchange rate	Quarterly growth rate	4,04%	3,86%
Bank system loans	Quarterly growth rate	0,23%	0,24%
Loan rate	Quarterly	0,30%	0,30%
Deposit rate	Quarterly	0,30%	0,31%
Bank system capital	Quarterly growth rate	0,19%	0,19%

11. Appendix C: model code for Dynare

/*
"DSGE Model of the Russian Economy with the Banking Sector and the Fiscal Rule"

This is the code for the extension of the model presented in

Kreptsev, D., Seleznev S. (2017), "DSGE model of the Russian economy with a banking sector", Bank of Russia Working Paper Series, WP 27. Available online: https://www.cbr.ru/Content/Document/File/87562/wp27_e.pdf

The "fiscal rule" extension differs from Kreptsev, Seleznev (2017) in equations 'Trade balance', 'Fiscal rule ', 'Dynamics for external fund', 'Government budget constraint', 'projected fiscal exchange rate'.

This code corresponds to the curve for the fiscal rule with AR(1) process for domestic debt ("F.rule, B=AR(1)") in the figures 5 and 6.

Note, that figures 5 and 6 show the deviations from a constant steady state, i.e. sum(x(tau),tau=1..t)) = cumsum(x), where x corresponds to the variables g_Y_obs, g_C_p_obs, g_I_obs, g_X_obs, g_G_obs, g_w_obs, g_e_obs, g_Y_star_obs, d_p_oil_obs, g_b_DC_obs, g_J_obs.
*/

var C R zeta_C pi_c R_NFA g_e w l K A_c Y p_Y z eps_h p_H pi_H eps_f pi_F p_F p_F_star rer p_H_star eps_h_star pi_H_star C_p C_H C_F I U I_H I_F p_I q K_bar K_bar_str u_DC zeta_I d_star dres IM p_oil p_oil_fast p_oil_slow S_oil Y_star R_star pi_c_star zeta_C_star l_star w_star A_star p_Y_star G g_A z_RP Y_H_star Y_H Im K_bar_DC R_k_DC z_DC omega_bar_DC R_en b_DC n_DC gamma_DC sigma_omega_DC z_w GAMMA_DC G_DC GAMMA1_DC G1_DC K_bar_DC_pr R_en1 R_en2 R_en3 J profit_b R_DC R_D omega_J eps_D R_D1 R_D2 R_D3 eps_cap BG_star BG_dom rer_G g_Y_obs g_C_p_obs g_I_obs g_X_obs g_G_obs g_w_obs MIACR_obs g_p_Y_obs g_CPI_obs g_p_F_star_obs g_p_X_obs g_p_I_obs dRes2X_obs g_e_obs FED_obs g_Y_star_obs g_CPI_star_obs d_p_oil_obs g_b_DC_obs R_en4_obs R_D4_obs g_J_obs;

varexo e_p_oil_slow e_p_oil_fast e_MP_star e_MP e_res e_g_A e_zeta_C e_zeta_I e_A_c e_U e_eps_h e_eps_f e_p_F_star e_eps_h_star e_S_oil e_z_RP e_zeta_C_star e_A_star e_G e_sigma_omega_DC e_gamma_DC e_omega_J e_eps_D e_eps_cap;

parameters beta_d h eps_w phi k_w iota_w pi_target alpha1 k_H iota_H k_F iota_F k_H_star iota_H_star pi_target_star gamma_C nu_C gamma_I nu_I delta k_I sigma_a_DC p_oil_ss_par beta_d_star h_star zeta_l_star phi_star eps_foreign k_star iota_star phi_R_star phi_pi_star phi_R phi_pi phi_nfa gamma_export

nu_export g_A_ss_par zeta_C_ss_par zeta_l_ss_par A_c_ss_par U_ss_par eps_h_ss_par eps_f_ss_par p_F_star_ss_par eps_h_star_ss_par S_oil_ss_par z_RP_ss_par zeta_C_star_ss_par A_star_ss_par G_ss_par phi_oil mu1 tr_e_DC sigma_omega_DC_ss_par gamma_DC_ss_par PHI o delta_b omega_J_ss_par k_K k_D k_DC iota_DC iota_D eps_DC eps_D_ss_par eps_cap_ss_par tau smooth_par str_def_par rho_p_oil_slow rho_p_oil_fast rho_g_A rho_zeta_C rho_zeta_l rho_A_c rho_U rho_eps_h rho_eps_f rho_p_F_star rho_eps_h_star rho_S_oil rho_z_RP rho_zeta_C_star rho_A_star rho_G rho_gamma_DC rho_sigma_omega_DC rho_omega_J rho_eps_D rho_eps_cap rho_BG_star tau_oil
C_ss R_ss zeta_C_ss pi_c_ss R_NFA_ss g_e_ss w_ss l_ss K_ss A_c_ss Y_ss p_Y_ss z_ss eps_h_ss p_H_ss pi_H_ss eps_f_ss pi_F_ss p_F_ss p_F_star_ss rer_ss p_H_star_ss eps_h_star_ss pi_H_star_ss C_p_ss C_H_ss C_F_ss I_ss U_ss I_H_ss I_F_ss p_I_ss q_ss K_bar_ss K_bar_str_ss u_DC_ss zeta_l_ss d_star_ss dres_ss IM_ss p_oil_ss p_oil_fast_ss p_oil_slow_ss S_oil_ss Y_star_ss R_star_ss pi_c_star_ss zeta_C_star_ss l_star_ss w_star_ss A_star_ss p_Y_star_ss G_ss g_A_ss z_RP_ss Y_H_star_ss Y_H_ss Im_ss K_bar_DC_ss R_k_DC_ss z_DC_ss omega_bar_DC_ss R_en_ss b_DC_ss n_DC_ss gamma_DC_ss sigma_omega_DC_ss z_w_ss GAMMA_DC_ss G_DC_ss GAMMA1_DC_ss G1_DC_ss K_bar_DC_pr_ss R_en1_ss R_en2_ss R_en3_ss J_ss profit_b_ss R_DC_ss R_D_ss omega_J_ss eps_D_ss R_D1_ss R_D2_ss R_D3_ss eps_cap_ss BG_star_ss BG_dom_ss rer_G_ss g_Y_obs_ss g_C_p_obs_ss g_I_obs_ss g_X_obs_ss g_G_obs_ss g_w_obs_ss MIACR_obs_ss g_p_Y_obs_ss g_CPI_obs_ss g_p_F_star_obs_ss g_p_X_obs_ss g_p_I_obs_ss dRes2X_obs_ss g_e_obs_ss FED_obs_ss g_Y_star_obs_ss g_CPI_star_obs_ss d_p_oil_obs_ss g_b_DC_obs_ss R_en4_obs_ss R_D4_obs_ss g_J_obs_ss;

