Probability of Default (PD) Model to Estimate Ex Ante Credit Risk

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Abstract

A genuine measure of an ex ante credit risk links borrowers' financial position with the odds of default. Comprehension of borrower's financial position is proxied by the derivatives of its filled financial statements, i.e. financial ratios. To measure an ex ante credit risk, one needs a forward-looking estimate. We identify statistically significant relationships between the shortlisted financial ratios and the subsequent default events.

To estimate the odds of the borrower to default on its obligations, we simulate its probability of default at a horizon of one year. We horse run the constructed PD model against the alternative measures of ex ante credit risk that the related literature on bank risk-taking widely uses: credit quality groups and credit spreads in interest rates. We compare the results obtained with the PD model, and with the alternative approaches. We find that the PD model predicts the default event more accurately at a horizon of one year.

We conclude that the developed measure of ex ante credit risk is feasible for estimating the risk-taking behaviour by banks and analysing the shifts in portfolio composition with the sufficient degree of granularity. The model could be used in applied research as the tool for measuring ex ante credit risk based on micro level data (credit registry).

JEL-classification: E44, E51, E52, E58, G21, G28

Keywords: ex ante probability of default, corporate credit, credit registry, probability of default mode, credit quality groups, credit spreads

Contents

1. Introduction

Measures of credit risk could be divided into the two categories: 1) self-reported by banks and 2) evaluated by outsiders (researchers, rating agencies etc.). Those self-reported by banks include *credit spreads* in loan-interest rates of newly issued loans and *quality groups (or an internal credit rating)* assigned by a bank to each new loan. Delis et al. (2017) construct a comprehensive U.S. credit-registry based dataset and use a loan interest rate spread as a measure of ex ante risk-taking. Ioannidou, et al. (2015) use not only internal ratings reported by banks as an ex ante measure of credit risk, but also observable past delinquencies. Proposing our measure of ex ante credit risk based on a PD-model we act as a “third party” relative to a firm and to a bank. The similar measure is mentioned by Van Roy et al. (2018), but for the purposes of timely monitoring of financial stability risks. We relate to the literature on estimating and validating a PD-model on granular loan-level data. Table 1 shortlist the papers over a wide time span with the different datasets used for PD models’ development and validation, and the different methodology applied. We refer to the model specification introduced by Moody’s Analytics that is built and tested with dataset of Russian companies (Moody’s KMV RiskCalc V3.1 Russia).

In order to estimate ex ante probability of default of the newly issued portfolio of loans we construct the PD model using comprehensive micro-level database of loans issued by domestic banks to the non-banking sector in Russia. Model specification includes interpretable methods, i.e. ordinary least squares (OLS), probit, and does not include the non-interpretable methods, e.g. ensemble methods (random forests) or deep-learning neural networks. The estimated regression residuals are studied. The list of variables and their transformations are chosen to avoid multicollinearity, heteroskedasticity, and non-Gaussian distribution to the extent feasible for real-world data. We do not run any adjustment for the through-the-cycle (TTC) PD as such a procedure has no theoretical ground, though is often used in practice and is often referred to, e.g. in (Ozdemir and Miu 2009). Thus, we may call our PD model a point-in-time (PIT) one.

When we apply the model in pseudo real-time, i.e. when we want to estimate a probability of default at a horizon of one year for the *newly issued* loans that comprise actual portfolio of a bank, we want to use the most recent financial information. When we use the most recent financial statements, we are confident to obtain an up-to-date status estimate. On this step for new loans we use the most recent financial statement. However, there are cases when financial data lags much behind the estimation date. Further research assume that we will calculate all adjustments to treat problems with financial data lags, that is described in Appendix A.

Table 1. Summary of PD estimation literature

Paper	Data source	Country	Period	Method
Altman (1968)	Financial statements, data from the National Bankruptcy Act	USA	1946-1965	Linear discriminant analysis
Ohlson (1980)	10-K financial statements	USA	1970-1976	Logistic regression
Odom and Sharda (1990)	Financial statements	USA	1975-1982	Neural networks
Shirata (1998)	Financial statements	Japan	1986-1996	Multivariate Discriminant Analysis
Shin, Lee, and Kim (2005)	Korea Credit Cuarantee Fund	Korea	1996-1999	Support vector machines
Shirata and Sakagami (2008)	Financial reports	Japan	2002	Text mining
Li and Sun (2009)	Financial statements	China	2000-2005	Case-based reasoning, Support vector machines
Dwyer D., Korableva I., Zhao J. (2010)	Financial statements, database with default information	Russia	2002-2009	Logistic regression
Brédart (2014)	Financial statements of quoted firms, American Bankruptcy code	USA	2000-2012	Logistic regression
Demeshev B. B., Tikhonova A. S. (2014)	Financial statements	Russia	2011-2012	Linear Discriminant Analysis, Quadratic Discriminant Analysis, Mixture Discriminant Analysis, Logistic Regression, Probit Regression, Tree and Random Forest
Ioannidou et al. (2015)	Public credit registry	Bolivia	1993-2003	Logistic regression
Тотьмянина, К. М. (2014)	Financial statements, external data for bankruptcy (FIRA PRO)	Russia	2005-2013	Logistic regression
Chatterjee, S. (2015)	Financial statements for structural models, for reduced form – default is an exogenous event	-	-	Structural models, reduced form models
Gupta, J., Gregoriou, A., & Healy, J. (2015)	Unique database from the Credit Management Research Center of the University of Leeds	UK	2000-2009	Survival analysis
Kalak, Hudson (2015)	Financial statements (Compustat)	US	1980-2013	Survival analysis
Sartori, Mazzucchelli, Gregorio (2016)	Financial statements	Italy	2013	Case-based reasoning
Miteski, et al. (2018)	The Credit Registry of the National Bank of the Republic of Macedonia, banks' balance sheets	Macedonia	2010-2017	OLS
Bank of Japan (2019)	Financial statements, Credit Risk Database	Japan	2001-2016	Logistic regression
Mogilat A. (2019)	Financial statements, external data for bankruptcy	Russia	2006-2016	Logistic regression
Hosaka (2019)	Consolidated BS, P&L from grayscale image	Japan	2002-2016	Neural networks
Albanesi, Vamossy (2020)	Credit file data from the Experian credit bureau	US	2004-2015	Deep learning

The approach suggested in this research paper is practically feasible and easy to implement. The simplicity of the method that we used in estimation allows us to quickly get the results without excess computing power. Moreover, our model is interpretable which means that we are able to assess default behavior in different aspects and how they vary over time.

2. Data

Granular information on defaults at the loan-level is usually unavailable. In that case a set of assumptions should be imposed, either backed up by the professional judgement or based on the generally acceptable practices. To overcome the subjectivity of assumptions, we construct a comprehensive micro-level database comprising the firm-level financial information (both quantitative and qualitative, extracted from the annual reports and financial statements), the firm-bank-loan-level information on new loans issued (extracted from the credit registry), and the firm-bank level information on defaults (extracted from the credit history bureaus).

We define default event as a case of payment overdue for more than 90 days or the case when firm officially was liquidated according to the information from the SPARK database. We see significant sectoral differences in relationship between independent variables and default rates.

Model specification includes firm-level public financial information available from the Russian Statistic Agency (Rosstat) and statistics on defaults available from the CHB dataset[1]. We use firm-level monthly data on defaults and the annual financial information that is translated into the monthly data. For the composition of dataset, refer to Table 2.