rho_p_oil_slow = .869284509229340; rho_p_oil_fast =0;
beta_d = .998960644884841; h = .638672165057401; eps_w = 10; phi = 2; k_w = 63.1097932008467; iota_w = .360373012995306; pi_target = 1.00985340654897; alpha1 = .333333333333333; k_H = 12.7914229866517; iota_H = .438486322164074; k_F = 25.5649158311541; iota_F = .467843427357989; k_H_star = 36.2342165537757; iota_H_star = .637344671473642; pi_target_star = 1.00496293157320; gamma_C = .656558592425033; nu_C = .820458053020706; gamma_I = .701675805112753; nu_I = .950440781332837; delta = 0.250000000000000e-1; k_I = 4.40575282501109; sigma_a_DC = 1.46982199207423; p_oil_ss_par = 1; beta_d_star = .997671304222037; h_star = .597960949000000; zeta_l_star = 2.22628661822021; phi_star = 2; eps_foreign = 10; k_star = 21.8597920900000; iota_star = .428820563000000; phi_R_star = .757151117000000; phi_pi_star = 1.72087583200000; phi_R = .891332252212934; phi_pi = 1.57834160288009; phi_nfa = 0.607325708106040e-3; gamma_export = .190468985577987; nu_export = .818971529646414; g_A_ss_par = 1.00372908893809; zeta_C_ss_par = 1; zeta_l_ss_par = 3.52199884486014; A_c_ss_par = 1; U_ss_par = 1; eps_h_ss_par = 10; eps_f_ss_par = 10; p_F_star_ss_par = 1; eps_h_star_ss_par = 10; S_oil_ss_par = .377101302006873; z_RP_ss_par = .998709317860111; zeta_C_star_ss_par = 1; A_star_ss_par = 1; G_ss_par = .443098500449163; mu1 = .200000000000000; tr_e_DC = -0.130526312945101e-1; sigma_omega_DC_ss_par = .269769855153010; gamma_DC_ss_par = .970000000000000; PHI = .795275543538090; o = .317320620300058; delta_b = 0.500000000000000e-2; omega_J_ss_par = .400000000000000; k_K = .505682948057596; k_D = 198.937457243176; k_DC = 174.661099859953; iota_DC = .404754186621960; iota_D = .628880677848861; eps_DC = 105.676870642002; eps_D_ss_par = 284.162275509075; eps_cap_ss_par =

1; tau = .193016170211303; smooth_par = 1; str_def_par = 0; rho_g_A = .533551526640626; rho_zeta_C = .808516287426451; rho_zeta_l = .701682227743090; rho_A_c = .846771992483363; rho_U = .848257725625904; rho_eps_h = .509432674380209; rho_eps_f = .496526640652574; rho_p_F_star = .875920623393800; rho_eps_h_star = .500353274634021; rho_S_oil = .862384708415610; rho_z_RP = .743816932882522; rho_zeta_C_star = .926858809000000; rho_A_star = .737826763000000; rho_G = .989466687557771; rho_gamma_DC = .961044980593985; rho_sigma_omega_DC = .625529769736866; rho_eps_D = .886461001807177; rho_eps_cap = .483488718063609; tau_oil = .550000000000000;
rho_BG_star = .9; phi_oil = 0.139138406248529e-1; rho_omega_J = 0;

model;
//////////// Model equations

//////////// Households

[name= 'Euler equation for household']
0 = beta_d*((C - h*C(-1)/g_A)/(g_A(1)*C(1) - h*C)*R_D/pi_c(1)*zeta_C(1)/zeta_C) - 1;

[name= 'UIP']
0 = beta_d*((C - h*C(-1)/g_A)/(g_A(1)*C(1) - h*C)*R_NFA/pi_c(1)*g_e(1)*zeta_C(1)/zeta_C) - 1;

[name= 'Labour supply of household']
0 = eps_w*zeta_l*l^phi/w + zeta_C/(C - h*C(-1)/g_A)*(1 - eps_w) - zeta_C/(C - h*C(-1)/g_A)*k_w*(w*pi_c/w(-1) - pi_c(-1)^iota_w*pi_target^(1-iota_w))*w*pi_c/w(-1) + beta_d*zeta_C(1)/(C(1)*g_A(1) - h*C)*k_w*(w(1)*pi_c(1)/w - pi_c^iota_w*pi_target^(1- iota_w))* w(1)^2*pi_c(1)/w^2*g_A(1)*l(1)/l;

//////////// Private production

[name= 'Private sector production function']
0 = - Y + A_c*K^alpha1*l^(1-alpha1) - PHI;

[name= 'Private wage demand']
0 = (1 - alpha1)*(Y + PHI)/l - w/p_Y;

[name= 'Private capital demand']
0 = alpha1*(Y + PHI)/K - z/p_Y;

//////////// Retailers

[name= 'Phillips curve for home goods']

```
0 = (1 - eps_h) + eps_h*p_Y/p_H - k_H*(pi_H - pi_H(-1)^iota_H*pi_target^(1 - iota_H))*pi_H + beta_d*k_H*(C - h*C(-1)/g_A)/(C(1)*g_A(1) - h*C)*zeta_C(1)/zeta_C*(pi_H(1) - pi_H^iota_H*pi_target^(1 - iota_H))*Y_H(1)/Y_H*g_A(1)*pi_H(1)^2/pi_c(1);

[name= 'Phillips curve for imported goods']
0 = (1 - eps_f) + eps_f*rer*p_F_star/p_F - k_F*(pi_F - pi_F(-1)^iota_F*pi_target^(1 - iota_F))*pi_F + beta_d*k_F*(C - h*C(-1)/g_A)/(C(1)*g_A(1) - h*C)*zeta_C(1)/zeta_C*(pi_F(1) - pi_F^iota_F*pi_target^(1 - iota_F))*Im(1)/Im*g_A(1)*pi_F(1)^2/pi_c(1);

[name= 'Phillips curve for exported goods']
0 = (1 - eps_h_star) + eps_h_star*p_Y/rer/p_H_star - k_H_star*(pi_H_star - pi_H_star(-1)^iota_H_star*pi_target_star^(1 - iota_H_star))*pi_H_star + beta_d*k_H_star*(C - h*C(-1)/g_A)/(C(1)*g_A(1) - h*C)*zeta_C(1)/zeta_C*(pi_H_star(1) - pi_H_star^iota_H_star*pi_target_star^(1 - iota_H_star))*Y_H_star(1)/Y_H_star*g_A(1)*pi_H_star(1)^2/pi_c(1)*g_e(1);

//////////// Aggregators

[name= 'Consumption technology']
0 = - C_p + (gamma_C^(1/nu_C)*C_H^(1-1/nu_C)+(1-gamma_C)^(1/nu_C)*C_F^(1-1/nu_C))^(nu_C/(nu_C-1));

[name= 'Home consumption demand']
0 = - C_H + gamma_C*p_H^(-nu_C)*C_p;

[name= 'Foreign consumption demand']
0 = - C_F + (1 - gamma_C)*p_F^(-nu_C)*C_p;

[name= 'Investment technology']
0 = - (I + z_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC-1))-1)*K_bar_DC(-1)/g_A)/U + (gamma_I^(1/nu_I)*I_H^(1-1/nu_I)+(1-gamma_I)^(1/nu_I)*I_F^(1-1/nu_I))^(nu_I/(nu_I-1));

[name= 'Home investment demand']
0 = - I_H + gamma_I*(p_H/(p_I*U))^(-nu_I)*(I + z_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC-1))-1)*K_bar_DC(-1)/g_A)/U;

[name= 'Foreign investment demand']
0 = - I_F + (1 - gamma_I)*(p_F/(p_I*U))^(-nu_I)*(I + z_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC-1))-1)*K_bar_DC(-1)/g_A)/U;

//////////// Financial frictions
[name= 'Balance']
```

0 = - q*K_bar_DC + b_DC + n_DC;

[name= 'Return on capital']

0 = - R_k_DC + (u_DC*z_DC - z_DC_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC-1))-1)*p_I)/q(-1)*pi_c + (1 - delta)*q/q(-1)*pi_c;

[name= 'Cut definition']

0 = - omega_bar_DC*R_k_DC*q(-1)*K_bar_DC(-1) + R_en*b_DC(-1);

[name= 'Zero profit condition']

0 = - GAMMA_DC + mu1*G_DC + R_DC(-1)/R_k_DC*b_DC(-1)/q(-1)/K_bar_DC(-1);

[name= 'Net worth dynamics']

0 = - n_DC + gamma_DC*(1 - GAMMA_DC)*R_k_DC*q(-1)*K_bar_DC(-1)/pi_c + tr_e_DC;

[name= 'Entrepreners decision']

0 = (1 - GAMMA_DC(1))*R_k_DC(1)/(R_DC) + GAMMA1_DC(1)/(GAMMA1_DC(1) - mu1*G1_DC(1))*(R_k_DC(1)/(R_DC)*(GAMMA_DC(1) - mu1*G_DC(1)) - 1);

[name= 'Utilization rate']

0 = z_DC/p_I - z_DC_ss/p_I_ss*exp(sigma_a_DC*(u_DC-1));

//////////// Banks

[name= 'Capital accumulation']