Table 2. Dataset composition and identification of defaults

Operation applied	No. of observations	No. of defaults
Dataset initialisation	16 114 889	
Entities with loans overdue of more than 90 days		535 575
Entities liquidated (identified from SPARK database)		2 509
Default mark assigned		134 481
Default mark 12m backward shift (defaults of next year are matched with firm IDs in current year)	(3 095 820)	(17 336)
Censored outliers at 0.5% and 99.5%	(669,292)	(44,686)
Total	**12,349,777**	**610,543**

Source: Bank of Russia, author's calculations.

[1] Here CHB dataset stands for the aggregate dataset comprising the default information from the three Credit History Bureaus operating in Russia.

3. Methodology

Default definition

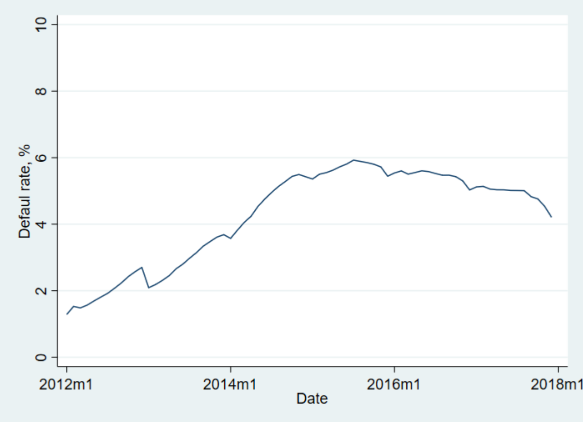
Probability of default (PD) model development starts from *dependent variable* definition, or default definition. We use 90 days past due in accordance with the Basel II (III) Internal ratings-based (IRB) methodology (BCBS 2017). We do not account for the unlikely to pay (UTP) criteria as those are not readily available for the entire set of borrowers (BCBS 2006, 100. par. 453). We define the *default event* as a case of payment overdue for more than 90 days or the case when firm officially was liquidated according to the information from the SPARK database[2]. We assign default mark to the entity in the month in which the default event took place. We then expand the default mark for the next 11 months to fix the year in which the entity has defaulted. We do not exclude defaulted entities form the rest of observations. We make a 12-month backward shift of the default mark to match the defaults of the next year with the firm financial characteristics of the current year. We do this in order to train the model to estimate the probability of default at the horizon of one year using most recent financial information of the borrowing entities. Once an entity has defaulted it receives the default mark. In order for the entity to have the default mark removed, it should proceed without the periods of debt overdue of more than 90 days during the subsequent 12 months, subject to entity's existence (confirmed via the SPARK database). We calculate the *default rate* with monthly frequency as the ratio of the number of firms that have defaulted in the subsequent 12 months starting from the month of observation, to the overall number of firms observed in that period. For graphical representation, refer to Figure 1.

To address the heterogeneity in the *operating* and *financial* conditions, we classify borrowing entities into the 9 aggregate industries. For the detailed industry composition, refer to Appendix A. The number of observations within each industry is presented in Table 3. To address the heterogeneity in the *borrowing* conditions, we classify the entities into the high-leverage and low-leverage class based on the median value of leverage[3] for all observations. The latter is done in (Bank of Japan 2019). For graphical representation of the industry-level default rates, refer to Figure 2.

[2] SPARK database contains financial (quantitative) and non-financial (qualitative) information on the business entities operating in Russia. The database is available from the Interfax News Group.

[3] Leverage is calculated as the sum of long-term debt and short-term debt normalized by the total assets.

Figure 1. Default rate at economy level



Source: Bank of Russia, author's calculations.

Table 3. Observations at industry level

Industry	No. of observations
Forestry & Agriculture	573,643
Mining	61,367
Manufacturing	1,603,842
Utilities	167,558
Construction	1,466,147
Wholesale & Retail	5,215,239
Hotels & Restaurants	322,813
Transportation	713,095
Other sectors	2,218,266
No information	7,807
Total	**12,349,777**

Independent variables

We proceed with a set of independent variables. The list consists of balance sheet and income statement elements as well as of their derivatives, i.e. financial ratios. We calculate financial ratios based on the information available from the financial statements and annual reports for the time period 2011 – 2018.

For our analysis we use *monthly data* that we construct from the annual financial statements. We assign each financial statement (FS) a weight proportionally to the number of months spent in the corresponding financial year, i.e.:

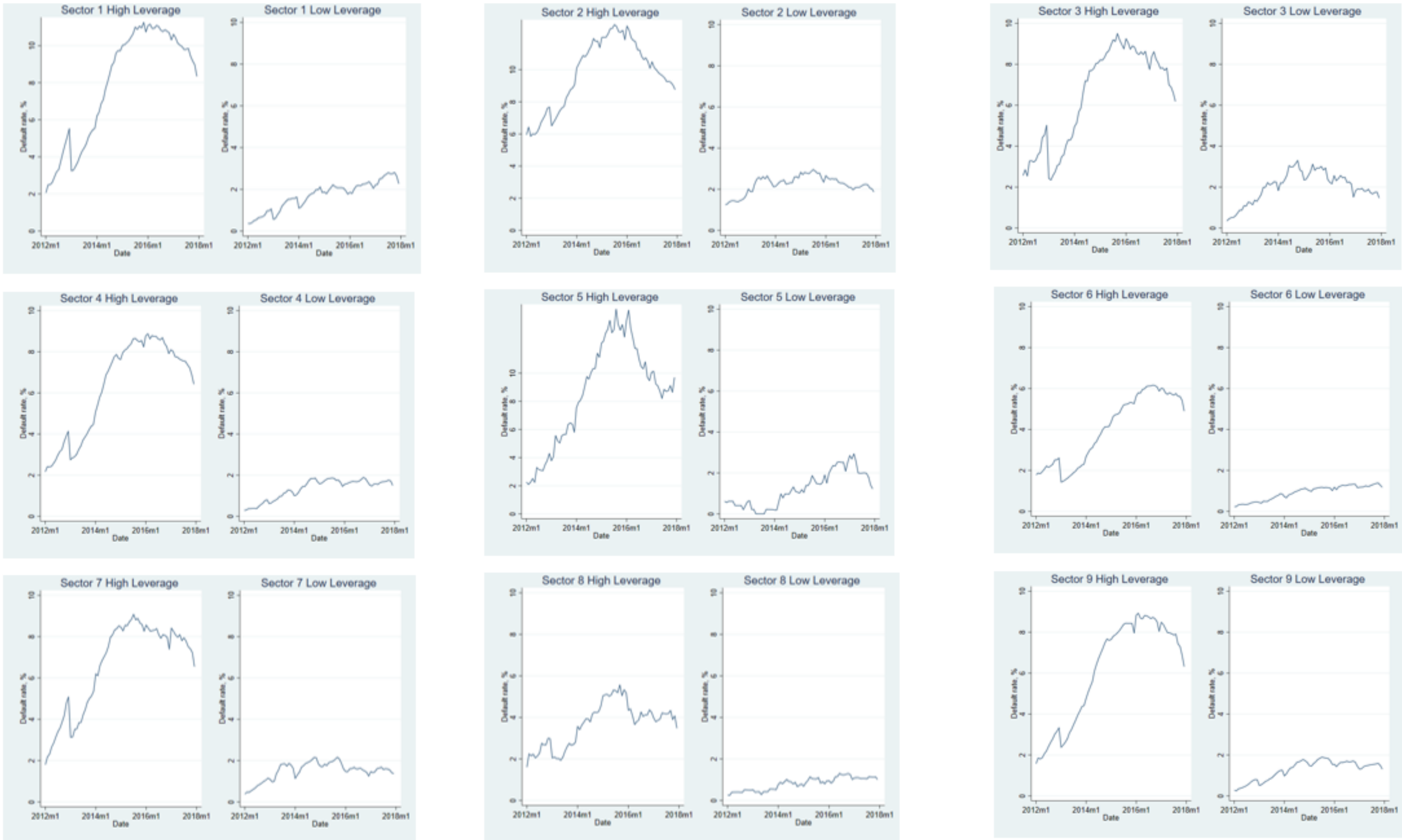
$$FS_{february\_2012} = \frac{2}{12} \times FS_{full\_2012} + \frac{10}{12} \times FS_{full\_2011}$$

Initial list of independent variables corresponds to the PD model by Moody's Risk Calc v3.1 methodology. In Appendix A you can see the detailed scheme of monthly data calculation and discussion about the risks of using future information. The selected ratios characterize entities performance in the dimensions of activity, solvency, growth, leverage, liquidity, and profitability.