0 = - J + J(-1)/g_A/pi_c/eps_cap + o*profit_b;

[name= 'Profit']

0 = - profit_b + (R_DC(-1) - R(-1))/g_A/pi_c*b_DC(-1) + (R(-1) - R_D(-1))/g_A/pi_c*(b_DC(-1)-J(-1)) + (R(-1) - delta_b - 1)/g_A/pi_c*J(-1);

[name= 'Loan rate decision']

0 = -(k_K*eps_DC*(J/b_DC - omega_J)*(J/b_DC)^2*1/R_DC + k_DC*(R_DC/(R_DC(-1))^iota_DC/R_DC_ss^(1 - iota_DC)-1)*1/(R_DC(-1))^iota_DC/R_DC_ss^(1 - iota_DC)) + beta_d/pi_c(1)/g_A(1)*((1 - eps_DC) + eps_DC*R/R_DC + k_DC*(R_DC(1)/(R_DC)^iota_DC/R_DC_ss^(1 - iota_DC)-1)*iota_DC*R_DC(1)/(R_DC)^iota_DC/R_DC_ss^(1 - iota_DC)*g_A(1)*pi_c(1)/R_DC);

[name= 'Deposit rate decision']

0 = -(k_D*(R_D/(R_D(-1))^iota_D/R_D_ss^(1 - iota_D)-1)*1/(R_D(-1))^iota_D/R_D_ss^(1 - iota_D)) + beta_d/pi_c(1)/g_A(1)*(- (1 + eps_D) + eps_D*R/R_D + k_D*(R_D(1)/(R_D)^iota_D/R_D_ss^(1 - iota_D)-1)*iota_D*R_D(1)/(R_D)^iota_D/R_D_ss^(1 - iota_D)*g_A(1)*pi_c(1)/R_D);

//////////// Investment firms

[name= 'Capital accumulation']
0 = - K_bar + K_bar_str + (1 - k_I/2*(I/I(-1) - 1)^2)*I;

[name= 'Investment decision']
0 = - p_I + q*(1 - k_I/2*(I/I(-1) - 1)^2) - q*k_I*(I/I(-1) - 1)*I/I(-1) + beta_d*(C - h*C(-1)/g_A)/(g_A(1)*C(1) - h*C)*zeta_C(1)/zeta_C*q(1)*k_I*(I(1)/I-1)*I(1)^2/I^2*g_A(1);

//////////// External sector

[name= 'Trade balance']
0= d_star - R_NFA(-1)/pi_c_star/g_A*d_star(-1) -0.28* (p_H_star_ss*Y_H_star_ss + p_oil_ss*S_oil_ss) + p_H_star*Y_H_star - p_F_star*IM + p_oil*S_oil - dres + BG_star(-1)/g_A/pi_c_star* R_star(-1) - BG_star;

//Kreptsev, Seleznev (2017) version:
//0 = d_star - R_NFA(-1)/pi_c_star/g_A*d_star(-1) - 0.28*(p_H_star_ss*Y_H_star_ss + p_oil_ss*S_oil_ss) + p_H_star*Y_H_star - p_F_star*IM + p_oil*S_oil - dres;

[name= 'Oil price dynamics']
0 = - p_oil + p_oil_ss_par*p_oil_slow*p_oil_fast;

[name= 'Slow oil price dynamics']
0 = - p_oil_slow + (p_oil_slow(-1))^rho_p_oil_slow*exp(e_p_oil_slow);

[name= 'Fast oil price dynamics']
0 = - p_oil_fast + (p_oil_fast(-1))^rho_p_oil_fast*exp(e_p_oil_fast);

[name= 'Euler equation']
0 = beta_d_star*((Y_star - h_star*Y_star(-1)/g_A)/(g_A(1)*Y_star(1) - h_star*Y_star)*R_star/pi_c_star(1)*zeta_C_star(1)/zeta_C_star) - 1;

[name= 'Labour supply']
0 = - zeta_l_star*l_star^phi_star/w_star + zeta_C_star/(Y_star - h_star*Y_star(-1)/g_A);

[name= 'Production function']
0 = - Y_star + A_star*l_star;

[name= 'Labour demand']
0 = p_Y_star*A_star - w_star;

[name= 'Phillips curve']
0 = (1 - eps_foreign) + eps_foreign*p_Y_star - k_star*(pi_c_star - pi_c_star(-1)^iota_star*pi_target_star^(1 - iota_star))*pi_c_star + beta_d_star*k_star*(Y_star - h_star*Y_star/g_A)/(Y_star(1)*g_A(1) - h_star*Y_star)*zeta_C_star(1)/zeta_C_star*(pi_c_star(1) - pi_c_star^iota_star*pi_target_star^(1 - iota_star))*Y_star(1)/Y_star*pi_c_star(1)*g_A(1);

[name= 'Interest rate rule']
0 = - R_star/R_star_ss + (R_star(-1)/R_star_ss)^phi_R_star*(pi_c_star/pi_target_star)^((1 - phi_R_star)*phi_pi_star)*(Y_star/Y_star(-1))^((1 - phi_R_star)*0.584861445)*exp(e_MP_star);

//////////// Monetary policy

[name= 'Interest rate rule']
0 = - R/R_ss + (R(-1)/R_ss)^phi_R*(pi_c/pi_target)^((1 - phi_R)*phi_pi)*exp(e_MP);

[name= 'Reserves rule']
0 = - dres + e_res;

//////////// Additional equations

[name= 'Import definition']
0 = - IM + Im + k_F/2*(pi_F - pi_F(-1)^iota_F*pi_target^(1 - iota_F))^2*Im*p_F/rer/p_F_star;

[name= 'Import demand']
0 = - Im + C_F + I_F;

[name= 'Home good demand']
0 = - Y_H + C_H + I_H;

[name= 'Consumption supply']
0 = - C_p + C + k_w/2*(w*pi_c/w(-1) - pi_c(-1)^iota_w*pi_target^(1-iota_w))^2*w*l;

[name= 'Intermediate good demand']
0 = - Y + Y_H + Y_H_star + G + k_H/2*(pi_H - pi_H(-1)^iota_H*pi_target^(1 - iota_H))^2*Y_H*p_H/p_Y + k_H_star/2*(pi_H_star - pi_H_star(-1)^iota_H_star*pi_target_star^(1 - iota_H_star))^2*Y_H_star*rer*p_H_star/p_Y;
[name= 'g_e definition']
0 = - g_e + rer/rer(-1)*pi_c/pi_c_star;

[name= 'pi_H definition']

```
0 = - pi_H + p_H/p_H(-1)*pi_c;

[name= 'pi_F definition']
0 = - pi_F + p_F/p_F(-1)*pi_c;

[name= 'pi_H_star definition']
0 = - pi_H_star + p_H_star/p_H_star(-1)*pi_c_star;

[name= 'Definition of R_NFA']
0 = - R_NFA + R_star*exp(phi_nfa*(d_star*rer - d_star_ss*rer_ss) - phi_oil*(p_oil - p_oil_ss))*z_RP;

[name= 'External demand']
0 = - Y_H_star + gamma_export*p_H_star^(-nu_export)*Y_star;

[name= 'Raw capital equilibrium']
0 = - g_A*K_bar_str + (1 - delta)*K_bar_DC(-1);
0 = - K_bar_DC + K_bar;

[name= 'Capital equilibrium']
0 = - g_A*K + u_DC*K_bar_DC(-1);
0 = - z + z_DC;

//////////// Other exogenious processes

[name= 'Dynamics of g_A']
0 = - g_A/g_A_ss_par + (g_A(-1)/g_A_ss_par)^rho_g_A*exp(e_g_A);
[name= 'Dynamics of zeta_C']
0 = - zeta_C/zeta_C_ss_par + (zeta_C(-1)/zeta_C_ss_par)^rho_zeta_C*exp(e_zeta_C);
[name= 'Dynamics of zeta_I']
0 = - zeta_I/zeta_I_ss_par + (zeta_I(-1)/zeta_I_ss_par)^rho_zeta_I*exp(e_zeta_I);
[name= 'Dynamics of A_p']
0 = - A_c/A_c_ss_par + (A_c(-1)/A_c_ss_par)^rho_A_c*exp(e_A_c);
[name= 'Dynamics of U']
0 = - U/U_ss_par + (U(-1)/U_ss_par)^rho_U*exp(e_U);
[name= 'Dynamics of eps_h']
0 = - eps_h/eps_h_ss_par + (eps_h(-1)/eps_h_ss_par)^rho_eps_h*exp(e_eps_h);
[name= 'Dynamics of eps_f']
0 = - eps_f/eps_f_ss_par + (eps_f(-1)/eps_f_ss_par)^rho_eps_f*exp(e_eps_f);
[name= 'Dynamics of p_F_star']
0 = - p_F_star/p_F_star_ss_par + (p_F_star(-1)/p_F_star_ss_par)^rho_p_F_star*exp(e_p_F_star);
[name= 'Dynamics of eps_h_star']
```

0 = - eps_h_star/eps_h_star_ss_par + (eps_h_star(-1)/eps_h_star_ss_par)^rho_eps_h_star*exp(e_eps_h_star);
[name= 'Dynamics of S_oil']
0 = - S_oil/S_oil_ss_par + (S_oil(-1)/S_oil_ss_par)^rho_S_oil*exp(e_S_oil);
[name= 'Dynamics of Z_RP']
0 = - z_RP/z_RP_ss_par + (z_RP(-1)/z_RP_ss_par)^rho_z_RP*exp(e_z_RP);
[name= 'Dynamics of zeta_C_star']
0 = - zeta_C_star/zeta_C_star_ss_par + (zeta_C_star(-1)/zeta_C_star_ss_par)^rho_zeta_C_star*exp(e_zeta_C_star);
[name= 'Dynamics of A_star']
0 = - A_star/A_star_ss_par + (A_star(-1)/A_star_ss_par)^rho_A_star*exp(e_A_star);

[name= 'Fiscal rule ']
0= -(1-rho_BG_star)*BG_star_ss*rer +BG_star(-1)/g_A/pi_c_star*rer* (R_star(-1)-rho_BG_star*g_A_ss*pi_c_star_ss) + ((1-rho_G)*BG_dom_ss+e_G) -BG_dom(-1)/g_A/pi_c*(R_D(-1)-rho_G*g_A_ss*pi_c_ss) + tau*p_H*Y_H + tau_oil*(p_oil-smooth_par*(p_oil-p_oil_ss)) *S_oil*rer_G -p_Y*G;