For the detailed list of independent variables and its derivation, refer to Table 4. We do not exclude the entities with missing values in the financial statements from the dataset. We put the mean values for variables in case of missing. We censor the outliers from 1 and 99 percentiles.

In Table 5 we report descriptive statistics of dataset after censoring over two groups: default firms and non-default firms. An overview of the descriptive statistics shows that there are differences between non-default and default firms. This fact supports our hypothesis that selected factors have an effect on a failure probability.

Figure 2. Default rate at industry and leverage levels



Sector 1 – Construction, Sector 2 – Forestry and Agriculture, Sector 3 – Hotels and Restaurants, Sector 4 – Manufacturing, Sector 5 – Mining, Sector 6 – Other services, Sector 7 – Transportation , Sector 8 – Utilities , Sector 9 – Wholesale and Retail Trade. ***Source: Bank of Russia, author’s calculations.***

Table 4. Definition of financial ratios

Code	Definition	Calculation
ACTIVITY	The ratio of account payable to sales	AP/SALES
DEBTCOVER	The ratio of operating profit to total amount of liabilities	PROFIT_OPERATING/(ST_DEBT+LT_DEBT+AP)
GROWTH	Growth rate of sales	$(SALES_t - SALES_{t-1})/(SALES_{t-1})$
LEV_EQ	The share of equity in total assets	EQUITY_TOTAL/ASSETS_TOTAL
LEV_RE	The ratio of retained earnings to current liabilities	RE/(ST_DEBT+AP)
LIQUIDITY	The ratio of cash to total assets	CASH/ASSETS_TOTAL
EBIT	Earnings before interest and tax	PROFIT_OPERATING+DIVIDEND_INCOME+ INTEREST_RECEIVABLE
ROA	EBIT divided to total assets	EBIT/ASSETS_TOTAL

Table 5. Descriptive statistics

Variable	Default mark	Mean	Std. Dev.	Min	Max
ACTIVITY	Default	1.770	6.942	0	127.179
	Non-default	0.804	3.946	0	127.195
DEBTCOVER	Default	0.360	0.980	-3.714	32.582
	Non-default	0.600	1.917	-3.718	32.701
GROWTH	Default	-0.017	0.081	-0.5	0.719
	Non-default	0.014	0.075	-0.5	0.720
LEV_EQ	Default	0.104	0.617	-8.703	1
	Non-default	0.277	0.455	-8.704	1
LEV_RE	Default	0.917	3.438	-5.293	111.771
	Non-default	1.380	5.580	-5.294	111.790
LIQUIDITY	Default	0.046	0.075	0	1
	Non-default	0.073	0.120	0	1
EBIT	Default	6 204.709	25 473.10	-87 302.3	783 263.4
	Non-default	10 355.370	43 602.83	-87 315.5	783 440.2
ROA	Default	0.076	0.202	-1.493	3.340
	Non-default	0.124	0.250	-1.494	3.342

Having both dependent and independent variables at hand, we undertake a single-factor analysis. The single-factor analysis consists of the following statistical procedures:

1. Testing for the variable's discriminatory power. This includes estimating pairwise correlation and its statistical significance between the default indicator and the independent variable of interest.

The *correlation matrix* is presented in Table 6. The results provide information about collinearity among the selected ratios.

Table 6. Correlation matrix

Covariates	Default	ACTIVITY	DEBTCOVER	GROWTH	LEV_EQ	LEV_RE	LIQUIDITY	ROA
Default	1							
ACTIVITY	0.05	1						
DEBTCOVER	-0.03	-0.05	1					
GROWTH	-0.08	-0.08	0.01	1				
LEV_EQ	-0.08	-0.13	0.26	-0.01	1			
LEV_RE	-0.02	-0.03	0.32	-0.03	0.23	1		
LIQUIDITY	-0.05	-0.05	0.13	0.08	0.05	0.00	1	
ROA	-0.04	-0.08	0.46	0.06	0.21	0.05	0.21	1

In order to check the relationship between ratios and default event we apply univariate regression analysis (Table 7).

Table 7. Univariate regression analysis

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ACTIVITY	0.014***						
	(0.000)						
DEBTCOVER		-0.059***					
		(0.001)					
GROWTH			-2.632***				
			(0.009)				
LEV_EQ				-0.252***			
				(0.001)			
LEV_RE					-0.011***		
					(0.000)		
LIQUIDITY						-1.432***	
						(0.008)	
ROA							-0.452***
							(0.003)
_cons	-1.695***	-1.652***	-1.678***	-1.625***	-1.667***	-1.595***	-1.634***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
N	12,938,679	12,938,679	12,938,679	12,938,679	12,938,679	12,938,679	12,938,679
R2_p	0.004	0.003	0.020	0.012	0.001	0.009	0.005

Standard errors in parentheses
$*$ p<0.10, $**$ p<0.05, $***$ p<0.01

We see that suggested variables are significant and have the expected sign except for the leverage ratios which could be explained by the heterogeneity in the high and low leverage groups.

2. *Visual analysis* via scatter plot of the independent variable against the dependent one, or against the *default rate*. The latter is done in (Bank of Japan 2019).

Pairwise representation demonstrates the relationship between average ratios and default rates among the industries and leverage levels. For example, Figure 4 demonstrates industry level relationship between the activity ratio and default rates for the high- and low-leverage groups. It shows strong significant dependence.

For detailed visual representation of the relationship between the default rate and independent variables at industry level for different leverage groups, refer to Appendix B.

PD model estimation results

To evaluate PD we use one of the conventional binary choice model types, i.e. the probit model (Cameron & Trivedi, 2010, p. 460). Y is the dependent variable. It is a Bernoulli trial. It reflects the default occurrence. It equals one when default occurs and zero otherwise. X is a vector of independent variables. β is the coefficient vector. We estimated it via maximum likelihood procedure. ε is the error component. We tend to fit data in a way that the error is independently identically distributed:

1) $P(Y = 1) = \frac{1}{1+e^{-X\beta+\varepsilon}}$

When the model is fit to the data, we may obtain the prediction of the default probability $\widehat{\mathrm{PD}}$. It is a floating variable such that $PD \in [0; 1]$.

2) $\widehat{PD} = P(Y = 1) = \frac{1}{1+e^{-X\hat{\beta}}}$,

where $\hat{\beta}$ are the estimated model coefficients. The model enables to predict defaults, i.e. the discrete events Y. It uses the Binomial distribution rule.

We train the model on 80% of the dataset and test the model on 20% subset of loans selected randomly. We implement validation procedures based on correspondence analysis, i.e. benchmark actual default marks to predicted ones. We use F1 score as a model performance metric:

3) $F1 = \frac{2TP}{2TP+FP+FN}$

where TP stands for the observation correctly classified as "default", FP – observation incorrectly classified as "default, FP – observations incorrectly classified as "non-default".

F1 score depends on the threshold level. Values of the predicted default probability in excess of the threshold correspond to discrete default events. Otherwise, the model signals the absence of the default event. We use the threshold of 0.01 ~ 0.5 to identify the level at which F1 score is the highest. We compare three PD model specifications: Model 1 as formulated by Moody's with the set of independent variables and without the control variables for industry or debt level; Model 2 with industry and leverage level controls, and Model 3 with triple interaction terms.

Figure 3 shows the evolution of F1 score depending on the threshold and model specification. Appendix C contains regression result for Model 3. It can be seen that almost all covariates are significant and have coefficients of the expected sign. According to the results *GROWTH* has the most explanatory power in identifying the default event. We find that the bigger growth rate of firm the smaller the probability of default.

The results also indicate that on average there are significant differences between industries in financial distress level. We find that all industry dummies (except industry 5 - Mining) are statistically significant. Almost all industries have on average much lower probability of default than in benchmark sector Construction. In Forestry and Agriculture (industry 2) and in Hotels and Restaurants (industry 3) we show that probability of default is significantly higher than in Construction. Furthermore, the estimation results indicate that between all industries there are clear differences in the links between PD level and financial ratios.

As we suggested according to the visual analysis there are a significant difference between firms with higher and lower leverage. The results show firms with leverage above median level are more likely to default than firms with lower leverage level. Moreover, between the two group of firms, those with high and low leverage, there are statistical differences in the relationship between financial ratios, that we included in the model, and the probability of default. In particular, for high leverage ratio group of firms, the more likely it is to default when debt cover ratio increases, ROA or liquidity ratio decrease.

4. Alternative measures of ex ante credit risk

We have developed the PD model as a measure of ex ante credit risk at a horizon of one year. We are now in a position to apply the model in pseudo real-time, i.e. to estimate ex ante credit risk for each new loan issued by bank in a given time period.

To validate the feasibility of our measure, we want to compare it with the alternative measures of ex ante credit risk, that are extensively used in the literature:

1. *Credit quality groups* (CQG) assigned by banks to each borrower at a time of loan issuance for the purpose of fulfilling capital regulation rules[4].
2. *Credit spreads* (CS) in interest rates, e.g. credit spread to key policy rate or money market rate (the transfer curve). It is considered as banks' mark-up above the minimum funding costs proxied by those interest rates[5].

In Table 8 we report descriptive statistics of dataset of alternative measures over two groups: default firms and non-default firms. An overview of the descriptive statistics shows that there are differences between non-default and default firms. Histogram of credit quality group and credit rates distributions are in Appendix B (Figure 3, Figure 4).

Table 8. Descriptive statistics

Variable	Default mark	Mean	Std. Dev.	Min	Max
RATE	Default	14.304	4.834	0	50
	Non-default	12.731	3.665	0	68.44

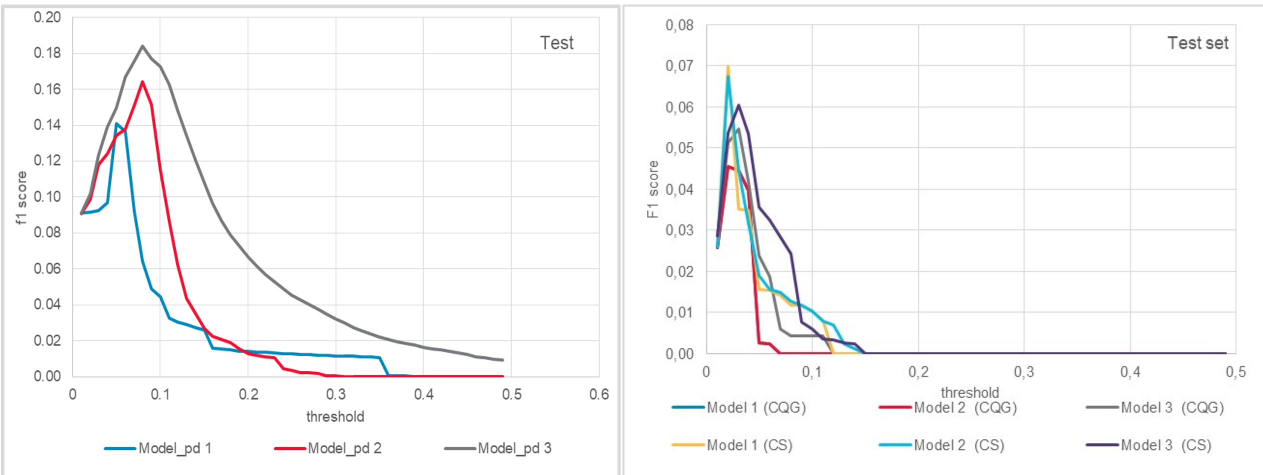
[4] For example, Dell'Ariccia et al. (2017) use "confidential data on individual U.S. banks' loan ratings from the Federal Reserve's Survey of Terms of Business Lending (STBL) and found support for risk-taking ("small, but economically meaningful. Another example of applying internal ratings is Ioannidou and Penas (2010). The latest example known to us is the paper by Miteski et al. (2018) that uses internal ratings and credit registry data on Macedonia.

[5] For example, Paligorova and Santos (2017) study how monetary policy affects the loan spread to LIBOR charged to borrowers with and without investment grade and find support to the hypothesis of risk-taking. Another example of applying credit spread in interest rate is Delis et al. (2017). They construct a comprehensive U.S. credit-registry based dataset and use loan interest rate spread over LIBOR as ex ante measures of credit risk. The authors find evidence supporting the presence of the risk-taking channel, especially before the global financial crisis.

We compare the performance of the alternative measures of ex ante credit risk in terms of their forecast of ex-post credit discipline. We run probit regression (in line with formula (1)) with the credit quality groups or credit spreads in interest rates as independent variables. In case of regression with credit spreads, we also control for the loan characteristics (maturity of loans issued). We make the preliminary assumption that the credit quality group reflects the creditworthiness of the borrower, and that the credit spread in interest rate reflects the borrower's inherent risk. In line with PD model construction, we define three specifications for alternative measures of ex ante credit risk: Model 1(CQG) and Model 1 (CS) contains only independent variables (quality group 1~5 or the credit spread respectively), Model 2 (CQG) and Model 2 (CS) are innovated with the control variables for industry and leverage groups, Model 3 (CQG) and Model 3 (CS) additionally contain interaction terms. We use 80x20 random division to construct train and test subsets. Our dataset is not balanced in terms of the number of observations in default and non-default groups. In this case it is more propriate to use F1 score to compare the performance of all alternative ex ante credit risk measures (Shibitov and Mamedli, 2019). F1 score takes into account the information concerning the predictions of default and non-default groups of companies. Figure 4 contains the results. PD model produced the highest F1 score compared with the alternative measures of ex ante credit risk.