//Kreptsev, Seleznev (2017) version:
//0 = - G/G_ss_par + (G(-1)/G_ss_par)^rho_G*exp(e_G);

[name= 'Risk shock ']
0 = - sigma_omega_DC/sigma_omega_DC_ss_par + (sigma_omega_DC(-1)/sigma_omega_DC_ss_par)^rho_sigma_omega_DC*exp(e_sigma_omega_DC);
[name= 'Wealth equation shock ']
0 = - gamma_DC/gamma_DC_ss_par + (gamma_DC(-1)/gamma_DC_ss_par)^rho_gamma_DC*exp(e_gamma_DC);
[name= 'Desired capital shock']
0 = - omega_J/omega_J_ss_par + (omega_J(-1)/omega_J_ss_par)^rho_omega_J*exp(e_omega_J);
[name= 'Dynamics of eps_D']
0 = - eps_D/eps_D_ss_par + (eps_D(-1)/eps_D_ss_par)^rho_eps_D*exp(e_eps_D);
[name= 'Dynamics of eps_D']
0 = - eps_cap/eps_cap_ss_par + (eps_cap(-1)/eps_cap_ss_par)^rho_eps_cap*exp(e_eps_cap);

//////////// Definitions for entrepreners
0 = - z_w + log(omega_bar_DC)/sigma_omega_DC(-1) + 0.5*sigma_omega_DC(-1);

//normcdfsym = @(x, mu, sigma)(1/2 - erf((2^(1/2)*(mu - x))/(2*(sigma^2)^(1/2)))/2);

//0 = - GAMMA_DC + omega_bar_DC*(1 - normcdfsym(z_w,0,1)) + normcdfsym(z_w - sigma_omega_DC(-1),0,1);

```
0 = - GAMMA_DC + omega_bar_DC*(1 - (1/2 - erf((2^(1/2)*(-z_w ))/(2*(1^2)^(1/2)))/2)) + (1/2 - erf((2^(1/2)*(-z_w + sigma_omega_DC(-1)))/(2*(1^2)^(1/2)))/2);
//0 = - G_DC + normcdfsym(z_w - sigma_omega_DC(-1),0,1);
0 = - G_DC + (1/2 - erf((2^(1/2)*(-z_w + sigma_omega_DC(-1)))/(2*(1^2)^(1/2)))/2);

//0 = - GAMMA1_DC + 1 - normcdfsym(z_w,0,1);
0 = - GAMMA1_DC + 1 - (1/2 - erf((2^(1/2)*(-z_w))/(2*(1^2)^(1/2)))/2);

0 = - G1_DC + 1/sigma_omega_DC(-1)*normpdf(z_w,0,1);
0 = - K_bar_DC_pr + K_bar_DC(-1)/g_A;

//////////// Definitions for rates
0 = - R_en1 + R_en(1);
0 = - R_en2 + R_en1(1);
0 = - R_en3 + R_en2(1);

0 = - R_D1 + R_D(1);
0 = - R_D2 + R_D1(1);
0 = - R_D3 + R_D2(1);

[name= 'Dynamics for external fund']
0=-BG_star +rho_BG_star*BG_star(-1)/g_A/pi_c_star*g_A_ss*pi_c_star_ss+(1-rho_BG_star)*BG_star_ss+tau_oil*smooth_par*(p_oil-p_oil_ss)*S_oil;

[name= 'Government budget constraint']
0 = -BG_star*rer + BG_star(-1)*R_star(-1)/g_A/pi_c_star*rer + BG_dom - BG_dom(-1)/g_A/pi_c*R_D(-1) + tau*p_H*Y_H + tau_oil*p_oil*S_oil*rer - p_Y*G;

[name= 'projected fiscal exchange rate']
0=-rer_G + rer;


////////////////  Observations //////////////////////


[name= 'Output growth']
0 = g_Y_obs - g_A*(C_p + p_Y(-1)*G + p_I(-1)*(I + z_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC-1))-1)*K_bar_DC(-1)/g_A) + rer(-1)*p_H_star(-1)*Y_H_star + rer(-1)*p_oil(-1)*S_oil - rer(-1)*p_F_star(-1)*IM)/(C_p(-1) + p_Y(-1)*G(-1) + p_I(-1)*(I(-1) + z_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC(-1)-1))-1)*K_bar_DC_pr(-1)) + rer(-1)*p_H_star(-1)*Y_H_star(-1) + rer(-1)*p_oil(-1)*S_oil(-1) - rer(-1)*p_F_star(-1)*IM(-1));
```

```
[name= 'Consumption growth']
0 = g_C_p_obs - g_A*C_p/C_p(-1);

[name= 'Investment growth']
0 = g_I_obs - g_A*(I + z_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC-1))-1)*K_bar_DC(-1)/g_A)/(I(-1) + z_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC(-1)-1))-1)*K_bar_DC_pr(-1));

[name= 'Export growth']
0 = g_X_obs - g_A*(rer(-1)*p_H_star(-1)*Y_H_star + rer(-1)*p_oil(-1)*S_oil)/(rer(-1)*p_H_star(-1)*Y_H_star(-1) + rer(-1)*p_oil(-1)*S_oil(-1));

[name= 'Government growth']
0 = g_G_obs - g_A*G/G(-1);

[name= 'Wage growth']
0 = g_w_obs - g_A*w/w(-1);

[name= 'Interest growth']
0 = MIACR_obs - R;

[name= 'GDP deflator']
0 = g_p_Y_obs - pi_c*(C_p + p_Y*G + p_I*(I + z_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC-1))-1)*K_bar_DC(-1)/g_A) + rer*p_H_star*Y_H_star + rer*p_oil*S_oil - rer*p_F_star*IM)/(C_p + p_Y(-1)*G + p_I(-1)*(I + z_ss/p_I_ss/sigma_a_DC*(exp(sigma_a_DC*(u_DC-1))-1)*K_bar_DC(-1)/g_A) + rer(-1)*p_H_star(-1)*Y_H_star + rer(-1)*p_oil(-1)*S_oil - rer(-1)*p_F_star(-1)*IM);

[name= 'CPI']
0 = g_CPI_obs - pi_c;

[name= 'Import deflator']
0 = g_p_F_star_obs - pi_c*p_F_star*rer/(p_F_star(-1)*rer(-1));

[name= 'Export deflator']
0 = g_p_X_obs - pi_c*(rer*p_H_star*Y_H_star + rer*p_oil*S_oil)/(rer(-1)*p_H_star(-1)*Y_H_star + rer(-1)*p_oil(-1)*S_oil);

[name= 'Investment deflator']
0 = g_p_I_obs - pi_c*p_I/p_I(-1);

[name= 'Reserves to export ratio']
```

```
0 = dRes2X_obs - dres/(p_H_star*Y_H_star + p_oil*S_oil);

[name= 'Exchange rate growth']
0 = g_e_obs - g_e;

[name= 'Fed Fund Rate']
0 = FED_obs - R_star;

[name= 'Foreign output']
0 = g_Y_star_obs - g_A*Y_star/Y_star(-1);

[name= 'Foreign inflation']
0 = g_CPI_star_obs - pi_c_star;

[name= 'Oil price growth']
0 = d_p_oil_obs - p_oil/p_oil(-1);

[name= 'Loan growth']
0 = g_b_DC_obs - pi_c*g_A*b_DC/b_DC(-1);

[name= 'Loan rate']
0 = R_en4_obs - (R_en(1)*R_en1(1)*R_en2(1)*R_en3(1))^0.25;

[name= 'Deposit rate']
0 = R_D4_obs - (R_D*R_D1*R_D2*R_D3)^0.25;

[name= 'Capital growth']
0 = g_J_obs - pi_c*g_A*J/J(-1);
end;




initval;
C = .942753255; R = 1.018244601; zeta_C = 1; pi_c = 1.009853407; R_NFA = 1.009760027; g_e = 1.004866324; w = 1.341803189; l = 1; K = 13.2271714; A_c = 1; Y = 1.569676438; p_Y = .851055243; z = 0.50721471e-1; eps_h = 10; p_H = .945616937; pi_H = 1.009853407; eps_f = 10; pi_F = 1.009853407; p_F = 1.111111111; p_F_star = 1; rer = 1; p_H_star = .945616937; eps_h_star = 10; pi_H_star = 1.004962932; C_p = .942753255; C_H = .648031557; C_F = .296967275; I = .380004584; U = 1; I_H = .279152055; I_F = .101817352; p_I = .992359877; q = .992359877; K_bar = 13.2764967; K_bar_str = 12.89649212; u_DC = 1; zeta_I = 3.521998845; d_star = 9.105806687; dres = 0; IM = .398784627; p_oil = 1; p_oil_fast = 1; p_oil_slow = 1; S_oil = .377101302; Y_star = 1; R_star = 1.01106499; pi_c_star = 1.004962932; zeta_C_star = 1; I_star
```