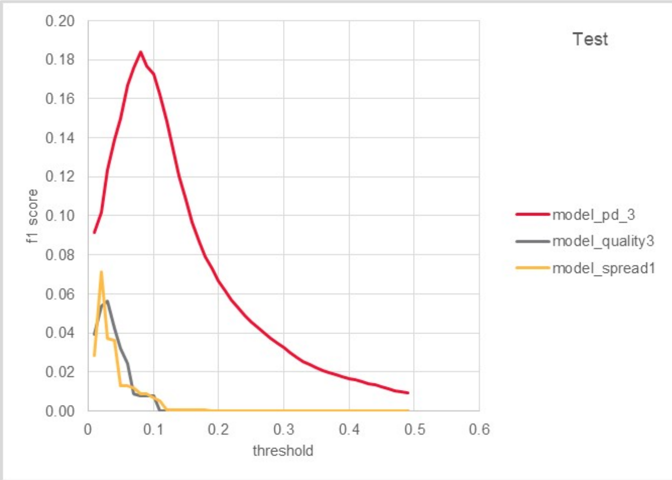
Figure 3. F1 score evolution for different threshold levels and model specifications

A) PD model **B) Credit Quality Group, Credit Spread**



Source: Bank of Russia, author's calculations.

Figure 4. F1 score evolution for different threshold levels and alternative measures of ex ante credit risk



Source: Bank of Russia, author's calculations.

Regression output tests

We perform the following tests for Model 3 (model specification contains the industry and leverage level dummy and interaction terms). Overall goodness-of-fit testing (we reject the models with low or negative adjusted R-squared values, we also run Pearson test for goodness-of-fit[6]). Individual regressors significance and economic adequateness (we need a variable coefficient to be statistically significant (i.e. not equal to zero) at least at the 10% confidence value and have an expected sign). Model residuals' normality (we run Shapiro-Wilk test to test model residuals' distribution). Model specification tests (we run link test[7] and Ramsey tests to check whether linear model is sufficient against non-linear (logarithmic) one and whether there are omitted (squared) variables). Table 9 summarises test results:

Table 9. Regression output tests

GOODNESS-OF-FIT		
r2 pseudo		
Pearson test	stat	17 771.49
RESIDUALS NORMALITY		
Shapiro-Wilk	stat p-value	1.8E-234 32.67%
MODEL SPECIFICATION TESTS		
Ramsey	stat p-value	2876.56 0%
Link test	stat p-value	480.52

[6] The sum of differences between observed and expected outcome frequencies; each squared and divided by the expectation. The resulting value of statistics can be compared with chi-squared distribution.

[7] The link test adds the squared independent variable to the model and tests for significance versus the non-squared model. A model without a link error will have a nonsignificant t-test versus the unsquared version.

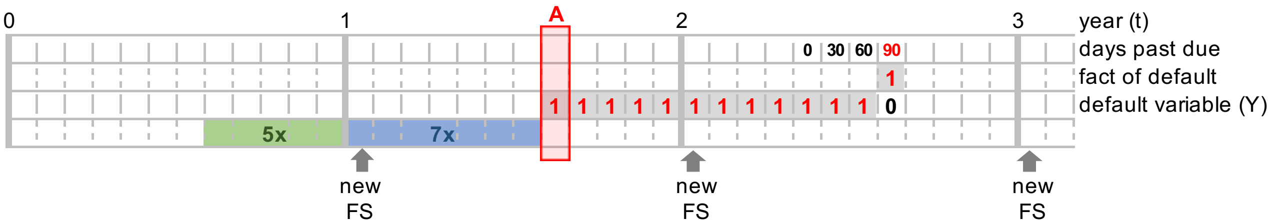
APPENDIX A. Identification

Calculation of monthly firm-level financial data

Let assume that we have 90 days past due in year 3. According to the default event identification we extended and made backward 12-month shift. Then in A we have $Y(A) = 1$, that means default event has occurred after 7th month year 2 (future event). In order to get monthly firm-level financial data for PD analysis we construct financial statement as assigned a weight proportionally to the number of months spent in corresponding financial year:

$$X(A) = \frac{5}{12} \cdot FS_1 + \frac{7}{12} \cdot FS_2$$

Here financial statement FS_2 is the past information for default. Under this construction we do not use future information when we estimate probability of default.



Probability of default adjustment for more accurate portfolio estimates

Basic financial statements are issued annually (quarterly financial statements are not prepared uniformly). Assuming financial statements becomes publicly available as of the year-end, means that in November of current year the most recent financial statements are 11 months old already. Nevertheless, the information on delinquent loans is available from the credit registry with monthly frequency. This allows us to introduce an equivalent to Bayesian adjustment to our monthly PD model estimate. Conceptually our goal is to reflect two stylised facts. First, we expect that in case the point-in-time (PIT; short-term) default rate (overall or industry-wise etc.) significantly deviates from its long-run average (through-the-cycle, TTC), the PD estimate should be proportionately adjusted. For example, if the recent default rate rose, the PD prediction should be higher than it was all else being equal. Second, the utility of the financial statements declines with the time going on. This means that the newly disclosed financial statements may need no adjustment during the month following the month of its disclosure (i.e. in January). On the contrary, at the end of the year (i.e. in November or December) the financials-based PD estimate should be adjusted much more compared with the PD estimate in January. For this reason, we hypothesise that the following rule for PD adjustment delivers more accurate portfolio estimates for the share of delinquent loans:

1) $PD^M_{adj.} = PD^M * \left[1 + \left(\frac{M-1}{11}\right) * \left(\frac{DR^{M-1}_{PIT}}{DR_{TTC}} - 1\right)\right]$

where M is the month counter ranging from 1 in January to 12 in December;

PD^M is the probability of default estimated with the PD model for loans issued in month M, in p.p.;

$PD_{adj.}^{M}$ is the *adjusted* probability of default estimated for loans issued in month M, in p.p.;

DR_{PIT}^{M-1} – is the point-in-time (short-term, monthly) default rate (in p.p.) from the last available month, i.e. $(M-1)$ as we do not know defaults for the current month M, in p.p.;

DR_{TTC} – is the through-the-cycle (long-term; e.g. 12 months at least) default rate that comes from the PD model development dataset, in p.p.

The idea of the formula above is that in January we do not adjust the model PD estimate, i.e. $[(M-1)\,/\,(11)] = [(1-1)\,/\,(11)] = 0$. We expect the financial statements are recent ones and fully reflect financial status of the borrower. We may still have deviance in PIT DR estimates, but those can be obtained when recent financials are put to PD model. On the contrary, in December $(M = 12)$ the last available financial statements are at least 11 months old. That is why we fully proportionately[8] adjust PD model estimates to the ratio of PIT and TTC default rates, i.e. $[(M-1)/\,(11)] = [(12-1)\,/\,(11)] = 1$.

Figure 1. Default event identification

	Days overdue	Mark of default event (to identify the month of default event)	***Extended*** mark of default event (to cover the year of default)	***12m backward shift*** of default event (to match with)
2012m01	0	0	0	0
2012m02	**0**	**0**	**0**	**1**
2012m03	0	0	0	1
2012m04	0	0	0	1
2012m05	0	0	0	1
2012m06	0	0	0	1
2012m07	0	0	0	1
2012m08	0	0	0	1
2012m09	0	0	0	1
2012m10	0	0	0	1
2012m11	0	0	0	1
2012m12	30	0	0	1
2013m01	60	0	0	1
2013m02	**90** →	**1** →	**1**	**0**
2013m03	0	0	1	0
2013m04	0	0	1	0
2013m05	0	0	1	0
2013m06	0	0	1	0
2013m07	0	0	1	0
2013m08	0	0	1	0
2013m09	0	0	1	0
2013m10	0	0	1	0
2013m11	0	0	1	0
2013m12	0	0	1	0
2014m01	0	0	1	0
2014m02	**0**	**0**	**0**	
2014m03	0	0	0	
2014m04	0	0	0	
2014m05	0	0	0	
2014m06	0	0	0	
2014m07	0	0	0	
2014m08	0	0	0	
2014m09	0	0	0	
2014m10	0	0	0	
2014m11	0	0	0	
2014m12	0	0	0	
2015m01	0	0	0	

12 months
without default event

[8] Alternatively, the weight parameters should be introduced as the ratio $[(M-1)\,/\,(11)]$ may not be the most optimal. Then maximum likelihood may be needed to calibrate them using historical data.

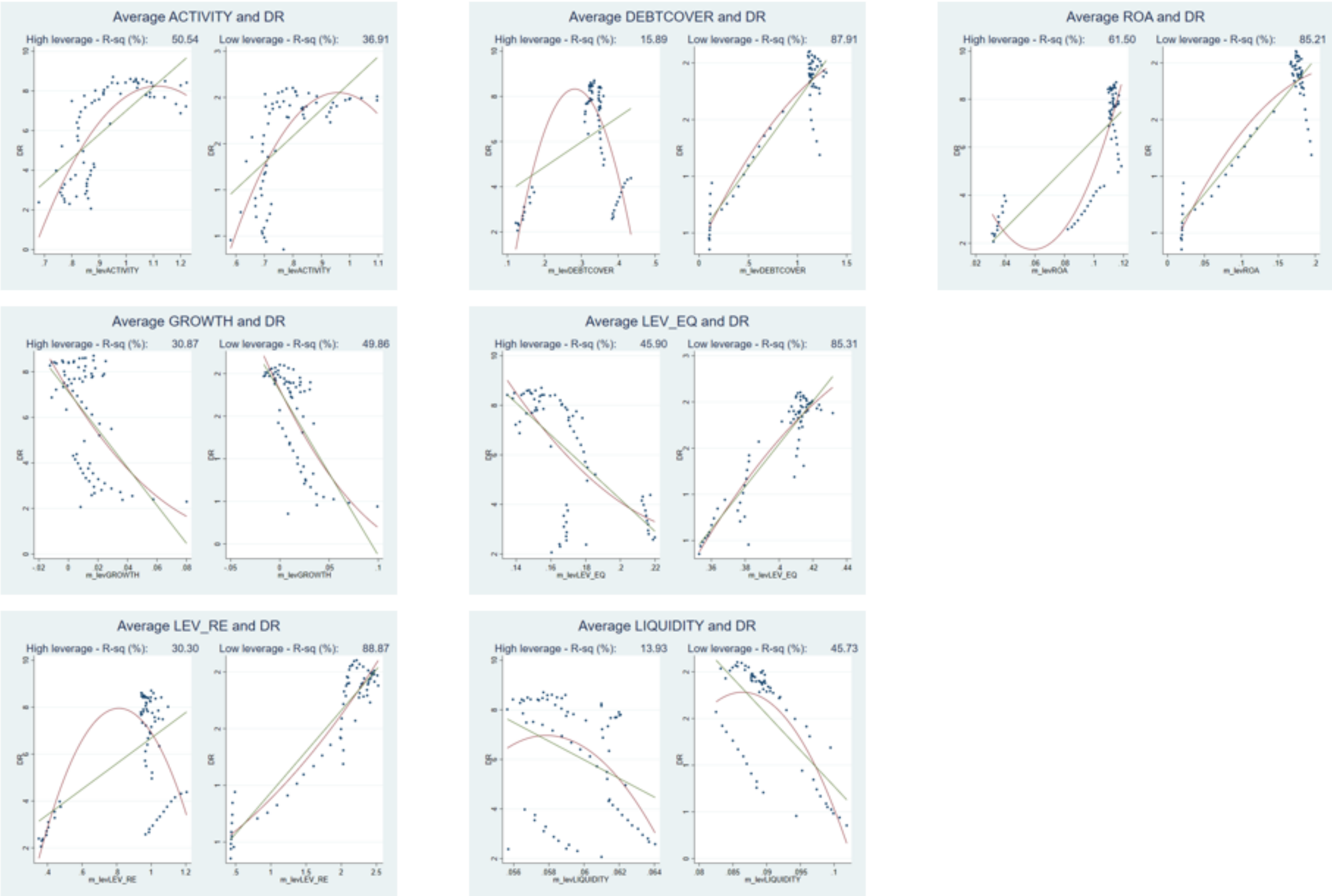
Table 1. Industry identification

	OKVED2 classification		Aggregate industry
A	01	Crop and animal production, hunting and related service activities	Forestry & Agriculture
	02	Forestry and logging	
	03	Fishing and aquaculture	
B	05	Mining of coal and lignite	Mining
	06	Extraction of crude petroleum and natural gas	
	07	Mining of metal ores	
	08	Other mining and quarrying	
	09	Mining support service activities	
C	10	Manufacture of food products	Manufacturing
	11	Manufacture of beverages	
	12	Manufacture of tobacco products	
	13	Manufacture of textiles	
	14	Manufacture of wearing apparel	
	15	Manufacture of leather and related products	
	16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials	
	17	Manufacture of paper and paper products	
	18	Printing and reproduction of recorded media	
	19	Manufacture of coke and refined petroleum products	
	20	Manufacture of chemicals and chemical products	
	21	Manufacture of basic pharmaceutical products and pharmaceutical preparations	
	22	Manufacture of rubber and plastic products	
	23	Manufacture of other non-metallic mineral products	
	24	Manufacture of basic metals	
	25	Manufacture of fabricated metal products, except machinery and equipment	
	26	Manufacture of computer, electronic and optical products	
	27	Manufacture of electrical equipment	
	28	Manufacture of machinery and equipment n.e.c	
	29	Manufacture of motor vehicles, trailers and semi-trailers	
	30	Manufacture of other transport equipment	
	31	Manufacture of furniture	
	32	Other manufacturing	
	33	Repair and installation of machinery and equipment	
D	35	Electricity, gas, steam and air conditioning supply	Utilities
E	36	Water collection, treatment and supply	
	37	Sewerage	
	38	Waste collection, treatment and disposal activities; materials recovery	
	39	Remediation activities and other waste management services	
F	41	Construction of buildings	Construction
	42	Civil engineering	
	43	Specialised construction activities	
G	45	Wholesale and retail trade and repair of motor vehicles and motorcycles	Wholesale & Retail
	46	Wholesale trade, except of motor vehicles and motorcycles	
	47	Retail trade, except of motor vehicles and motorcycles	