= 1; w_star = .9; A_star = 1; p_Y_star = .9; G = .4430985; g_A = 1.003729089; z_RP = .998709318; Y_H_star = .199394325; Y_H = .927183612; Im = .398784627; K_bar_DC = 13.2764967; R_k_DC = 1.036222671; z_DC = 0.50721471e-1; omega_bar_DC = .496941246; R_en = 1.029883572; b_DC = 6.587531319; n_DC = 6.587531319; gamma_DC = .97; sigma_omega_DC = .269769855; z_w = -2.45726339; GAMMA_DC = .496657988; G_DC = 0.319533e-2; GAMMA1_DC = .993; G1_DC = 0.72235465e-1; K_bar_DC_pr = 13.2271714; R_en1 = 1.029883572; R_en2 = 1.029883572; R_en3 = 1.029883572; J = 2.635012528; profit_b = .111573847; R_DC = 1.027972104; R_D = 1.014673846; omega_J = .4; eps_D = 284.1622755; R_D1 = 1.014673846; R_D2 = 1.014673846; R_D3 = 1.014673846; eps_cap = 1; BG_star = .423812638; BG_dom = .50229646; rer_G=1; g_Y_obs = 1.003729089; g_C_p_obs = 1.003729089; g_I_obs = 1.003729089; g_X_obs = 1.003729089; g_G_obs = 1.003729089; g_w_obs = 1.003729089; MIACR_obs = 1.018244601; g_p_Y_obs = 1.009853407; g_CPI_obs = 1.009853407; g_p_F_star_obs = 1.009853407; g_p_X_obs = 1.009853407; g_p_I_obs = 1.009853407; dRes2X_obs = 0; g_e_obs = 1.004866324; FED_obs = 1.01106499; g_Y_star_obs = 1.003729089; g_CPI_star_obs = 1.004962932; d_p_oil_obs = 1; g_b_DC_obs = 1.01361924; R_en4_obs = 1.029883572; R_D4_obs = 1.014673846; g_J_obs = 1.01361924;
end;