H	49	Land transport and transport via pipelines	Transportation
	50	Water transport	
	51	Air transport	
	52	Warehousing and support activities for transportation	
	53	Postal and courier activities	
I	55	Accommodation	Hotels & Restaurants
	56	Food and beverage service activities	
J	58	Publishing activities	Other sectors
	59	Motion picture, video and television programme production, sound recording and music publishing activities	
	60	Programming and broadcasting activities	
	61	Telecommunications	
	62	Computer programming, consultancy and related activities	
	63	Information service activities	
K	64	Financial service activities, except insurance and pension funding	
	65	Insurance, reinsurance and pension funding, except compulsory social security	
	66	Activities auxiliary to financial services and insurance activities	
L	68	Real estate activities	
M	69	Legal and accounting activities	
	70	Activities of head offices; management consultancy activities	
	71	Architectural and engineering activities; technical testing and analysis	
	72	Scientific research and development	
	73	Advertising and market research	
	74	Other professional, scientific and technical activities	
	75	Veterinary activities	
N	77	Rental and leasing activities	
	78	Employment activities	
	79	Travel agency, tour operator reservation service and related activities	
	80	Security and investigation activities	
	81	Services to buildings and landscape activities	
	82	Office administrative, office support and business support activities	
O	84	Public administration and defence; compulsory social security	
P	85	Education	
Q	86	Human health activities	
	87	Residential care activities	
	88	Social work activities without accommodation	
R	90	Creative, arts and entertainment activities	
	91	Libraries, archives, museums and other cultural activities	
	92	Gambling and betting activities	
	93	Sports activities and amusement and recreation activities	
S	94	Activities of membership organisations	
	95	Repair of computers and personal and household goods	
	96	Other personal service activities	
T		Activities of households as employers; Undifferentiated goods-and services-producing activities of private households for own use	

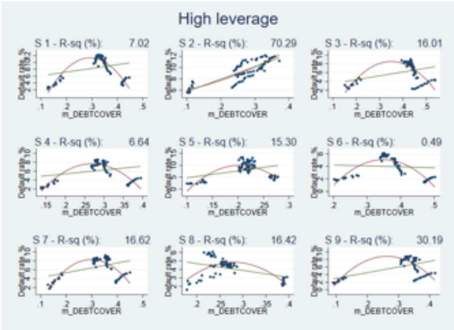
APPENDIX B. Detalisation

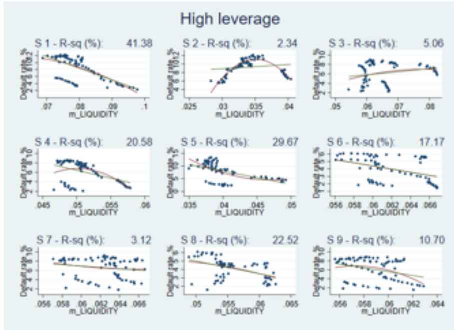
Table 1. Relationship between the average ratio and monthly default rate at High- and Low- leverage group

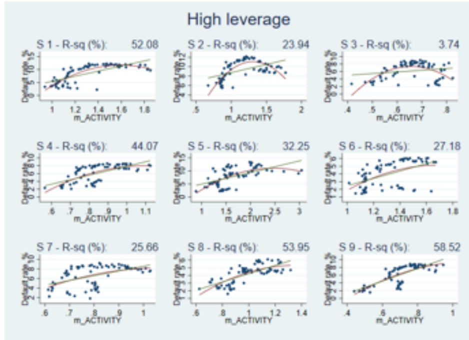


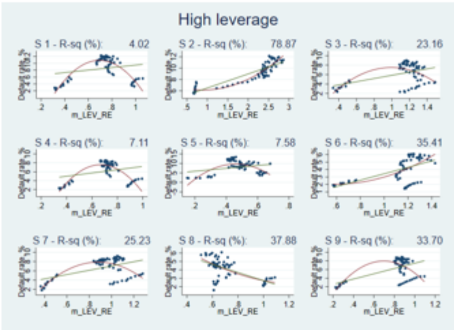
Sector 1 – Construction, Sector 2 – Forestry and Agriculture, Sector 3 – Hotels and Restaurants, Sector 4 – Manufacturing, Sector 5 – Mining, Sector 6 – Other services, Sector 7 – Transportation , Sector 8 – Utilities , Sector 9 – Wholesale and Retail Trade. ***Source: Bank of Russia, author's calculations.***

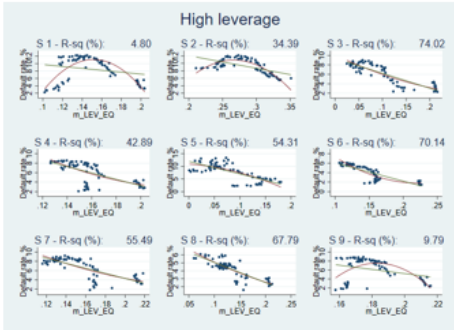
Table 2A. Relationship between the average ratio and monthly default rate at industry level for high-leverage group

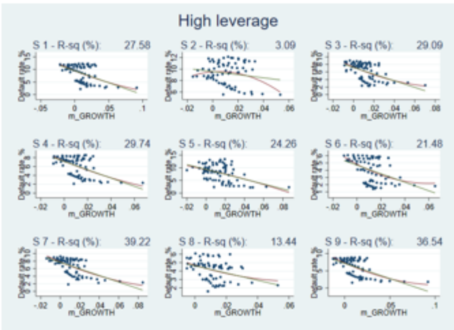


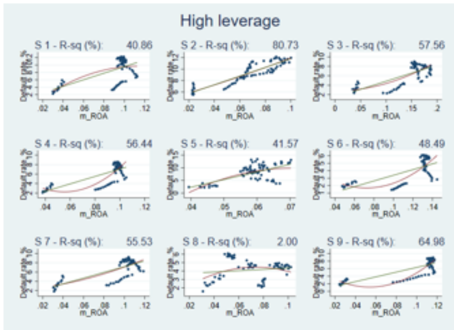












Sector 1 – Construction, Sector 2 – Forestry and Agriculture, Sector 3 – Hotels and Restaurants, Sector 4 – Manufacturing, Sector 5 – Mining, Sector 6 – Other services, Sector 7 – Transportation , Sector 8 – Utilities , Sector 9 – Wholesale and Retail Trade. ***Source: Bank of Russia, author's calculations.***

Table 2B. Relationship between the average ratio and monthly default rate at industry level for low-leverage group

Sector 1 – Construction, Sector 2 – Forestry and Agriculture, Sector 3 – Hotels and Restaurants, Sector 4 – Manufacturing, Sector 5 – Mining, Sector 6 – Other services, Sector 7 – Transportation , Sector 8 – Utilities , Sector 9 – Wholesale and Retail Trade. ***Source: Bank of Russia, author's calculations.***

Figure 3. Quality group distribution

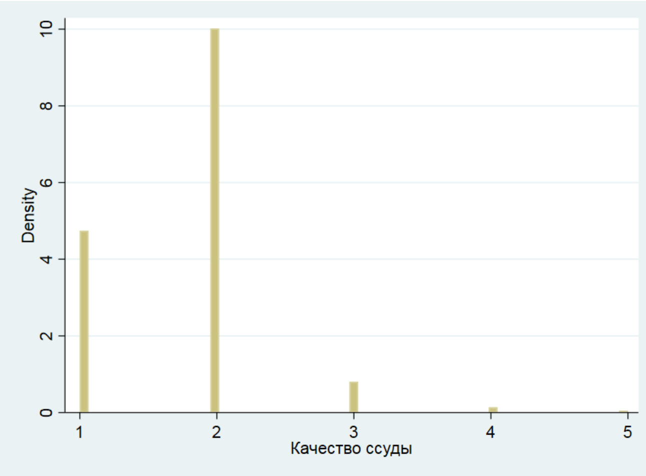
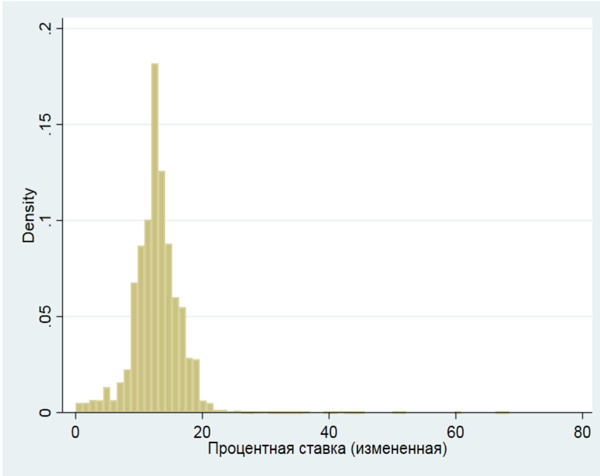


Figure 4. Credit rates distribution



Source: Bank of Russia, author's calculations.

APPENDIX C. Regression results

Table 1. Regression results

VARIABLES	Model 1	Interaction terms identification	
ACTIVITY	**-0.000**	Industry x Leverage	Yes
	(0.000)	Industry x ACTIVITY	Yes
DEBTCOVER	**-0.000***	Industry x DEBTCOVER	Yes
	(0.000)	Industry x GROWTH	Yes
GROWTH	**-0.934*****	Industry x LEV_EQ	Yes
	(0.012)	Industry x LEV_RE	Yes
LEV_EQ	**0.000**	Industry x LIQUIDITY	Yes
	(0.000)	Industry x ROA	Yes
LEV_RE	**-0.000*****	Leverage x ACTIVITY	Yes
	(0.000)	Leverage x DEBTCOVER	Yes
LIQUIDITY	**.**	Leverage x GROWTH	Yes
		Leverage x LEV_EQ	Yes
ROA	**.**	Leverage x LEV_RE	Yes
		Leverage x LIQUIDITY	Yes
Industry2	**0.104*****	Leverage x ROA	Yes
	(0.008)	Industry x Leverage x ACTIVITY	Yes
Industry3	**0.122*****	Industry x Leverage x DEBTCOVER	Yes
	(0.011)	Industry x Leverage x GROWTH	Yes
Industry4	**-0.118*****	Industry x Leverage x LEV_EQ	Yes
	(0.007)	Industry x Leverage x LEV_RE	Yes
Industry5	**-0.031**	Industry x Leverage x LIQUIDITY	Yes
	(0.025)	Industry x Leverage x ROA	Yes
Indusrty6	**-0.232*****		
	(0.006)		
Indusrty7	**-0.029*****		
	(0.008)		
Indusrty8	**-0.113*****		
	(0.016)		
Indusrty9	**-0.137*****		
	(0.005)		
Leverage dummy	**0.627*****		
	(0.006)		
Intercept	-1.907***		
	(0.004)		
Observations	10 826 964		
Pseudo R-Squared	0.067		
Area under ROC curve (training)	0.7248		
Area under ROC curve (test)	0.7244		

*Standard errors in parentheses * p<0.10, ** p<0.05, *** p<0.01*

Table 1 (cont). Regression results: double interaction terms

VARIABLES	Industry 2	Industry 3	Industry 4	Industry 5	Industry 6	Industry 7	Industry 8	Industry 9	Leverage
Leverage	-0.003	-0.348***	-0.098***	-0.075***	-0.173***	-0.139***	-0.217***	-0.099***	
	(0.010)	*(0.014)*	*(0.009)*	*(0.030)*	*(0.008)*	*(0.011)*	*(0.024)*	*(0.007)*	
ACTIVITY	0.000***	-0.000	0.000	-0.000	-0.000	0.000***	0.000**	0.000	0.006***
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*
DEBTCOVER	0.000***	0.000***	0.000	-0.000	0.000***	0.000***	0.005***	0.000***	0.039***
	(0.000)	*(0.000)*	*(0.000)*	*(0.008)*	*(0.000)*	*(0.000)*	*(0.001)*	*(0.000)*	*(0.008)*
GROWTH	0.544***	0.133***	-0.355***	0.082	0.068***	-0.436***	0.934***	0.834***	-1.581***
	(0.023)	*(0.061)*	*(0.027)*	*(0.150)*	*(0.029)*	*(0.044)*	*(0.121)*	*(0.018)*	*(0.042)*
LEV_EQ	-0.000***	-0.000	0.000	0.000	.	-0.000***	-0.001	-0.000***	-0.200***
	(0.000)	*(0.000)*	*(0.000)*	*(0.003)*	.	*(0.000)*	*(0.001)*	*(0.000)*	*(0.006)*
LEV_RE	0.000***	0.000***	0.000	-0.001***	0.000***	0.000***	-0.001***	0.000***	0.004***
	(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.001)*
LIQUIDITY	-0.943***	0.067***	-0.523***	-1.323***	.	-0.021*	-3.250***	-0.056***	-1.724***
	(0.066)	*(0.034)*	*(0.038)*	*(0.350)*	.	*(0.031)*	*(0.256)*	*(0.008)*	*(0.039)*
ROA	0.001***	0.000***	-0.000***	-0.754***	.	0.000***	-0.065***	0.000	-0.314***
	(0.001)	*(0.000)*	*(0.000)*	*(0.002)*	.	*(0.000)*	*(0.005)*	*(0.000)*	*(0.023)*

*Standard errors in parentheses * p<0.10, ** p<0.05, *** p<0.01*

Table 1 (cont). Regression results: triple interaction terms

VARIABLES	Industry 2 x Leverage	Industry 3 x Leverage	Industry 4 x Leverage	Industry 5 x Leverage	Industry 6 x Leverage	Industry 7 x Leverage	Industry 8 x Leverage	Industry 9 x Leverage
ACTIVITY	0.005***	0.004***	0.006***	0.002***	-0.000	0.004***	0.003**	0.004***
	(0.016)	*(0.015)*	*(0.011)*	*(0.057)*	*(0.010)*	*(0.013)*	*(0.039)*	*(0.009)*
DEBTCOVER	-0.041***	-0.042***	0.129***	0.408***	-0.096**	0.010	0.200***	0.123***
	(0.076)	*(0.120)*	*(0.064)*	*(0.240)*	*(0.062)*	*(0.086)*	*(0.238)*	*(0.049)*
GROWTH	-1.048***	-0.637***	-0.343***	-0.436***	0.286***	0.116*	-1.337***	-1.671***
	(0.009)	*(0.010)*	*(0.008)*	*(0.022)*	*(0.007)*	*(0.009)*	*(0.018)*	*(0.007)*
LEV_EQ	0.014*	0.133***	-0.003	-0.040**	0.070***	0.037***	-0.036***	-0.053***
	(0.002)	*(0.002)*	*(0.002)*	*(0.014)*	*(0.002)*	*(0.002)*	*(0.010)*	*(0.001)*
LEV_RE	-1.017***	-0.005***	-0.020***	-0.036***	-0.013***	-0.009***	-0.056***	0.001
	(0.119)	*(0.087)*	*(0.074)*	*(0.434)*	*(0.054)*	*(0.079)*	*(0.393)*	*(0.047)*
LIQUIDITY	0.312***	0.648***	0.567***	1.547***	0.412***	0.257***	1.875***	0.282***
	(0.046)	*(0.034)*	*(0.033)*	*(0.111)*	*(0.028)*	*(0.036)*	*(0.085)*	*(0.027)*
ROA	-0.285***	0.242***	-0.177***	0.167*	0.114***	0.163***	-0.055	-0.055**
	(0.001)	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*	*(0.002)*	*(0.001)*

*Standard errors in parentheses * p<0.10, ** p<0.05, *** p<0.01*

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