C_ss = C; R_ss = R; zeta_C_ss = zeta_C; pi_c_ss = pi_c; R_NFA_ss = R_NFA; g_e_ss = g_e; w_ss = w; l_ss = l; K_ss = K; A_c_ss = A_c; Y_ss = Y; p_Y_ss = p_Y; z_ss = z; eps_h_ss = eps_h; p_H_ss = p_H; pi_H_ss = pi_H; eps_f_ss = eps_f; pi_F_ss = pi_F; p_F_ss = p_F; p_F_star_ss = p_F_star; rer_ss = rer; p_H_star_ss = p_H_star; eps_h_star_ss = eps_h_star; pi_H_star_ss = pi_H_star; C_p_ss = C_p; C_H_ss = C_H; C_F_ss = C_F; I_ss = I; U_ss = U; I_H_ss = I_H; I_F_ss = I_F; p_I_ss = p_I; q_ss = q; K_bar_ss = K_bar; K_bar_str_ss = K_bar_str; u_DC_ss = u_DC; zeta_I_ss = zeta_I; d_star_ss = d_star; dres_ss = dres; IM_ss = IM; p_oil_ss = p_oil; p_oil_fast_ss = p_oil_fast; p_oil_slow_ss = p_oil_slow; S_oil_ss = S_oil; Y_star_ss = Y_star; R_star_ss = R_star; pi_c_star_ss = pi_c_star; zeta_C_star_ss = zeta_C_star; l_star_ss = l_star; w_star_ss = w_star; A_star_ss = A_star; p_Y_star_ss = p_Y_star; G_ss = G; g_A_ss = g_A; z_RP_ss = z_RP; Y_H_star_ss = Y_H_star; Y_H_ss = Y_H; Im_ss = Im; K_bar_DC_ss = K_bar_DC; R_k_DC_ss = R_k_DC; z_DC_ss = z_DC; omega_bar_DC_ss = omega_bar_DC; R_en_ss = R_en; b_DC_ss = b_DC; n_DC_ss = n_DC; gamma_DC_ss = gamma_DC; sigma_omega_DC_ss = sigma_omega_DC; z_w_ss = z_w; GAMMA_DC_ss = GAMMA_DC; G_DC_ss = G_DC; GAMMA1_DC_ss = GAMMA1_DC; G1_DC_ss = G1_DC; K_bar_DC_pr_ss = K_bar_DC_pr; R_en1_ss = R_en1; R_en2_ss = R_en2; R_en3_ss = R_en3; J_ss = J; profit_b_ss = profit_b; R_DC_ss = R_DC; R_D_ss = R_D; omega_J_ss = omega_J; eps_D_ss = eps_D; R_D1_ss = R_D1; R_D2_ss = R_D2; R_D3_ss = R_D3; eps_cap_ss = eps_cap; BG_star_ss = BG_star; BG_dom_ss = BG_dom; rer_G_ss = rer_G; g_Y_obs_ss = g_Y_obs; g_C_p_obs_ss = g_C_p_obs; g_I_obs_ss = g_I_obs; g_X_obs_ss = g_X_obs; g_G_obs_ss = g_G_obs; g_w_obs_ss = g_w_obs; MIACR_obs_ss = MIACR_obs; g_p_Y_obs_ss = g_p_Y_obs; g_CPI_obs_ss = g_CPI_obs; g_p_F_star_obs_ss = g_p_F_star_obs; g_p_X_obs_ss = g_p_X_obs; g_p_I_obs_ss = g_p_I_obs; dRes2X_obs_ss = dRes2X_obs; g_e_obs_ss = g_e_obs; FED_obs_ss = FED_obs; g_Y_star_obs_ss = g_Y_star_obs; g_CPI_star_obs_ss = g_CPI_star_obs; d_p_oil_obs_ss = d_p_oil_obs; g_b_DC_obs_ss = g_b_DC_obs; R_en4_obs_ss = R_en4_obs; R_D4_obs_ss = R_D4_obs; g_J_obs_ss = g_J_obs;

resid(1);

```
steady;
check;

shocks;
var e_p_oil_slow; stderr 0.17455; //estimated value
var e_p_oil_fast; stderr 0.01;
var e_MP_star; stderr 0.01;
var e_MP; stderr 0.01;
var e_res; stderr 0.01;
var e_g_A; stderr 0.01;
var e_zeta_C; stderr 0.01;
var e_zeta_I; stderr 0.01;
var e_A_c; stderr 0.01;
var e_U; stderr 0.01;
var e_eps_h; stderr 0.01;
var e_eps_f; stderr 0.01;
var e_p_F_star; stderr 0.01;
var e_eps_h_star; stderr 0.01;
var e_S_oil; stderr 0.01;
var e_z_RP; stderr 0.01;
var e_zeta_C_star; stderr 0.01;
var e_A_star; stderr 0.01;
var e_G; stderr 0.01;
var e_sigma_omega_DC; stderr 0.01;
var e_gamma_DC; stderr 0.01;
var e_omega_J; stderr 0.01;
var e_eps_D; stderr 0.01;
var e_eps_cap; stderr 0.01;
end;

stoch_simul(irf=20,order=1, nograph, nodisplay);
```